Quantum computation

in the hall of mirrors

Niel de Beaudrap (Oxford)
CVQT, Edinburgh

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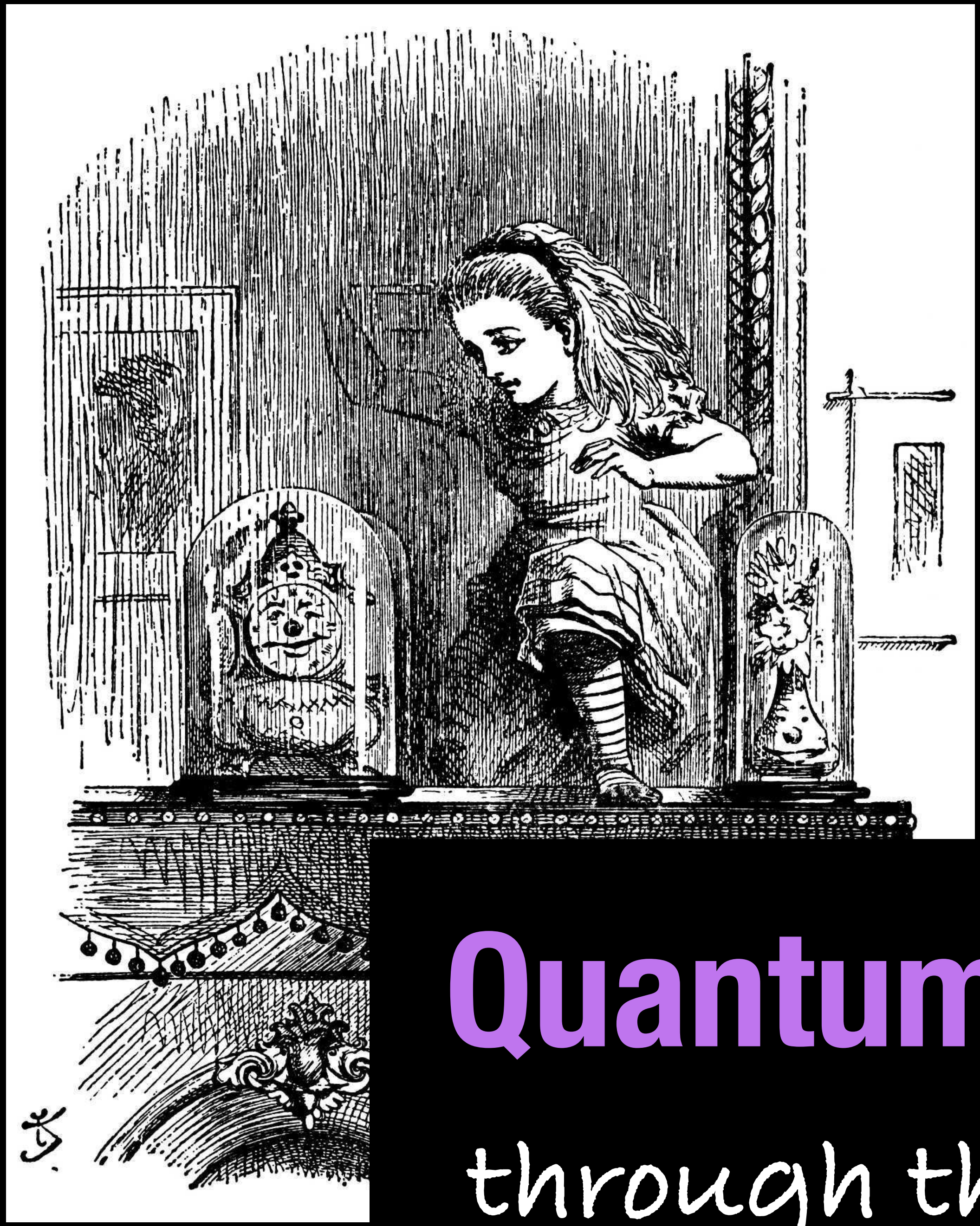

Quantum computation

through the looking-glass

Niel de Beaudrap (Oxford)

CVQT, Edinburgh

The power of quantum

- It is commonly believed that quantum computation is a powerful model
- I don't disagree — but:

why?

- Three kinds of arguments:
  - Fundamental strangeness
  - Lack of known classical simulation techniques
  - Conjectures leading to "quantum advantage"

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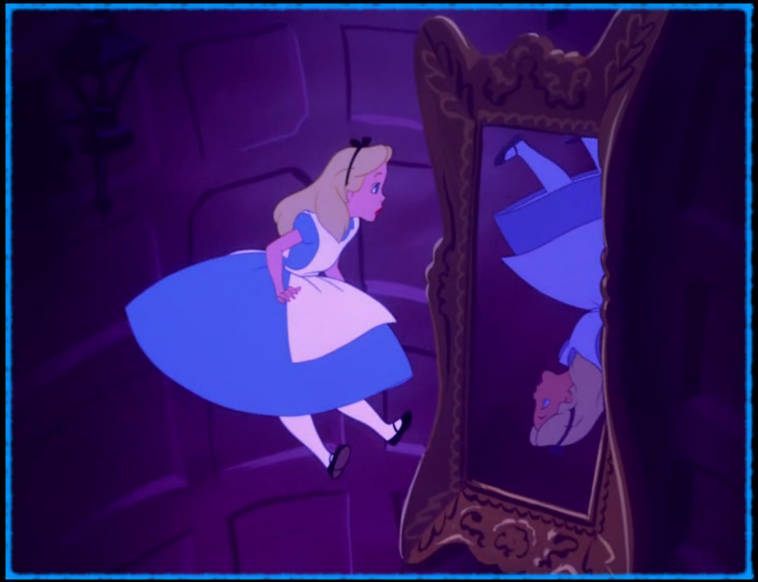

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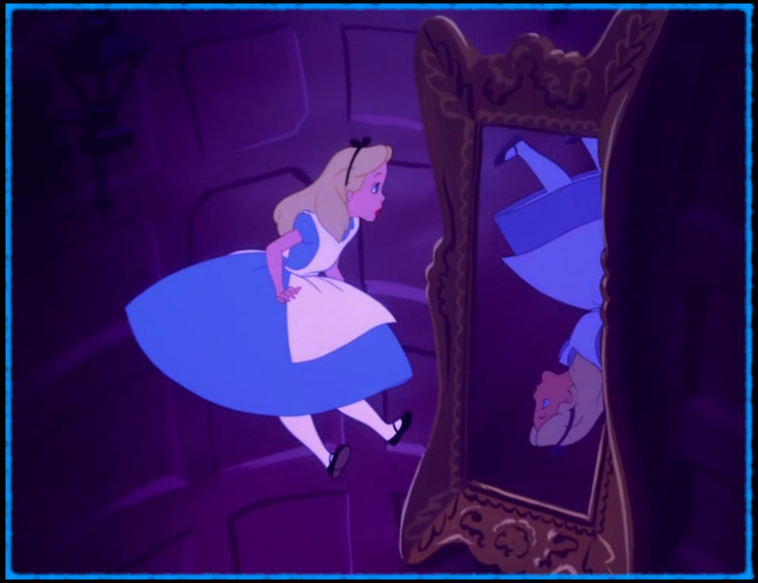

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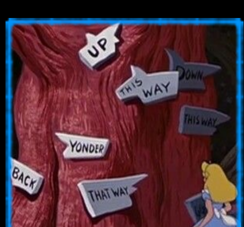

Multiple bases which can store information

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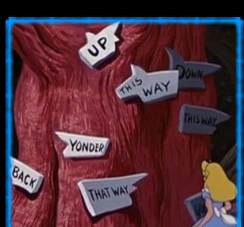

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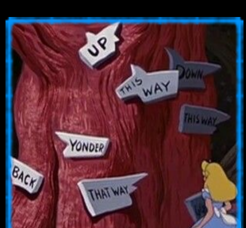

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“Destructive Interference”?

- **Interference**
  - *of wave patterns or external forces:*
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  - *by extension, motivated by quantum computation:*
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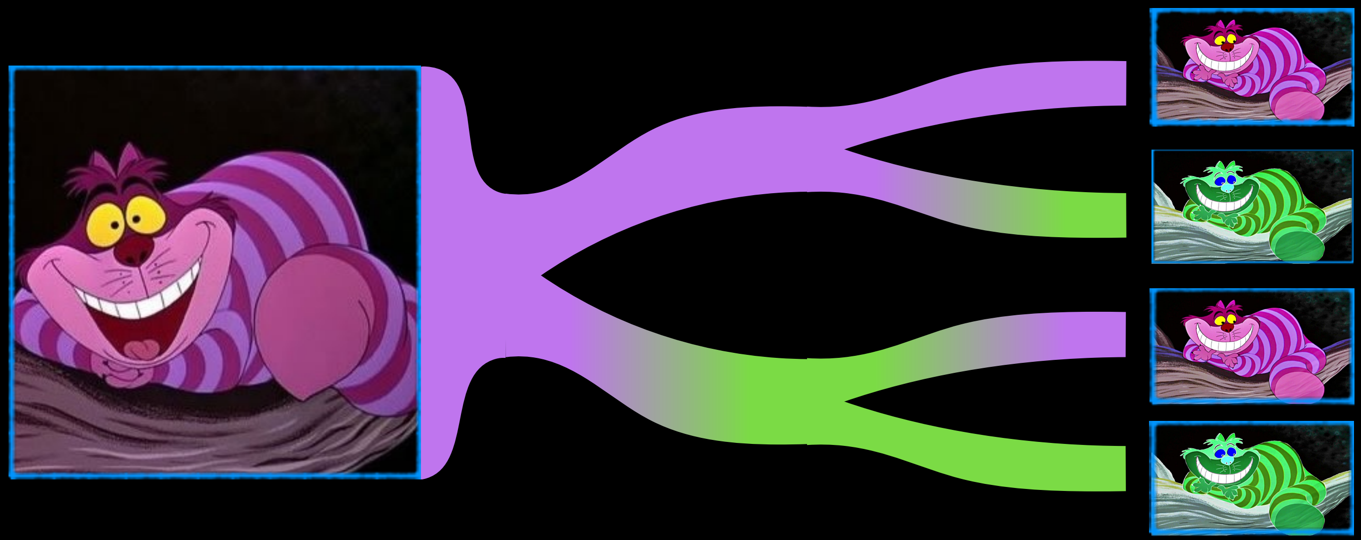

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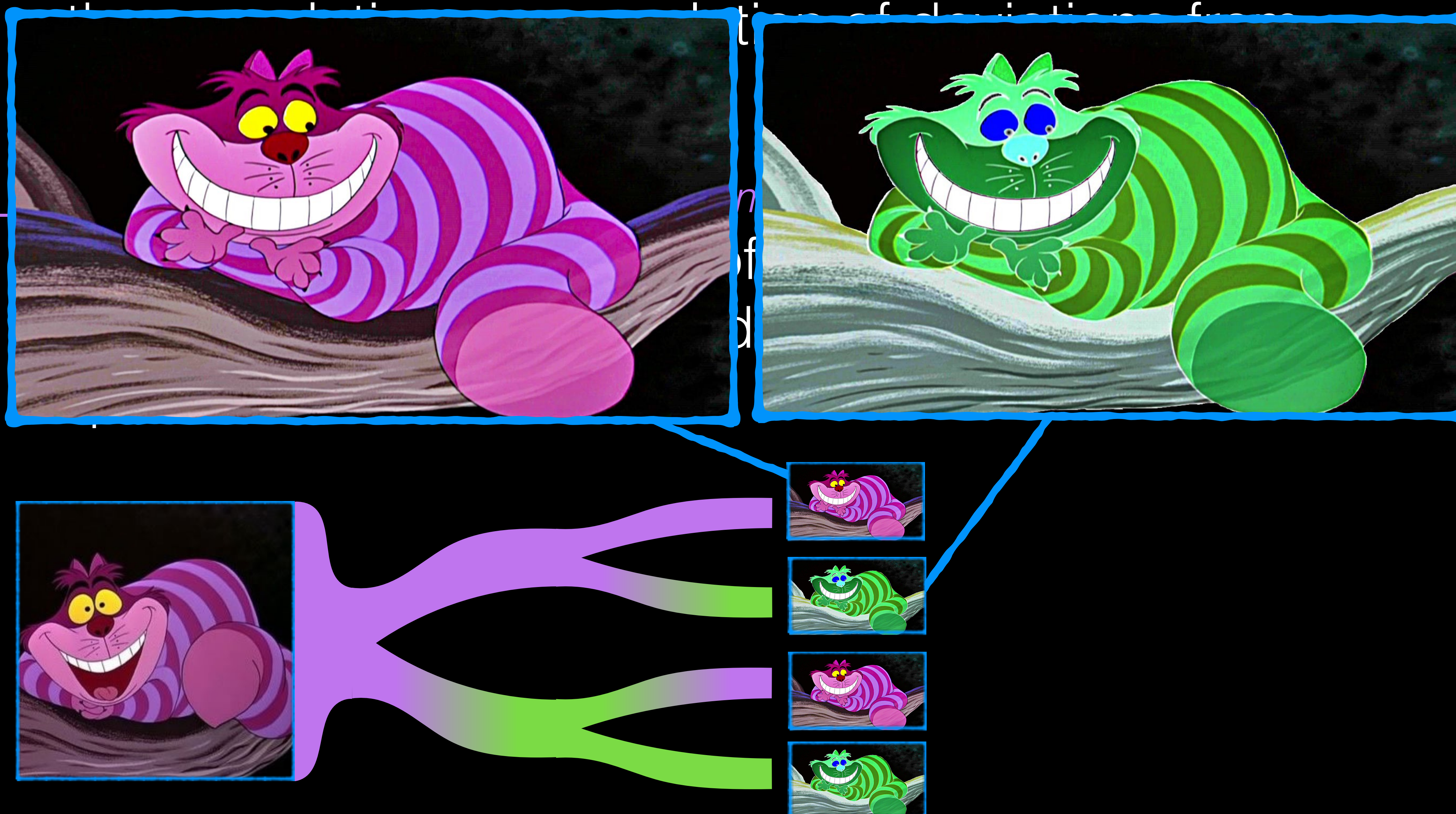

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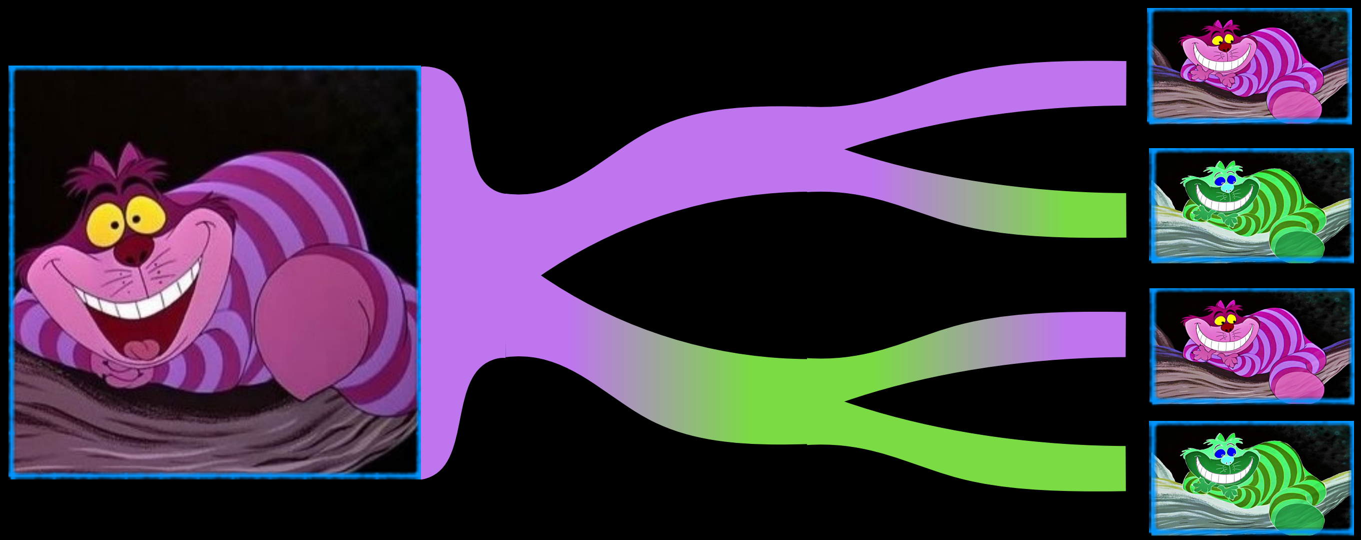

