Duality for Non-monotonic Consequence Relations and Antimatroids

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We present an extension of the Stone duality that relates non-monotonic consequence relations over Boolean algebras, satisfying the axioms of System P, to antimatroids over Stone spaces. This result gives new insights into existing completeness theorems for System P.

Non-monotonic consequence relations are used in artificial intelligence as a framework for defeasible reasoning [5]. They also correspond to the non-nested fragment of a conditional logic developed in philosophy and linguistics [6, 8]. However, the restriction to non-nested formulas is not essential for most properties of the logic. A third application of similar formal systems is in the theory of belief revision [3, 1].

The standard semantics for System P is given on posets. The idea is that a defeasible inference from A to C holds in a poset iff the minimal elements in the poset that satisfy A also satisfy C. Completeness proofs for System P with respect to its order semantics are provided by [5] and [2, 8]. These proofs are technical and it is not clear how they can be seen as extensions of the Stone duality between Boolean algebras and Stone spaces. The reason for these difficulties is a mismatch between the order semantics and System P. It it noticed in [8] that the addition of a rather complex condition, called coherence, to System P greatly simplifies the completeness proof. This condition has also been observed in the context of belief revision where it is the postulate (K-8r) [7].

The problem with the coherence condition is that it can not be employed in most applications of System P, as it is for instance not expressible in the language of conditional logic. By increasing the expressivity of the language and assuming an analogue of coherence [9] obtains a general representation result that is based on the Stone duality.

In our approach we remove the mismatch between System P and its order semantics by weakening the semantics instead of adding the coherence condition on the syntactic side. We use antimatroids as the additional structure on the Stone spaces. Antimatroids are closure operators that satisfy an additional separation property. They generalise the order semantics because one can consider the closure operator that maps a set of elements in a poset to its upset.

In the finite case antimatroids have already been studied as a combinatorial approach to the notion of convexity [4, ch. 2]. In this setting it has been shown that every antimatroid arises as the image of the antimatroid of some poset under some suitable notion of morphism. This construction can be adapted to the infinite topological case and thus allows to proof completeness of System P with respect to its more restrictive order semantics.

References

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