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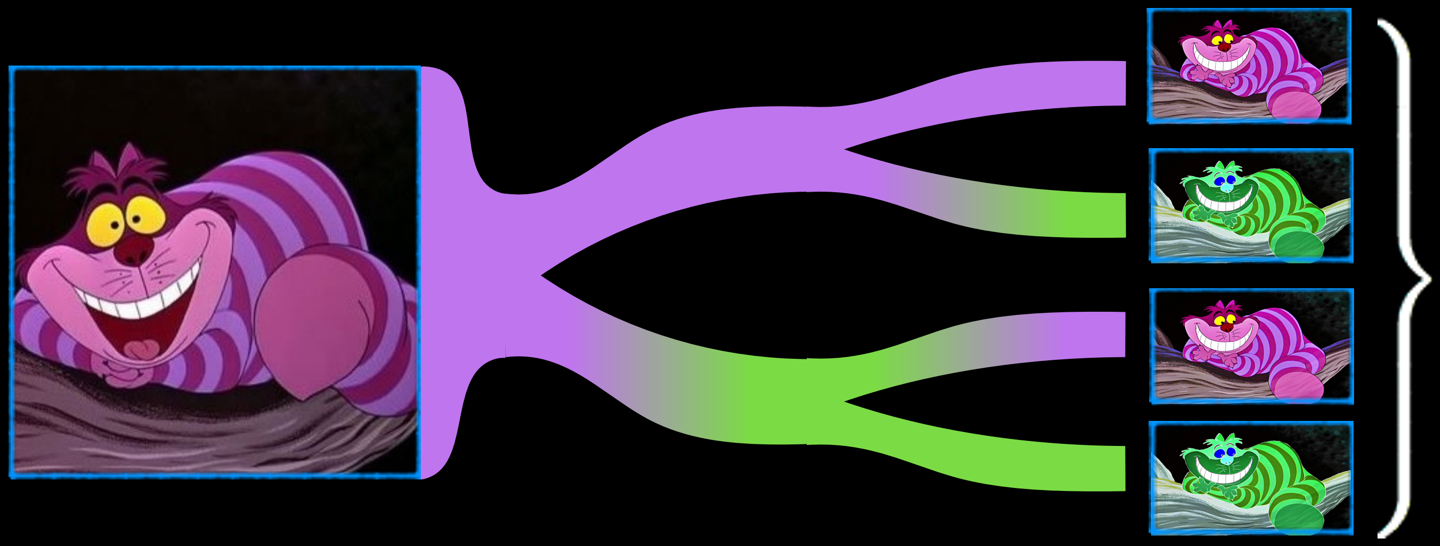

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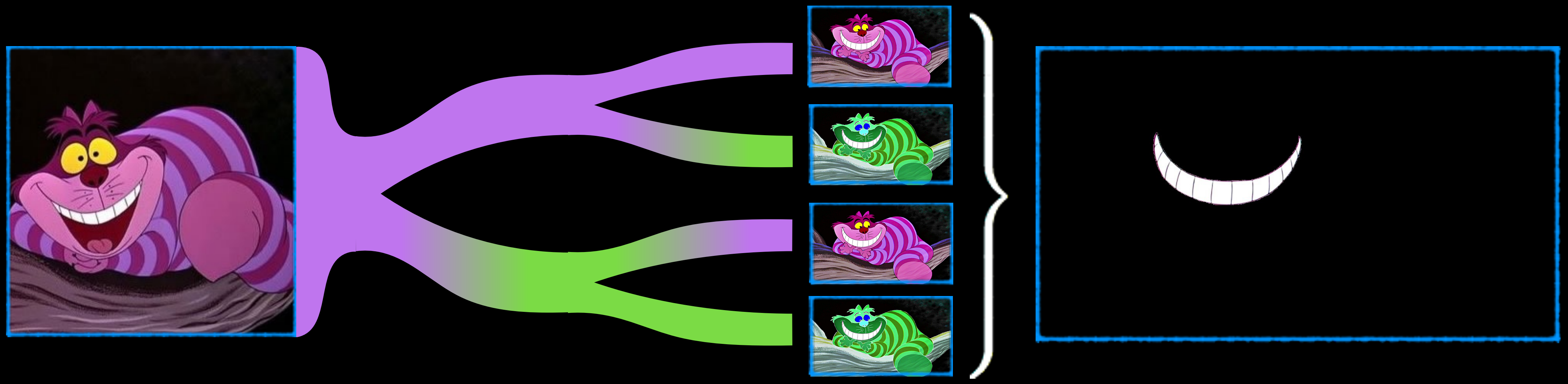

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Results

- An approach to studying **quasi-quantum** models of computation — funhouse-mirror physics
- Characterisations of some quasi-quantum models, in terms of *counting complexity*

Preliminaries

sums over paths
& counting complexity

Sums over paths

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- Evolution given by **sum of the amplitudes of each path,** associated with reaching each possible endpoint — a form of Huygen's principle for computation
- This is the principle behind all known relations between quantum computation and **counting classes**

Sums over paths *for NTMs*

- **Counting complexity** —
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AWPP — is the $|\text{total}|^2 \leqslant \frac{1}{3}$, or is it $\geqslant \frac{2}{3}$? ($w_A = +1/h(N)$, $w_R = -1/h(N)$)

Counting complexity

$P^{\#P}$

PP

AWPP

"very very hard"

"hard" NP

BPP

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"easy"

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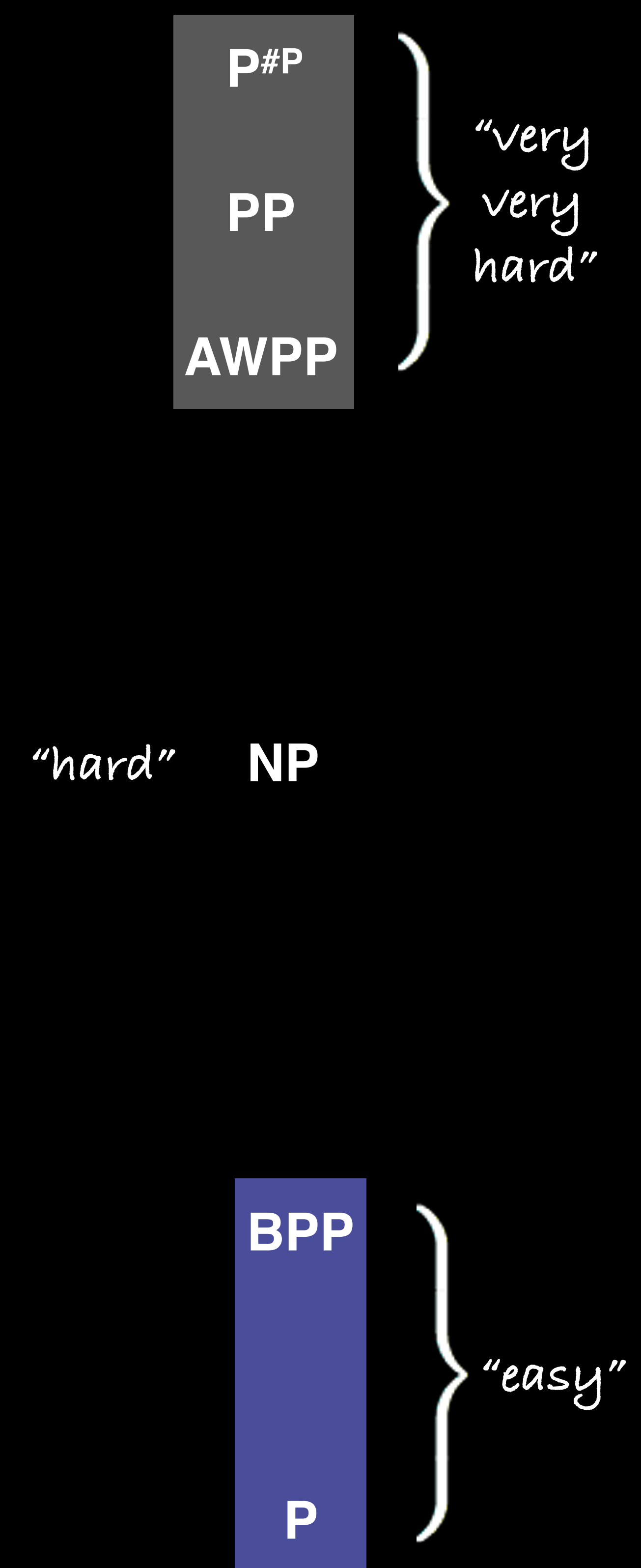

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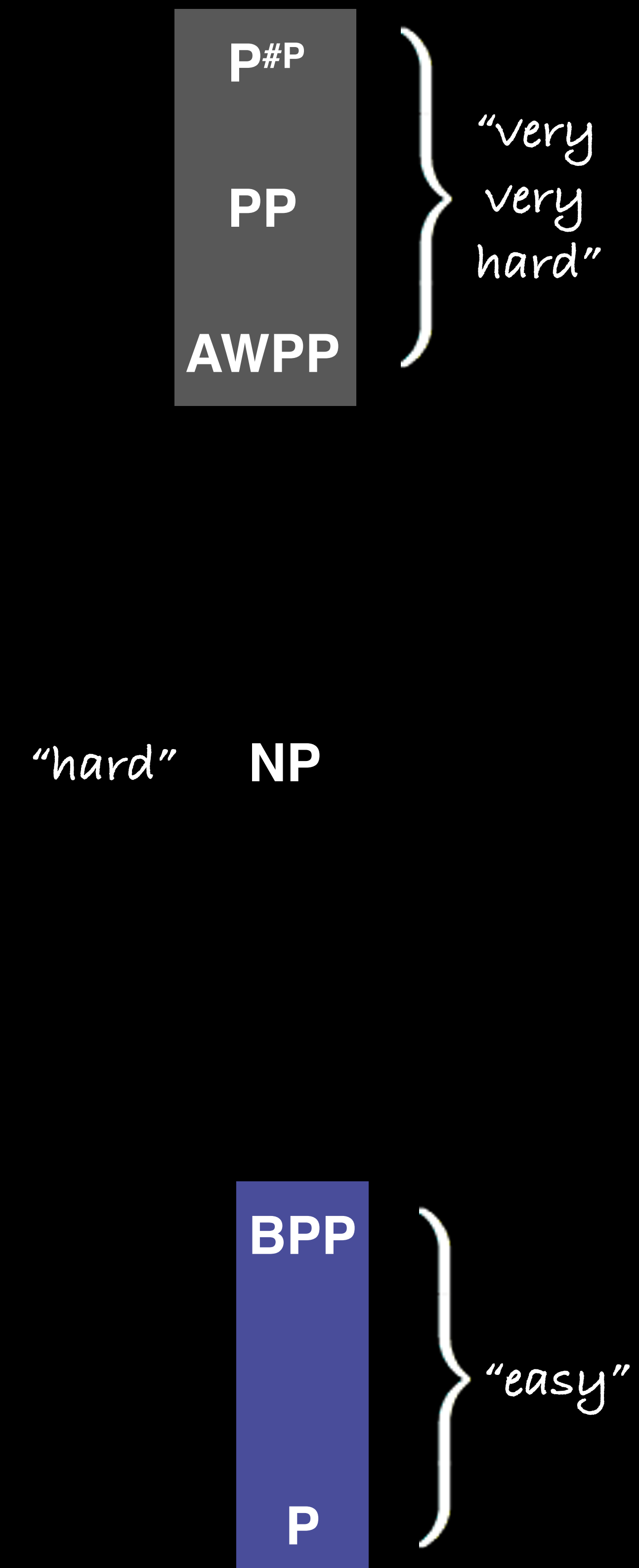

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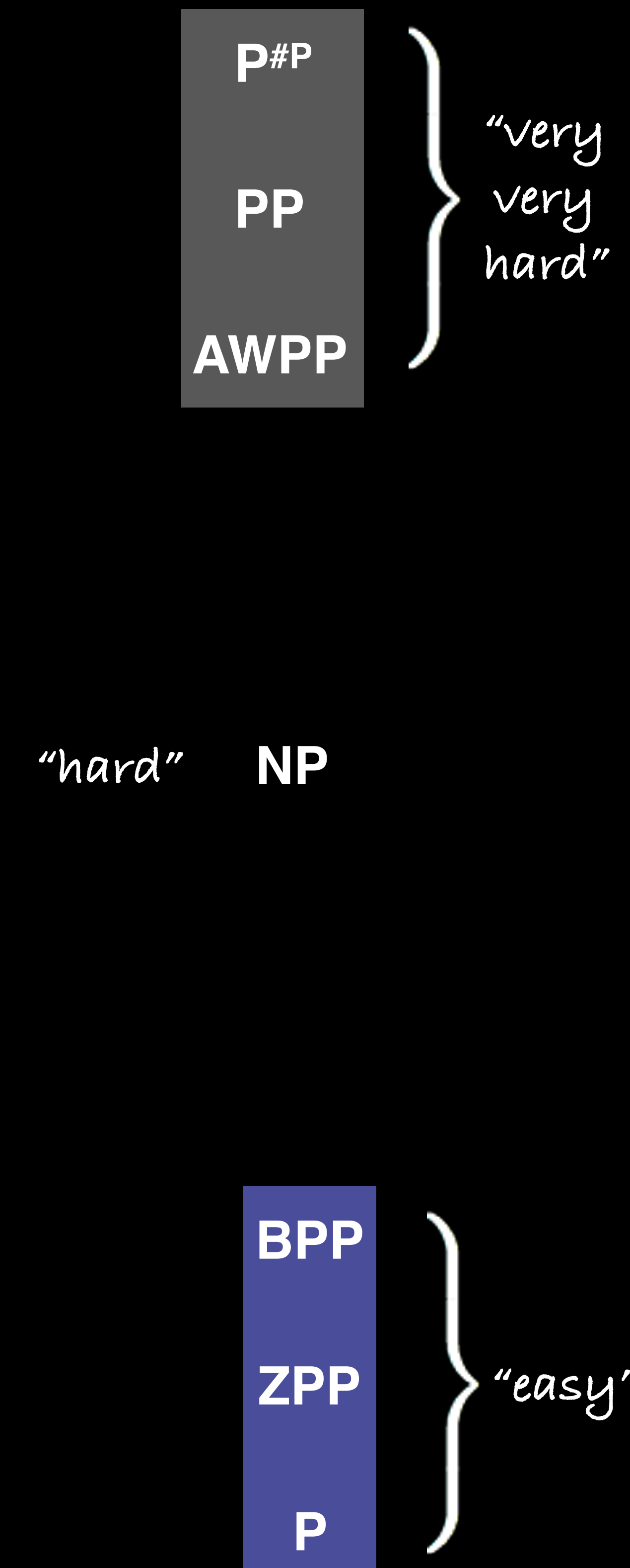

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"very very hard"

"hard" **NP**

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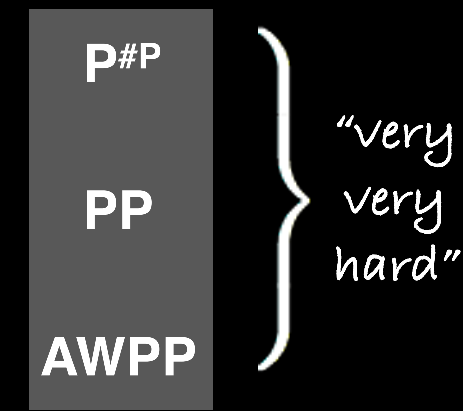

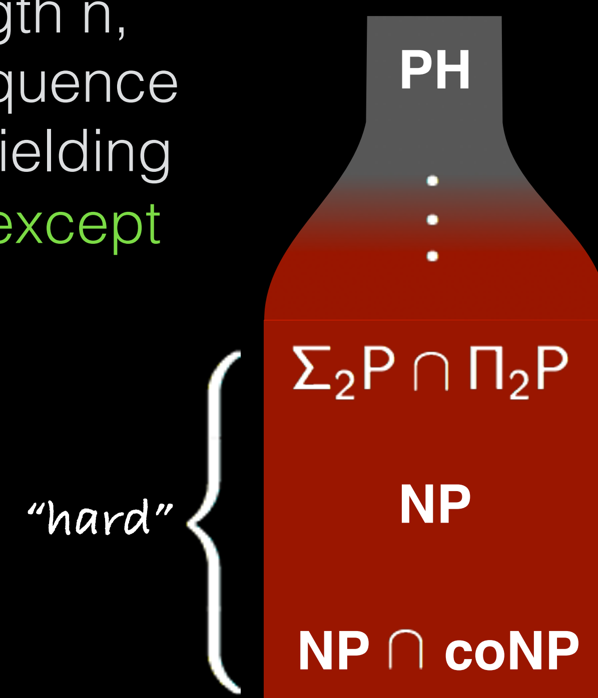

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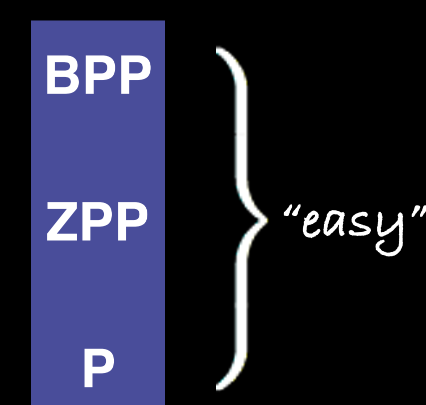

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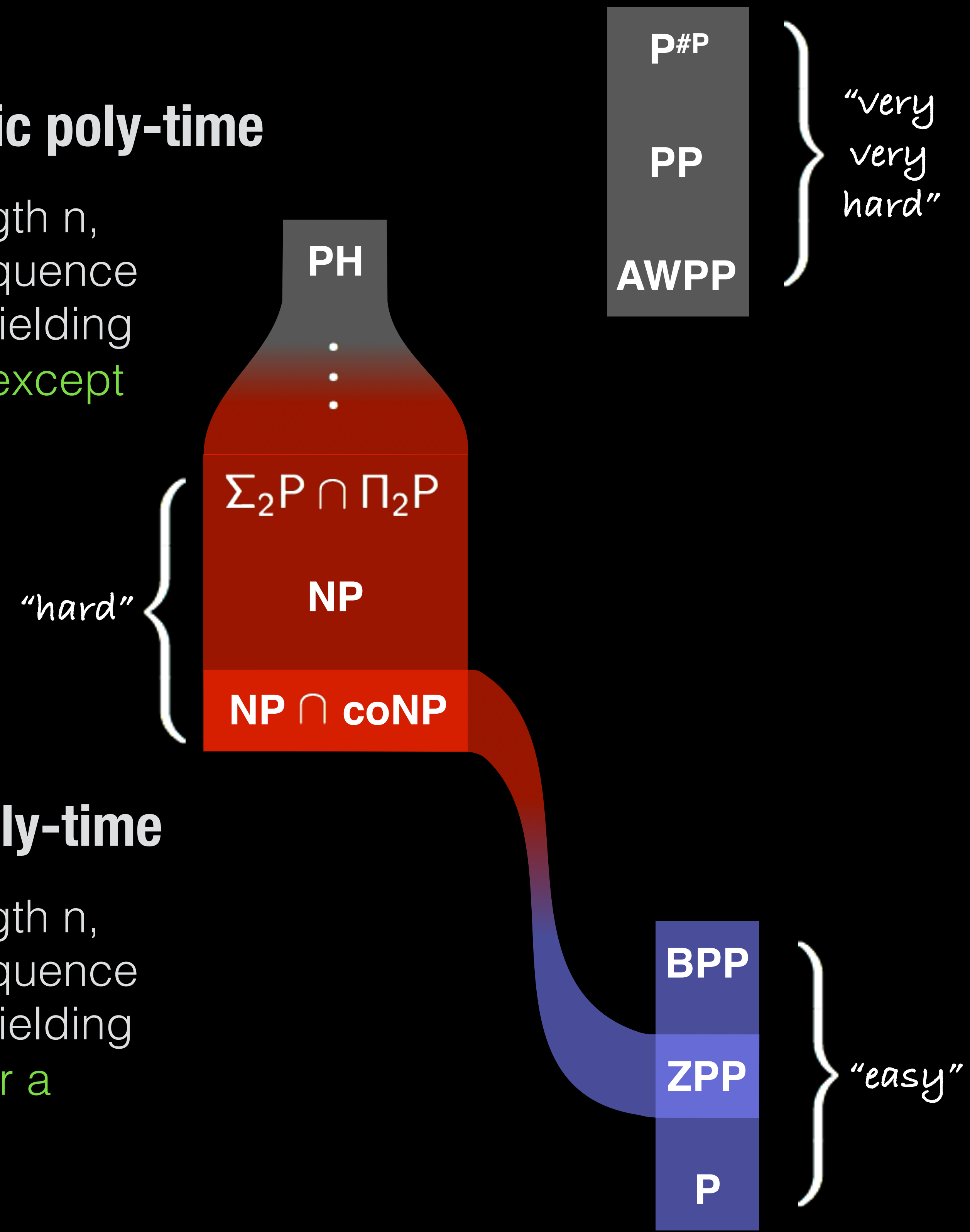

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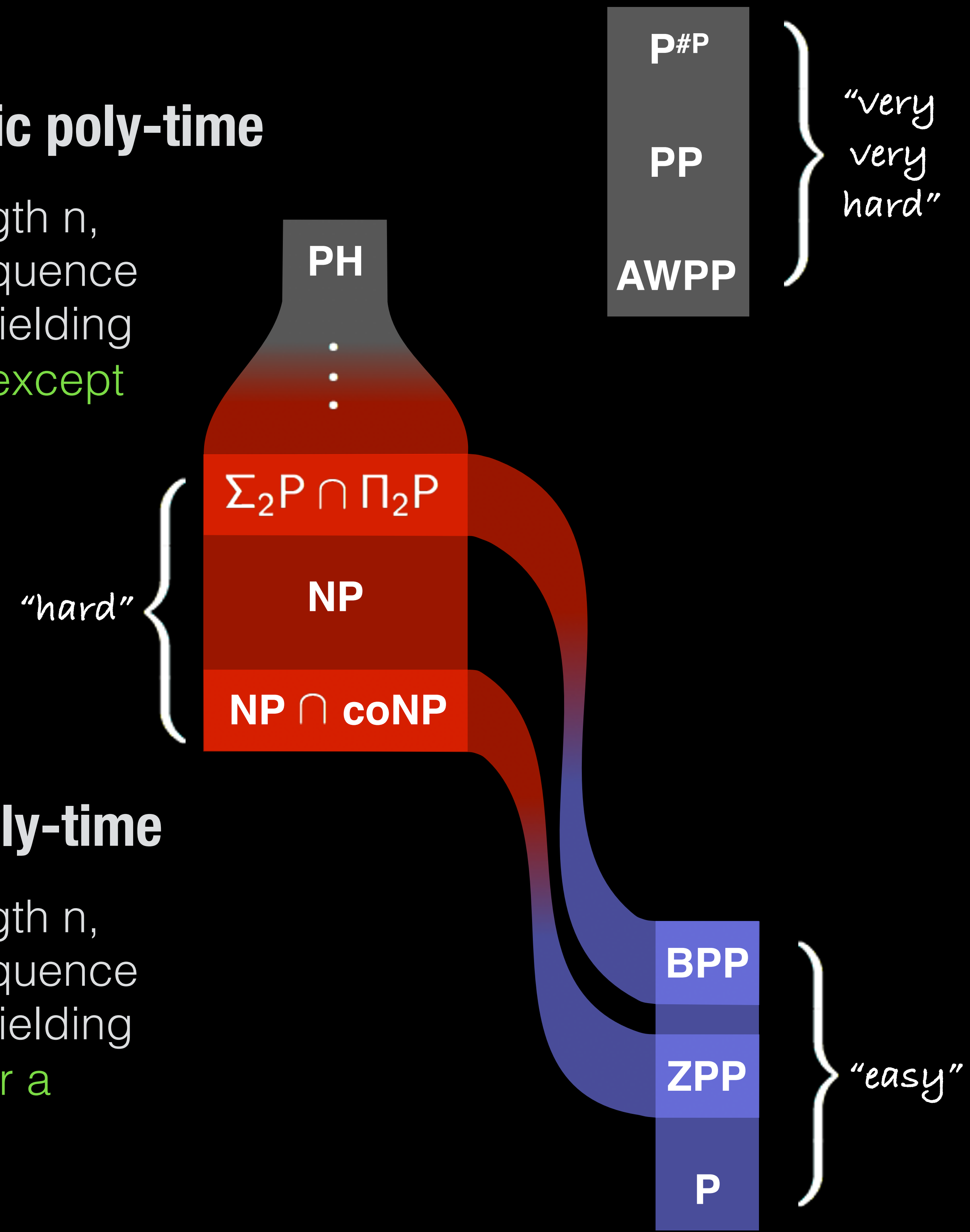

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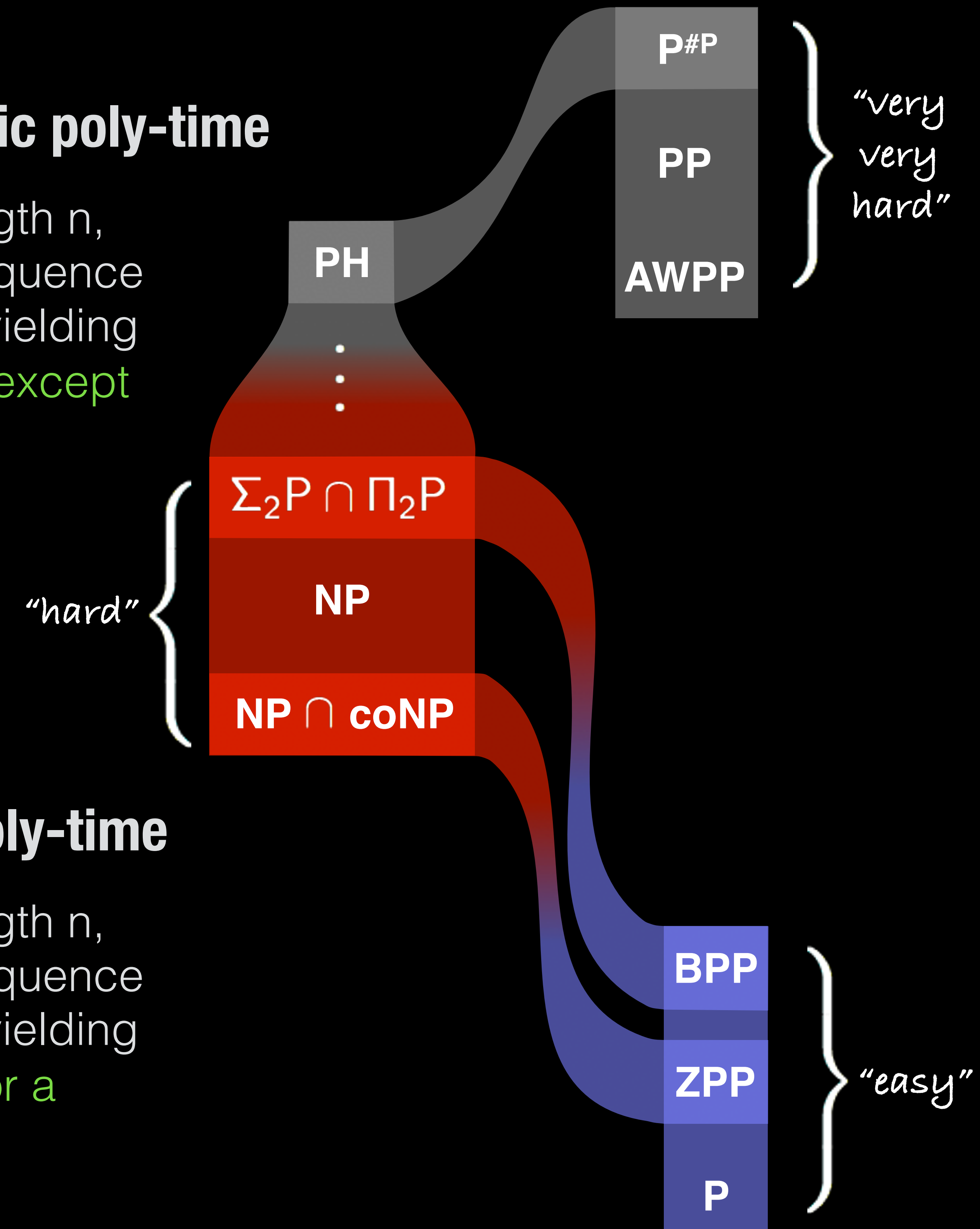

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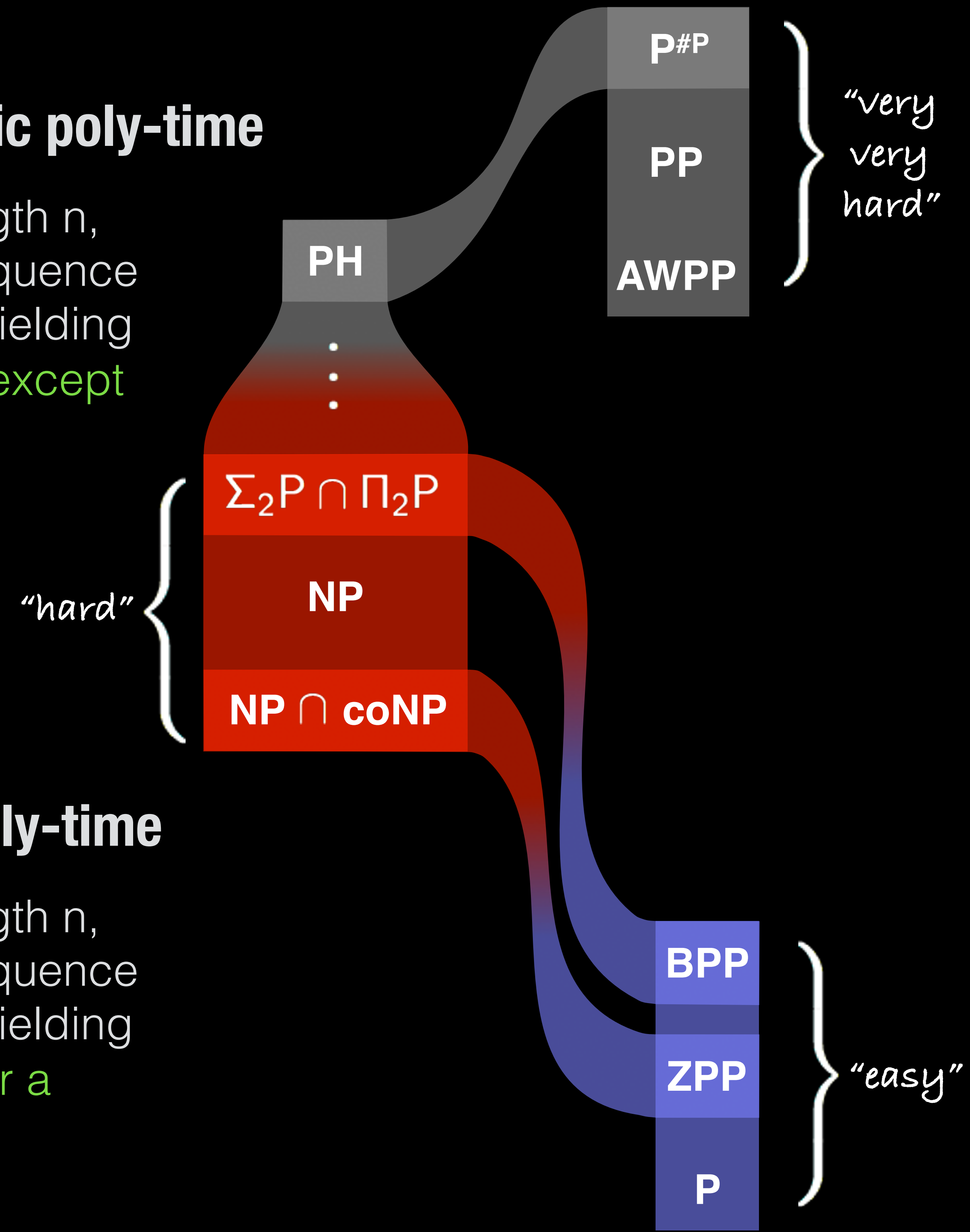

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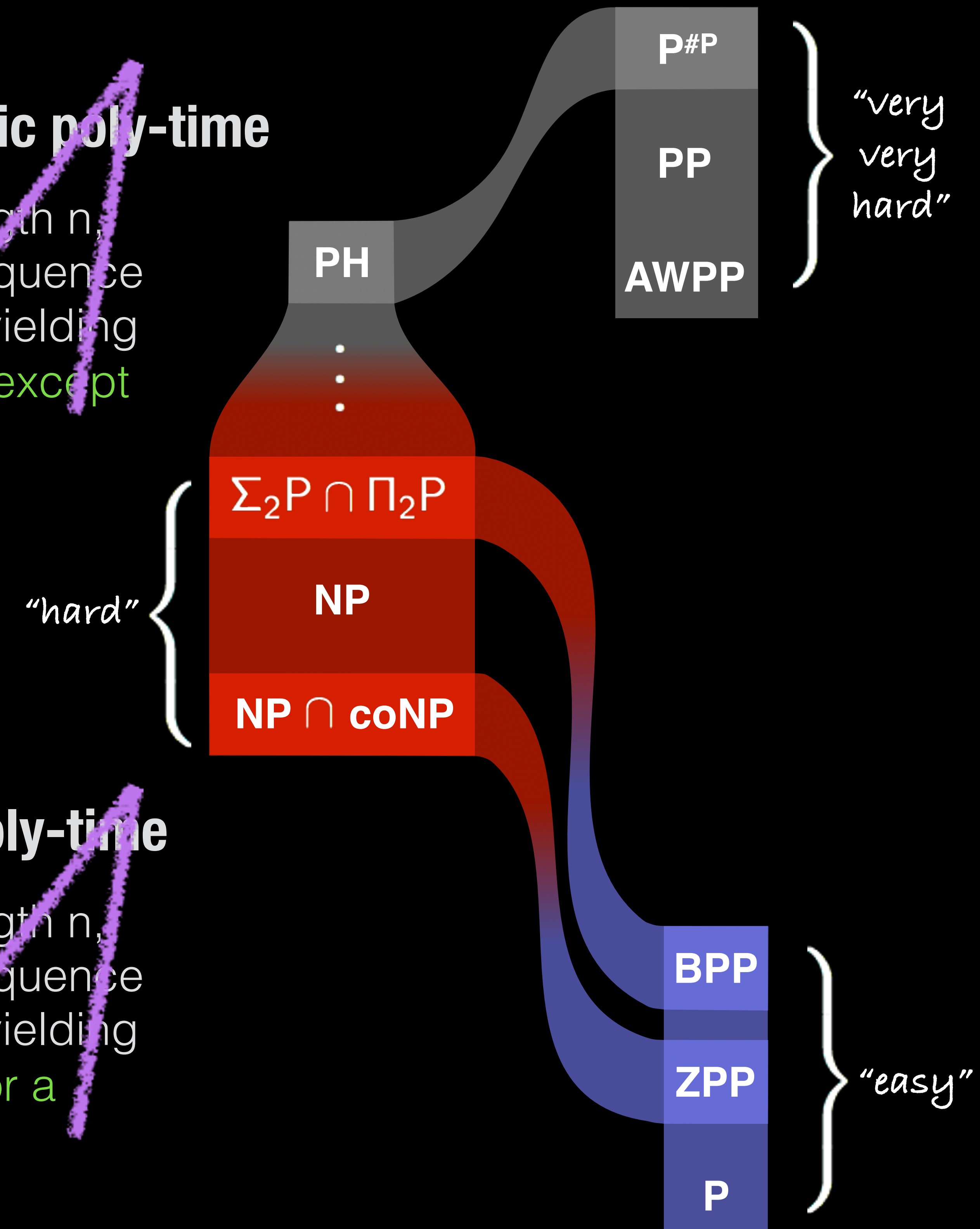

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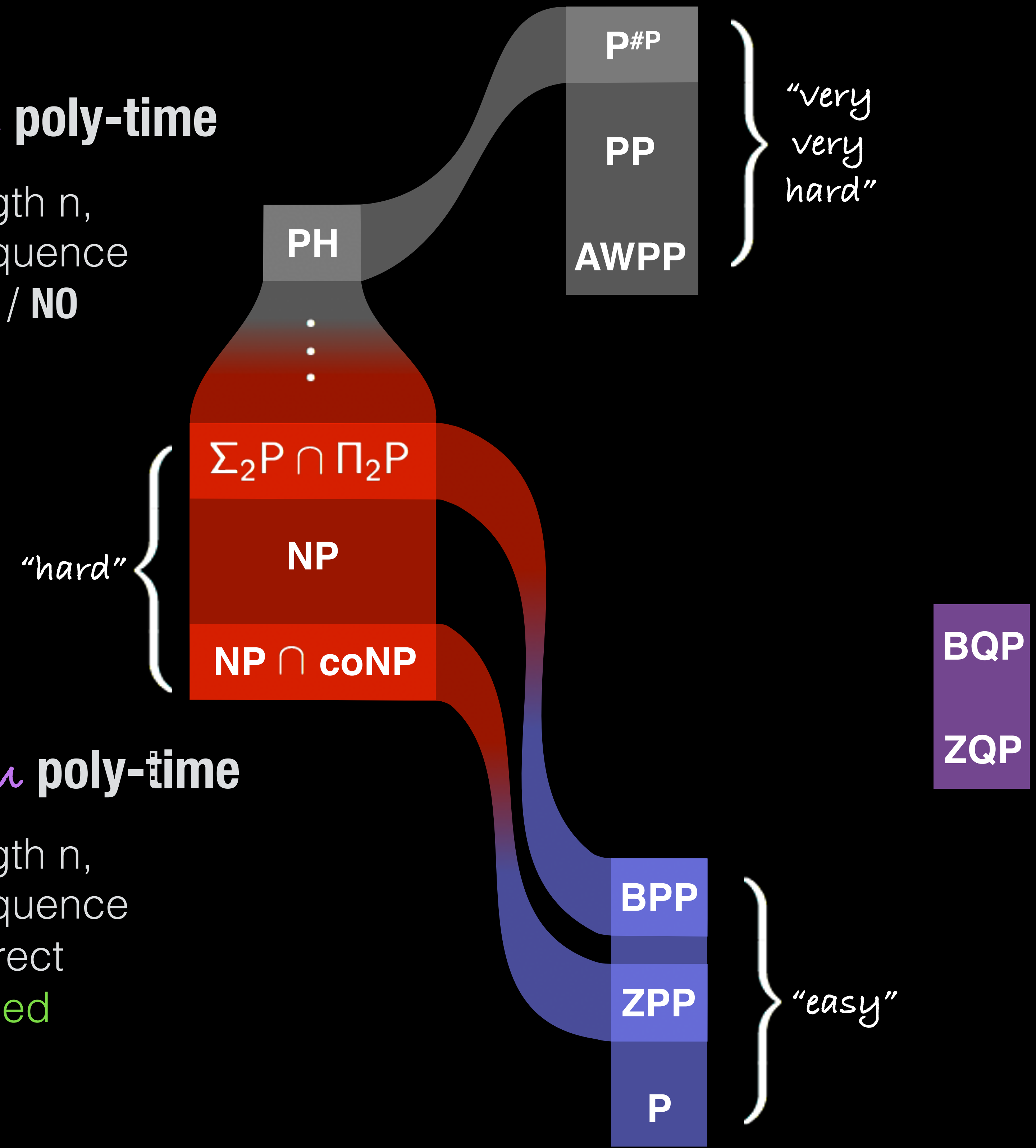

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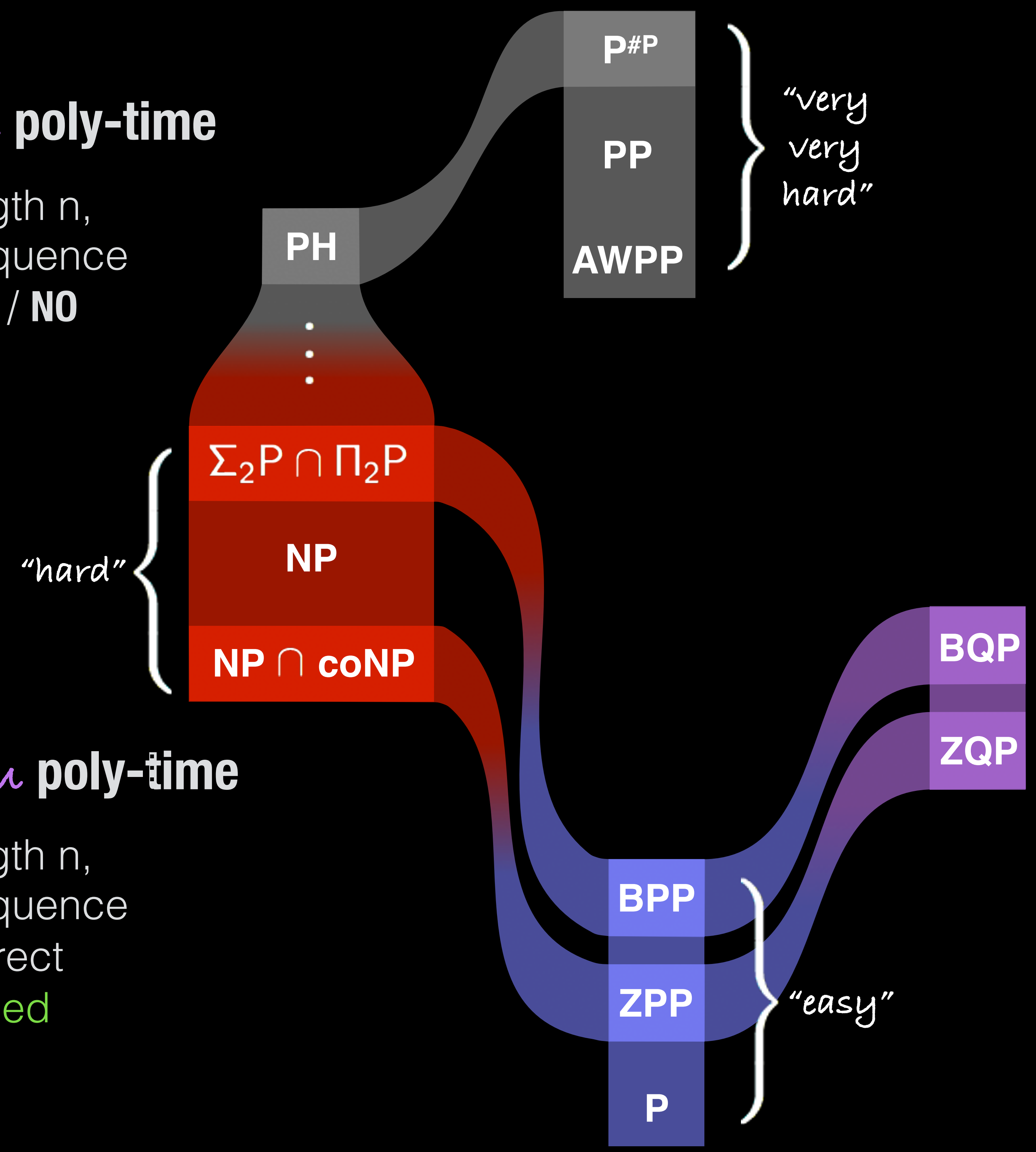

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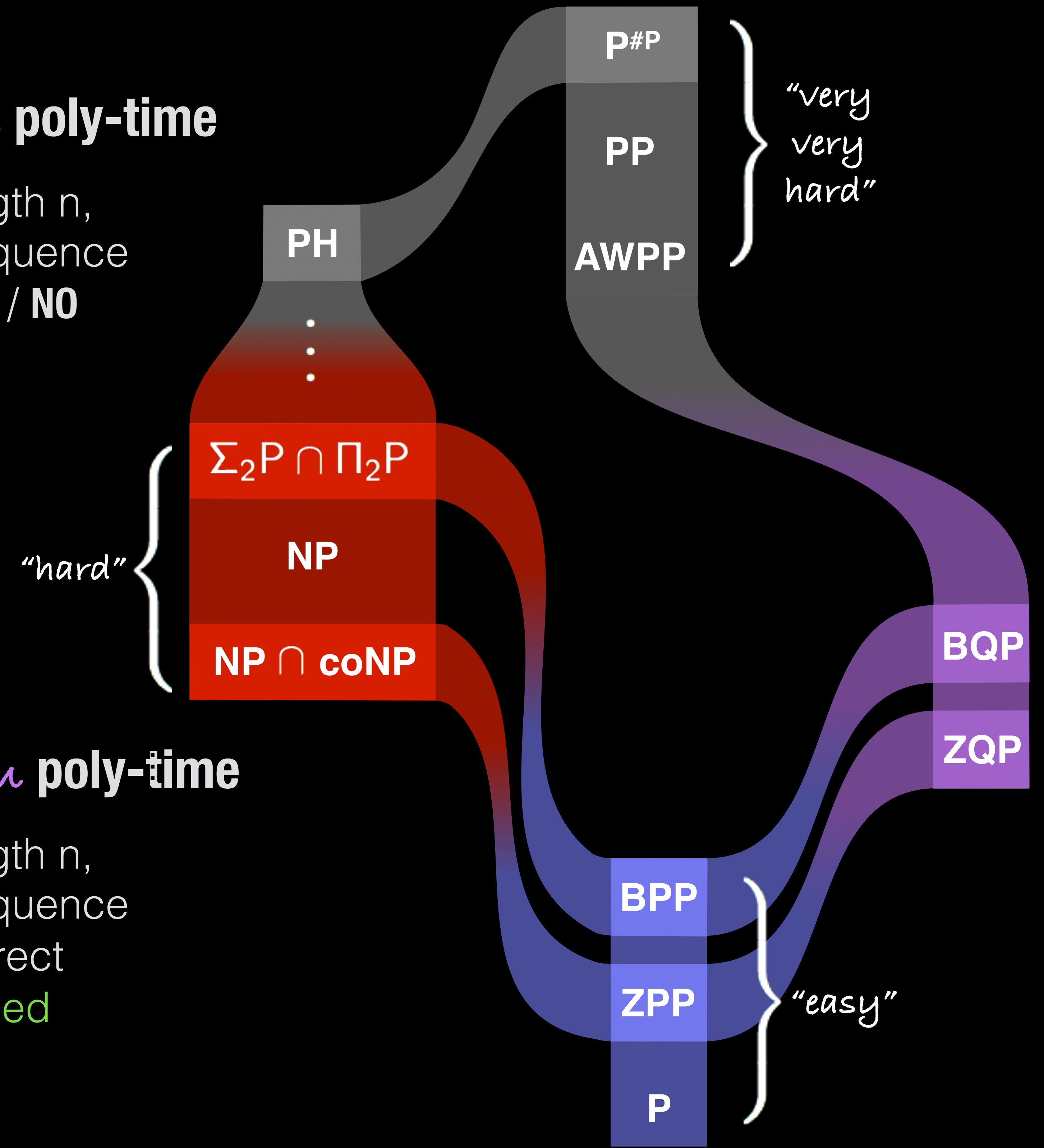

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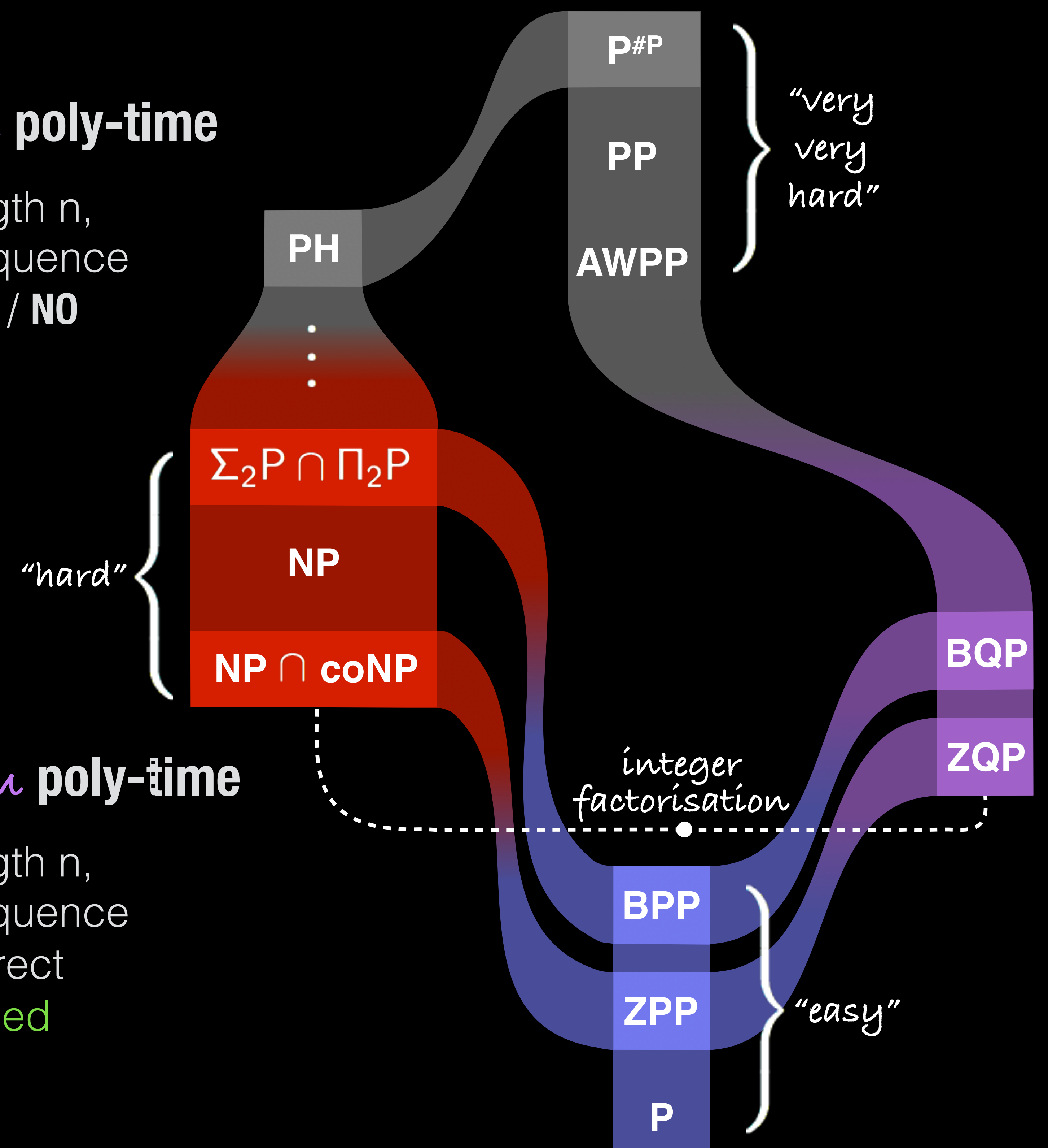
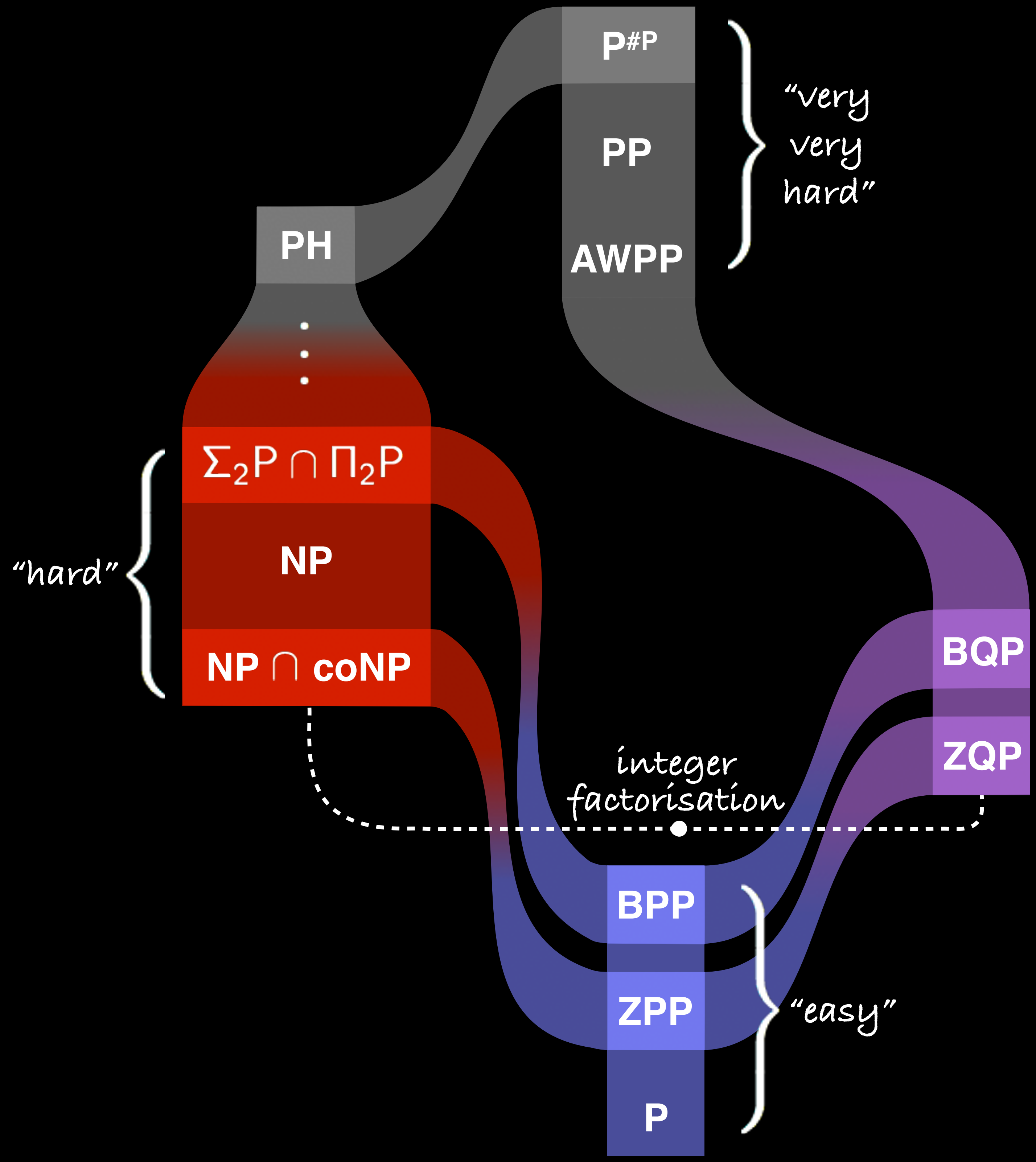


Quantum complexity

- **Observation:**
  - We know much less than we would like — *e.g.* unlike the "easy" classes, there is no known relationship yet between quantum classes and **PH**



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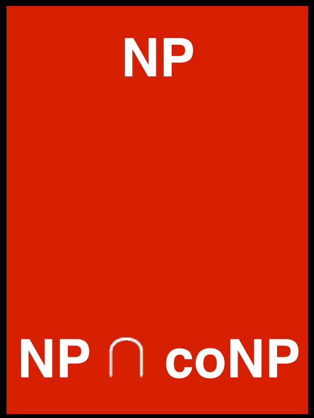

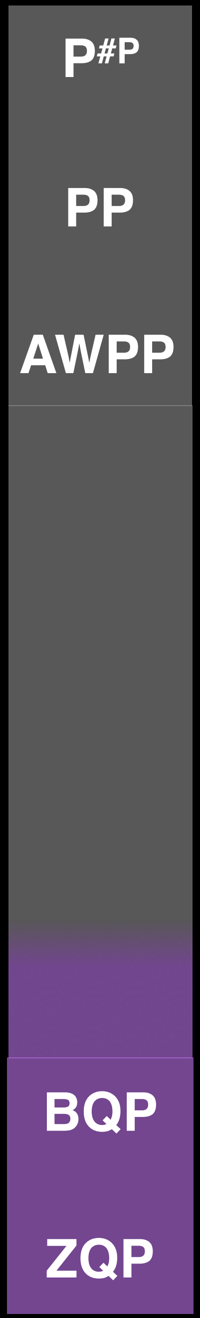

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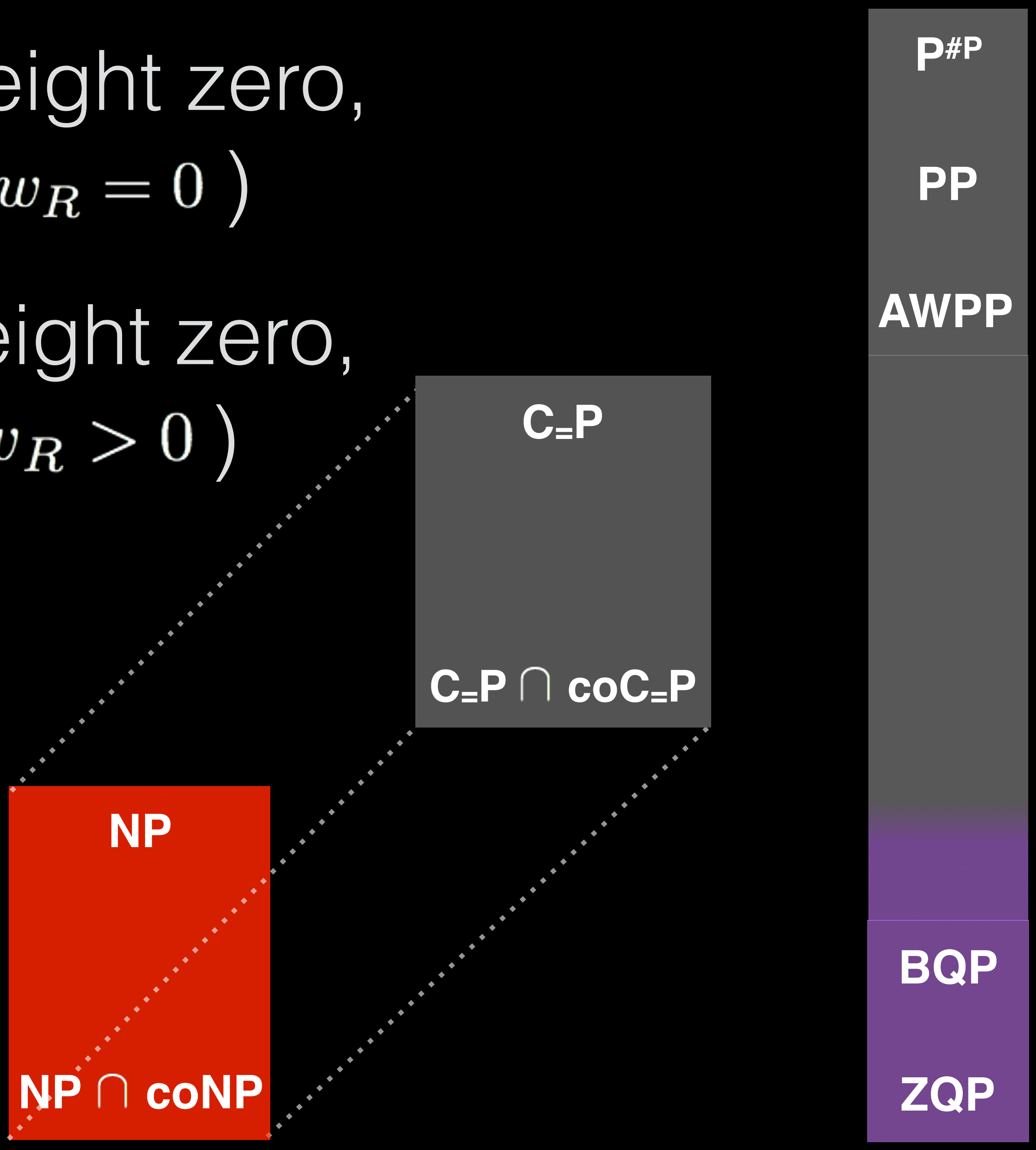

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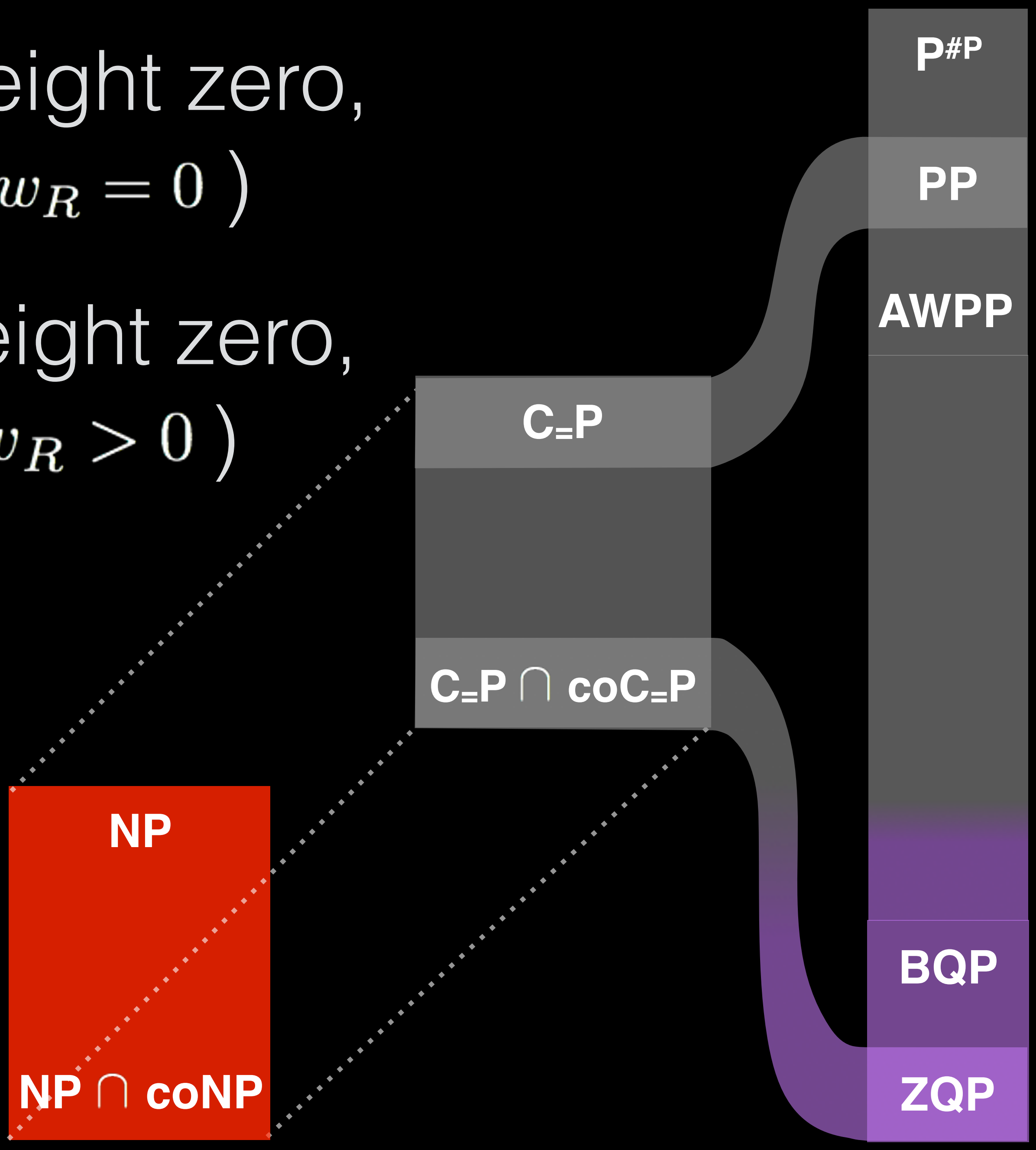

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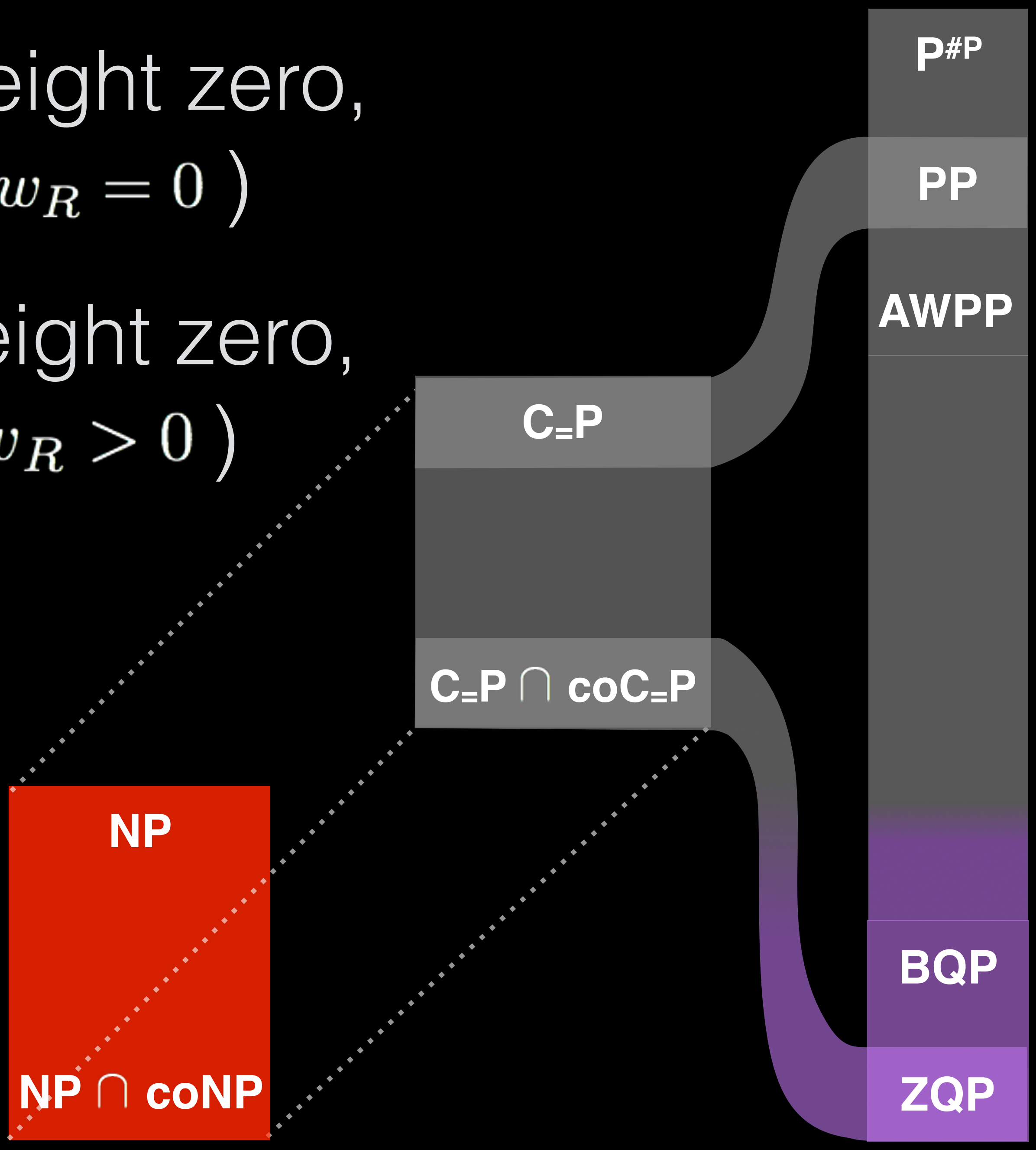

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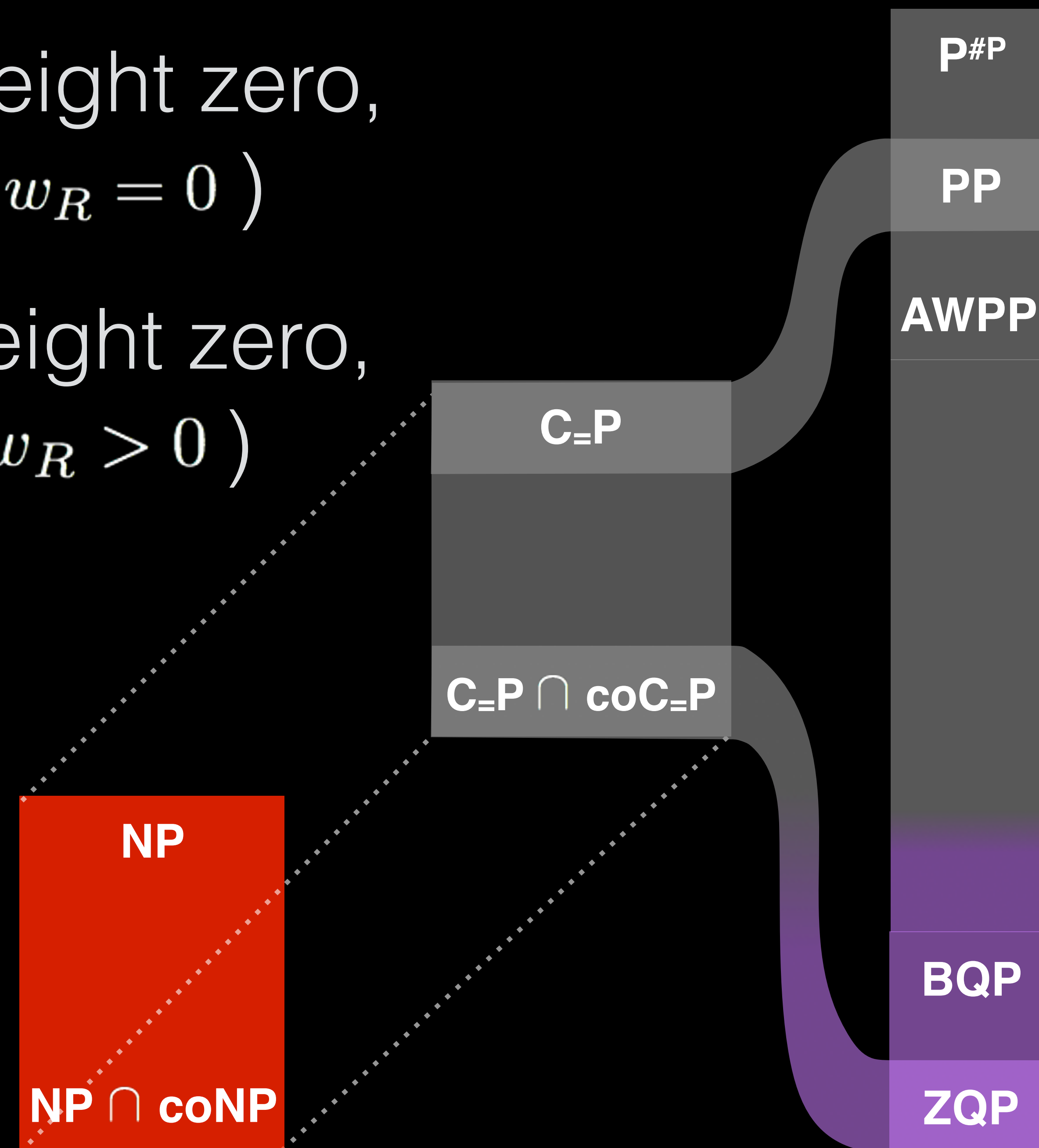
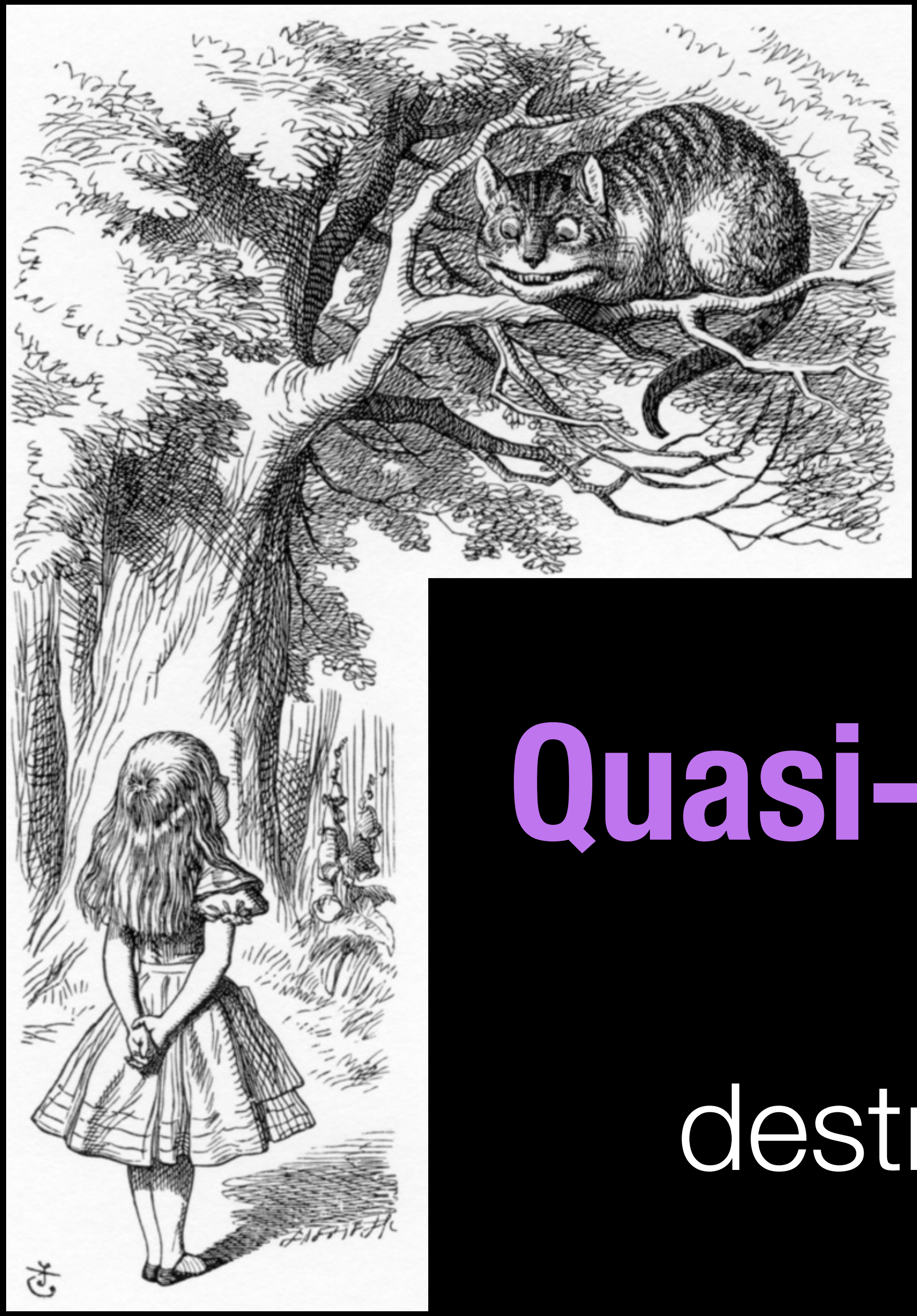


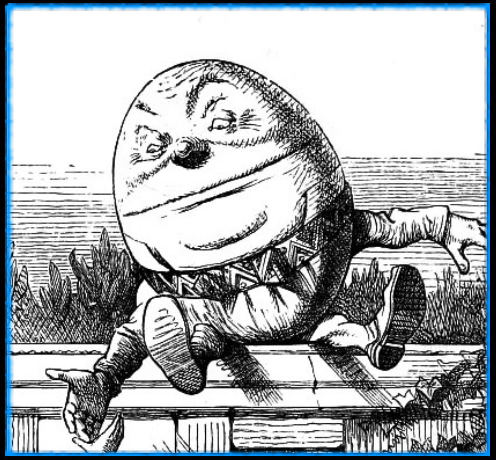
Can we describe the power of interference, without the notion of unitarity?



Quasi-quantum theories

to explore
destructive interference

“Quasi-quantum”?



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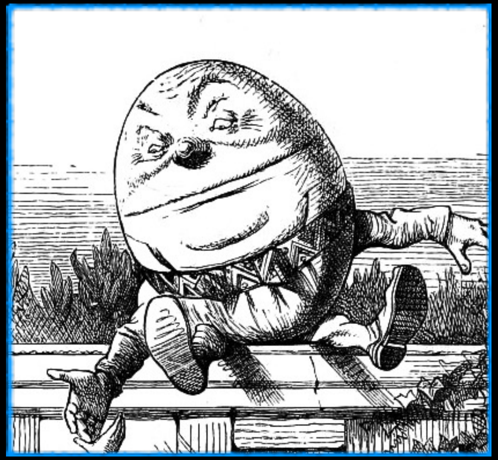
Like quantum computation (from a formal perspective) — but without the annoying adherence to physics



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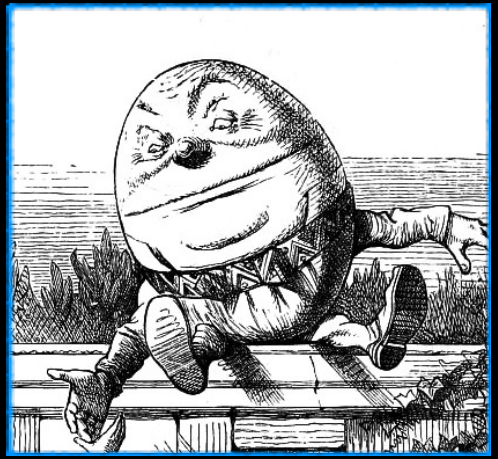

- Key ideas:
  - ❖ Computational states are a subclass of some **distributions** kX over “classical” outcomes X
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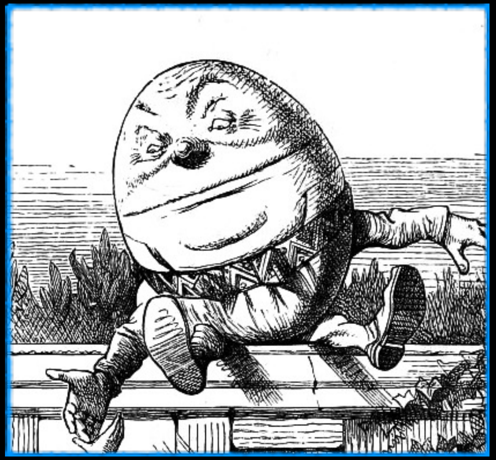

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- Key ideas:
  - Computational states are a subclass of some **distributions** kX over “classical” outcomes X
  - The states include X itself, and exclude the null distribution **0**
  - Transformations act linearly on all distributions, and map each state to some other state
  - Measurement consists of “sampling” labels from the distribution (*only one primitive notion of measurement*)

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- Stranger models of computation:
  - Distributions $\psi \in \mathbb{Z}_2 X$ satisfying $\psi^{\mathsf{T}} \psi = 1$

    — ***i.e.*** amplitudes are **integers mod 2** (not complex numbers)

What are such models ***for***?

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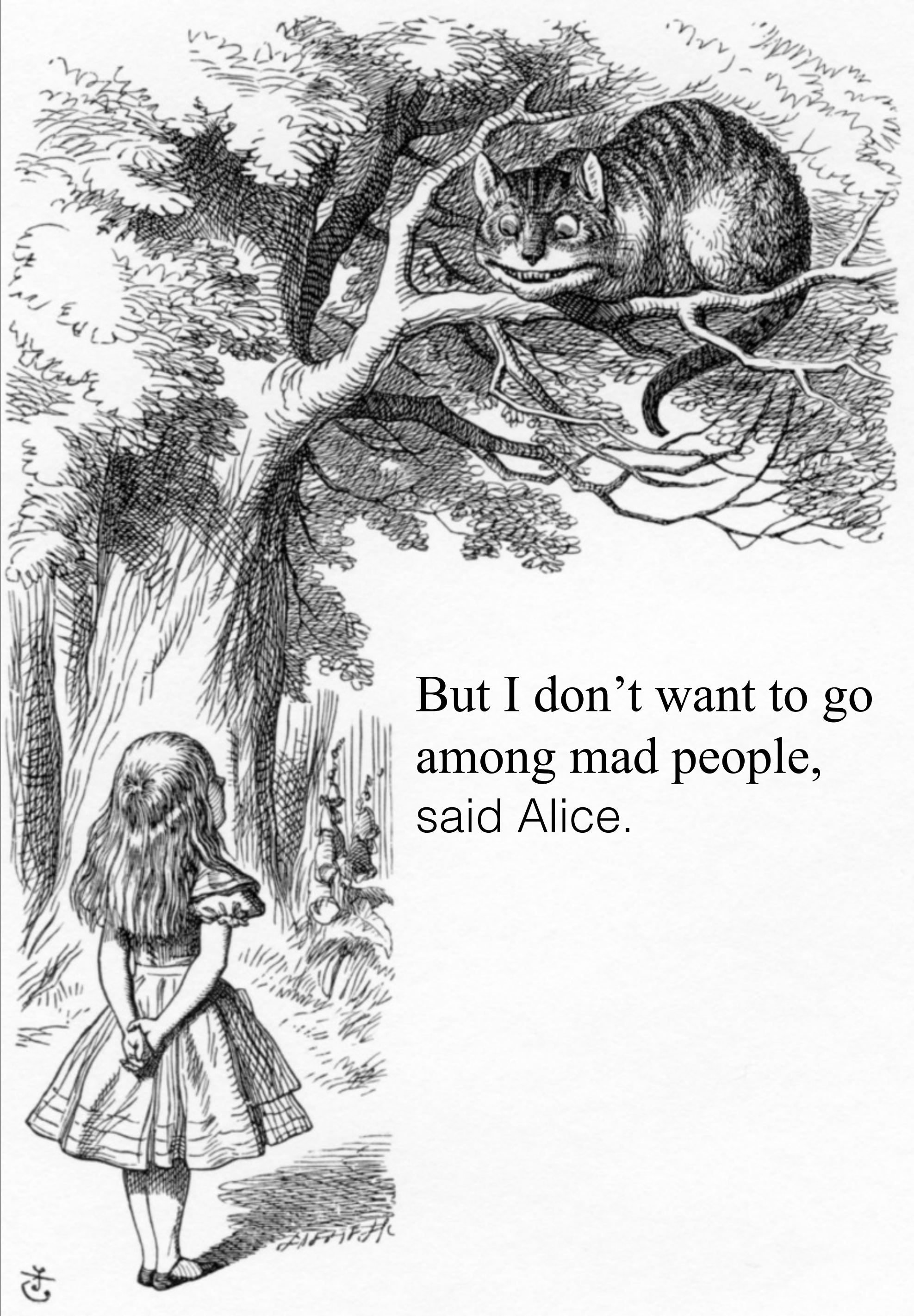

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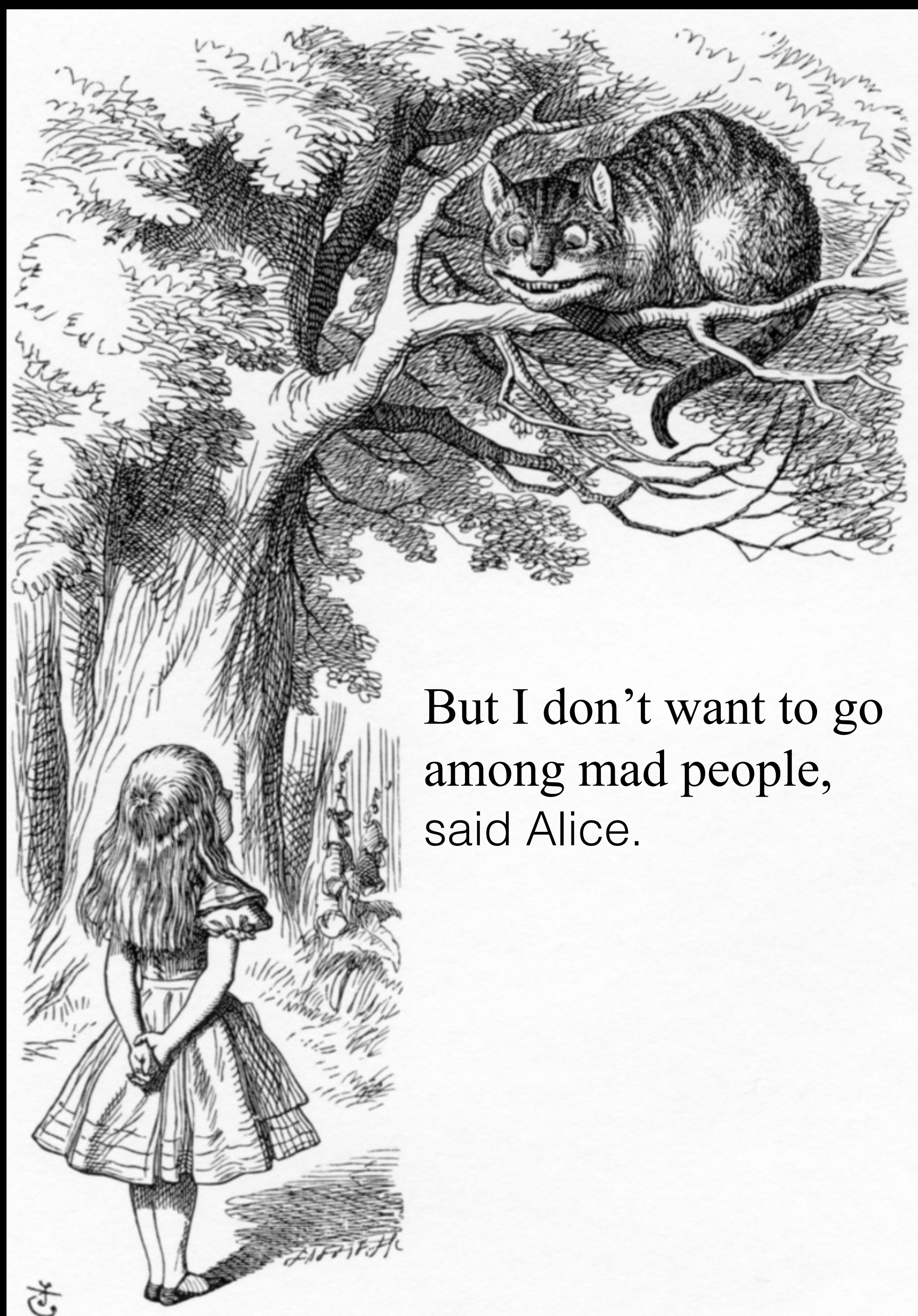

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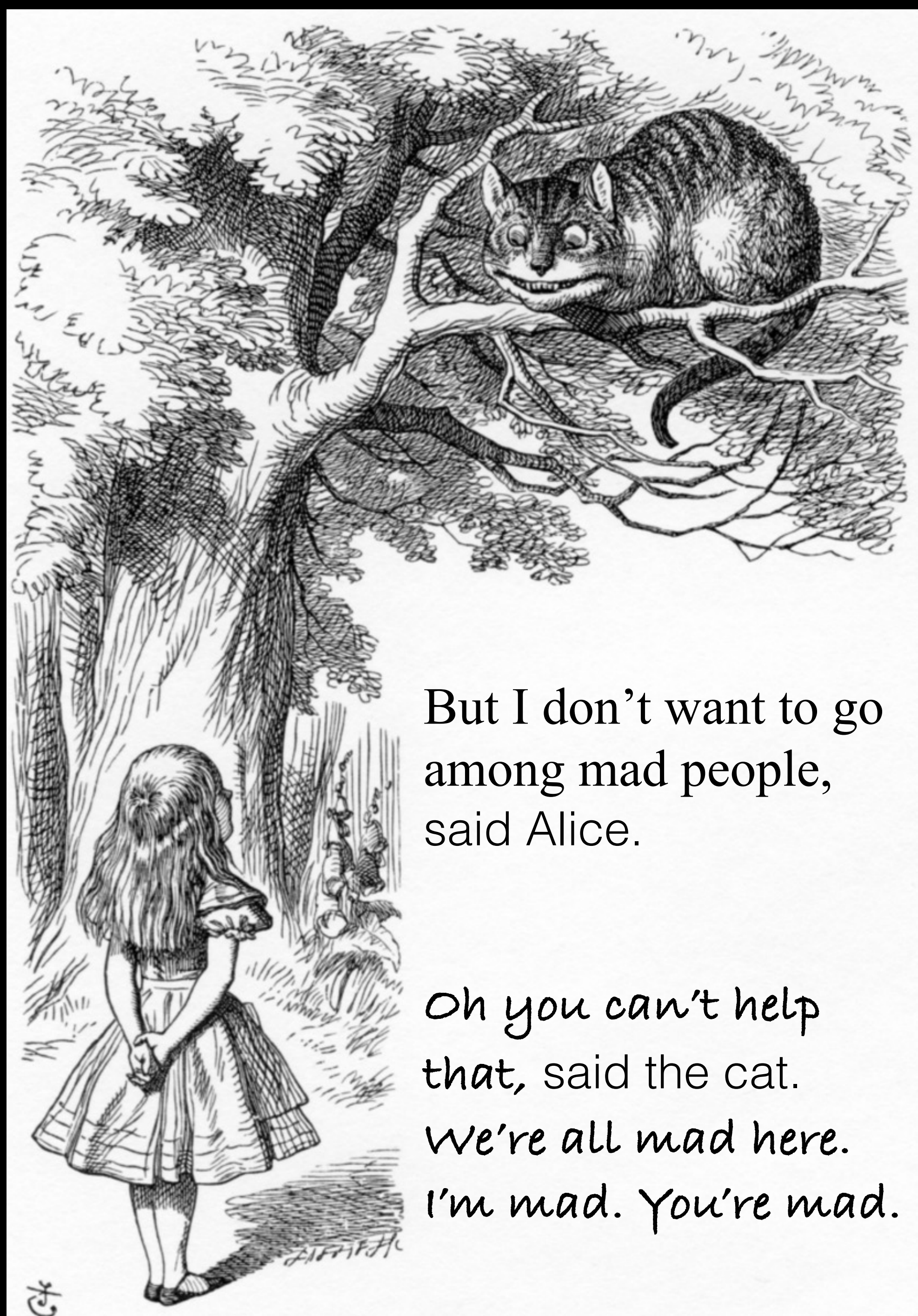

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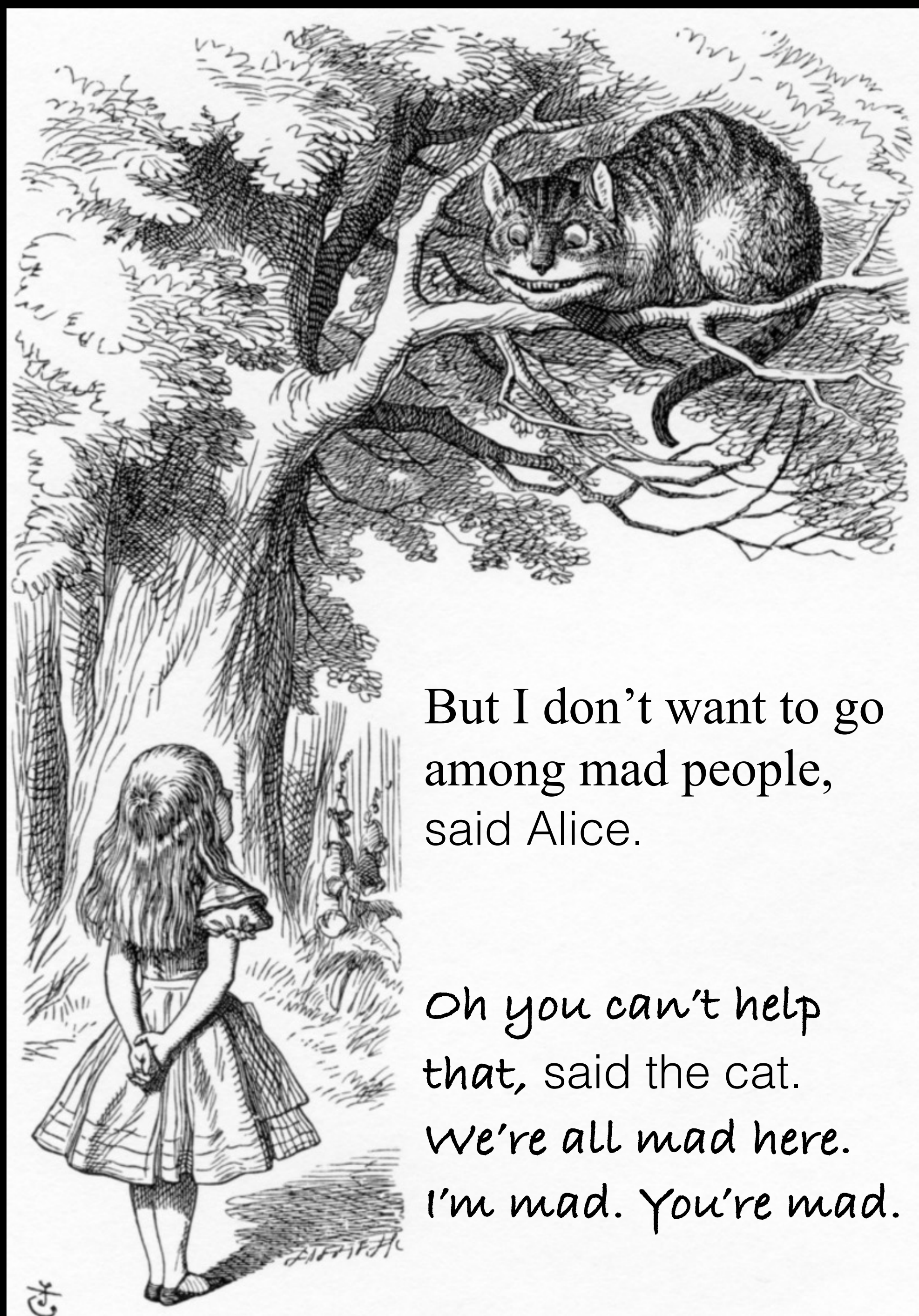

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Question: what happens when distributions have amplitudes which could cancel?

Approach

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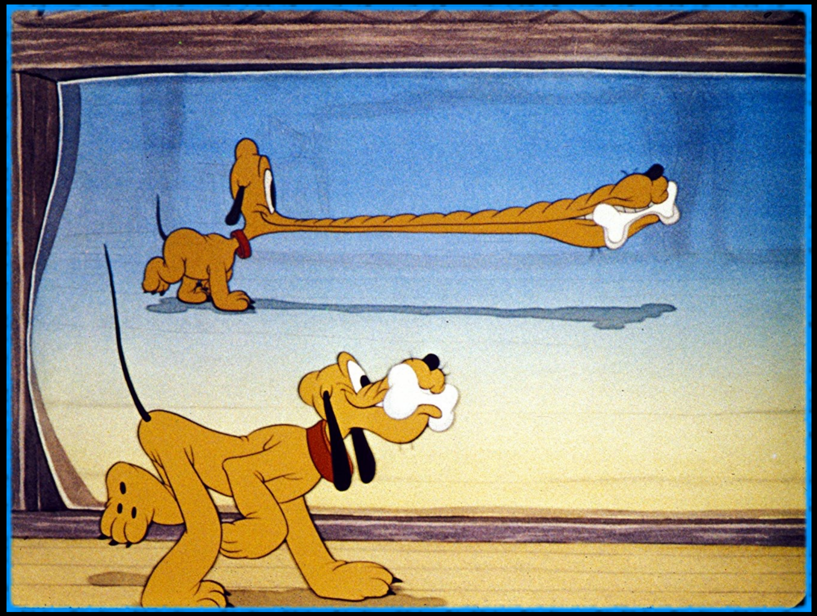

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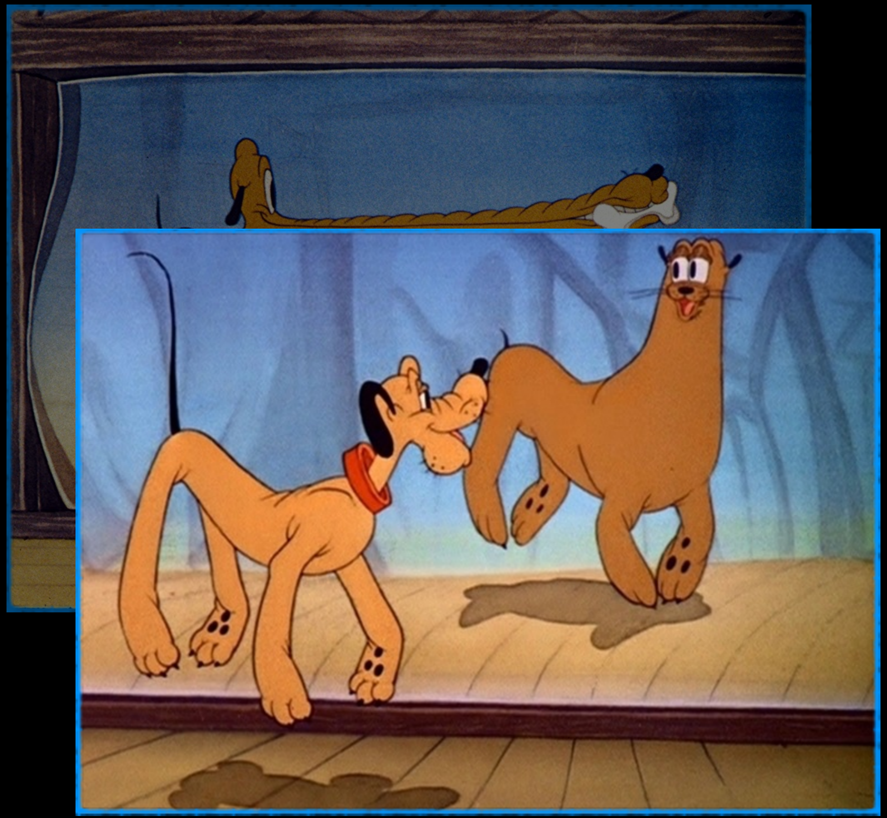

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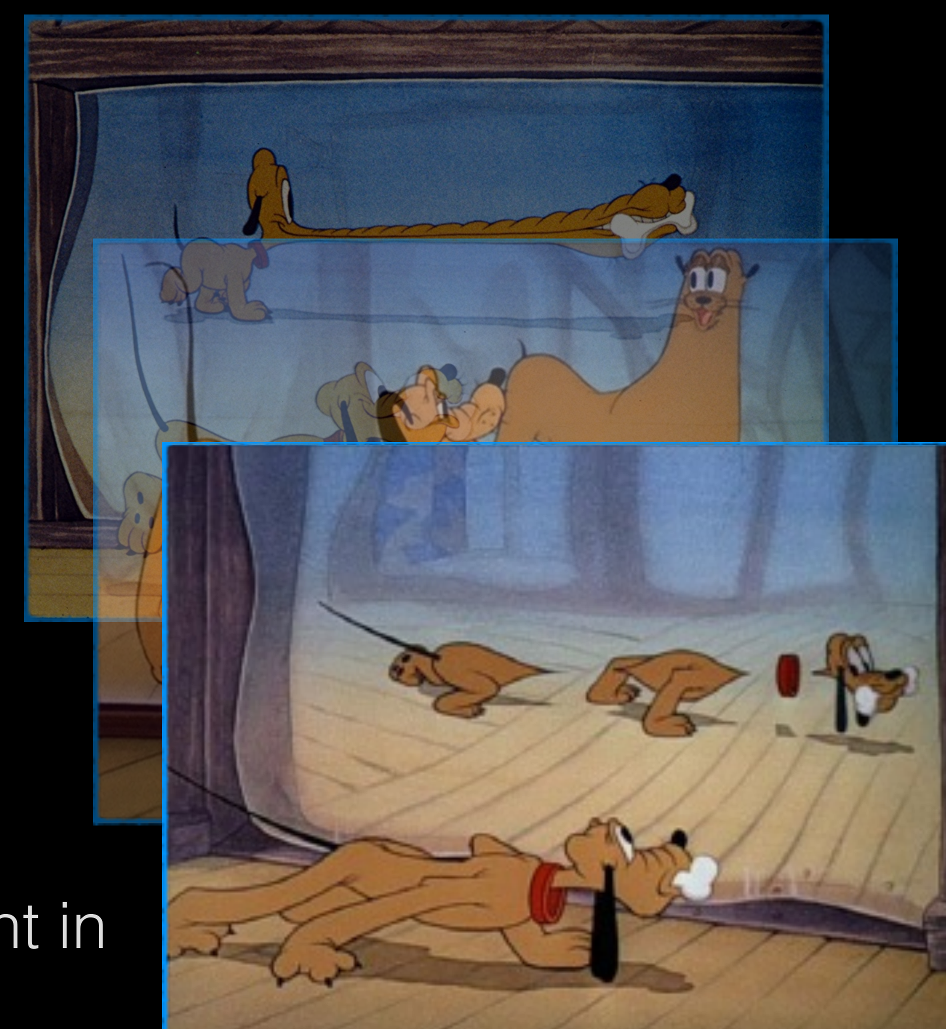

Results

a quick peek through
the looking glass

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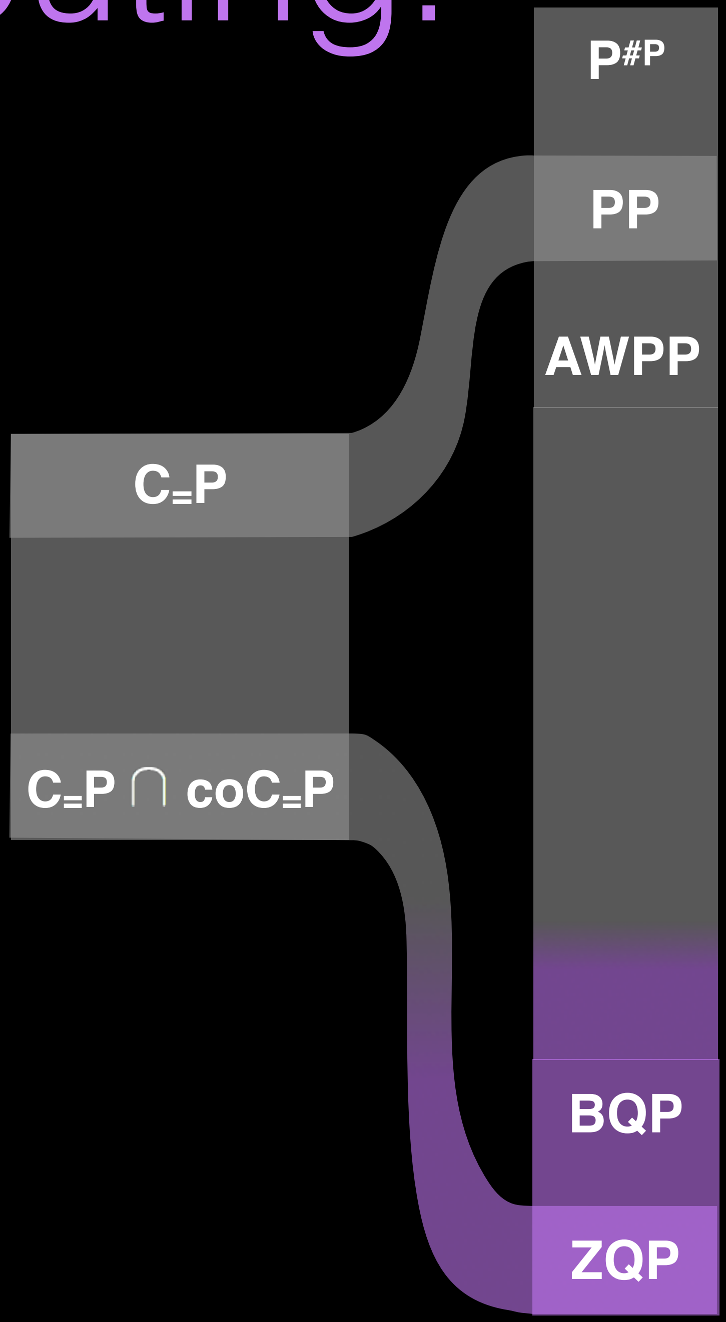

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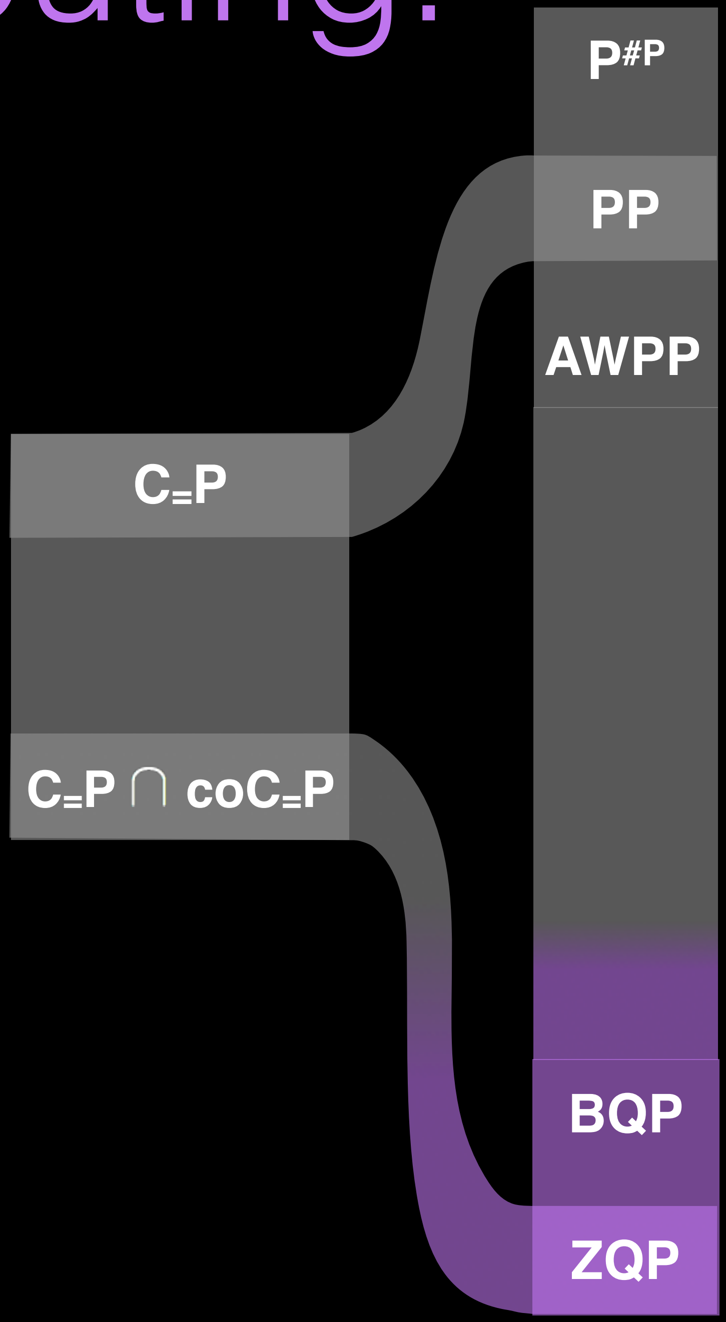

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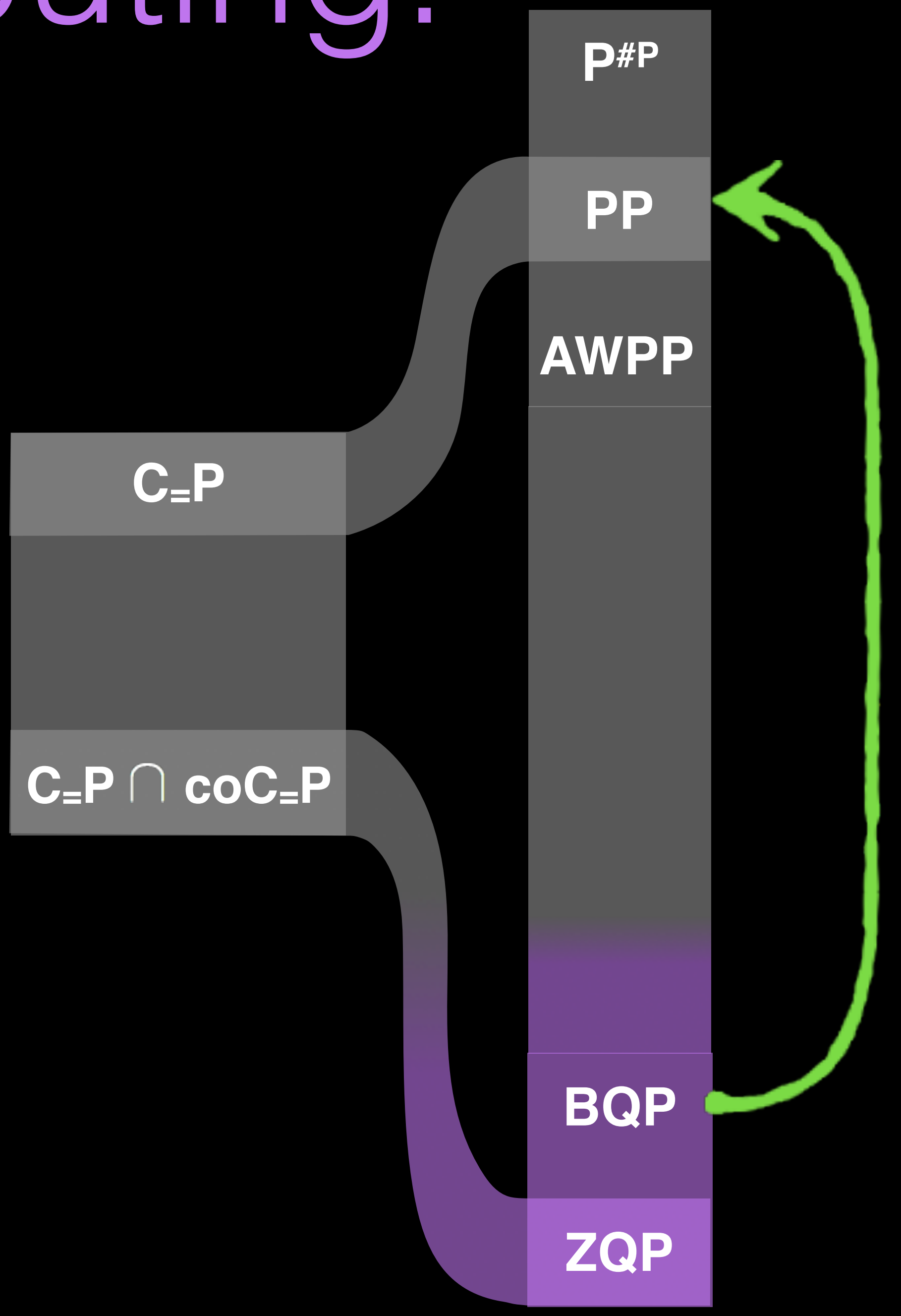

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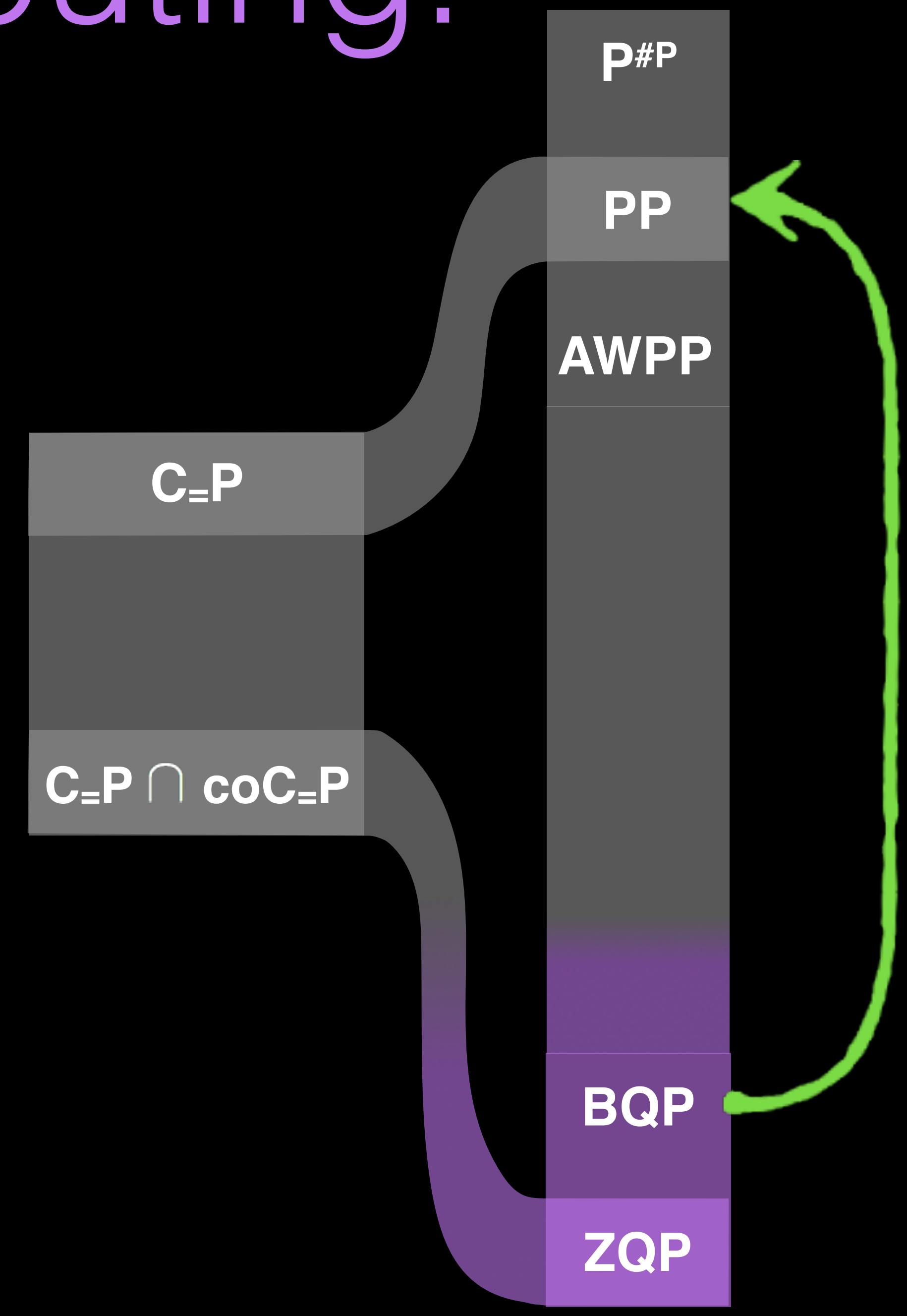

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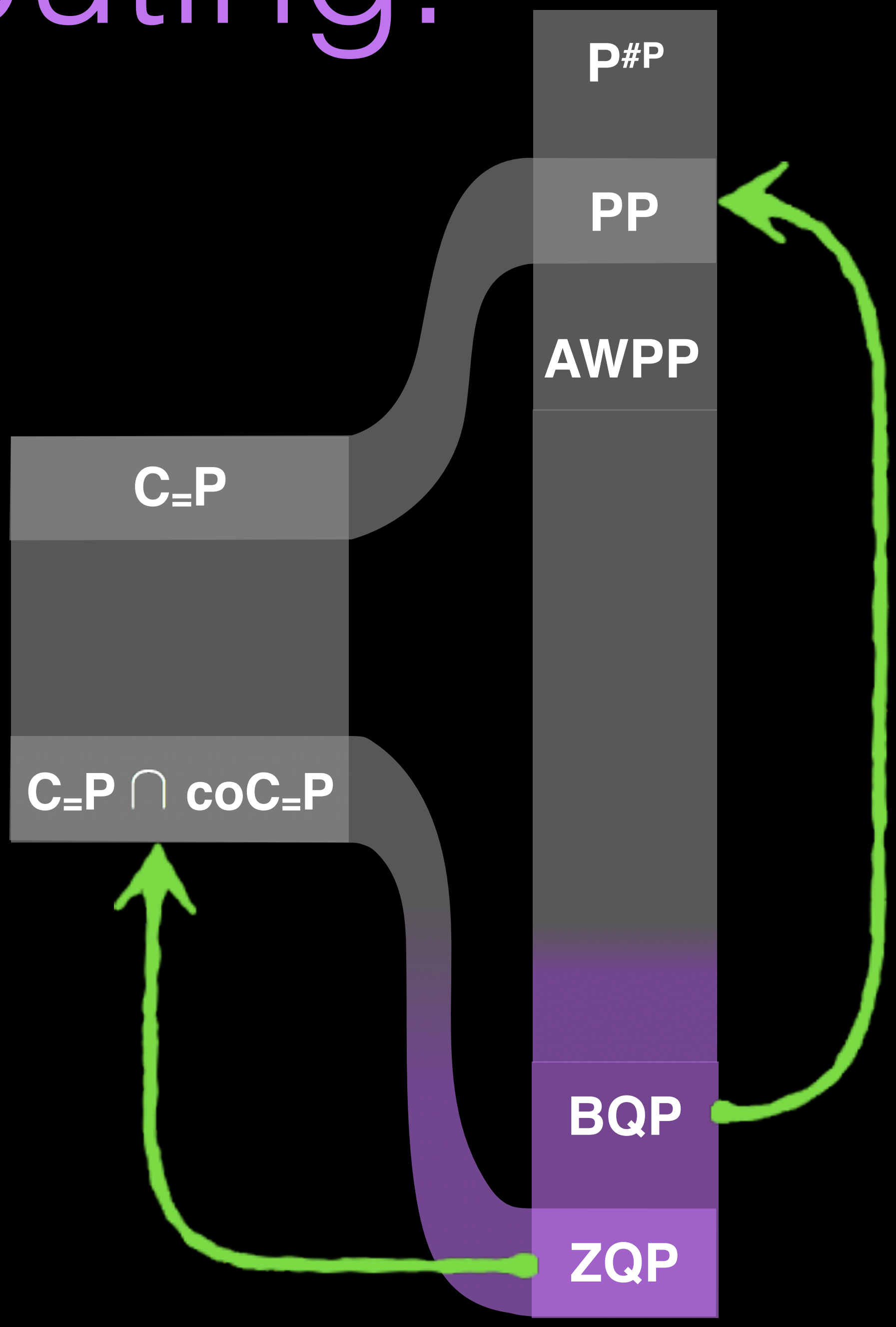

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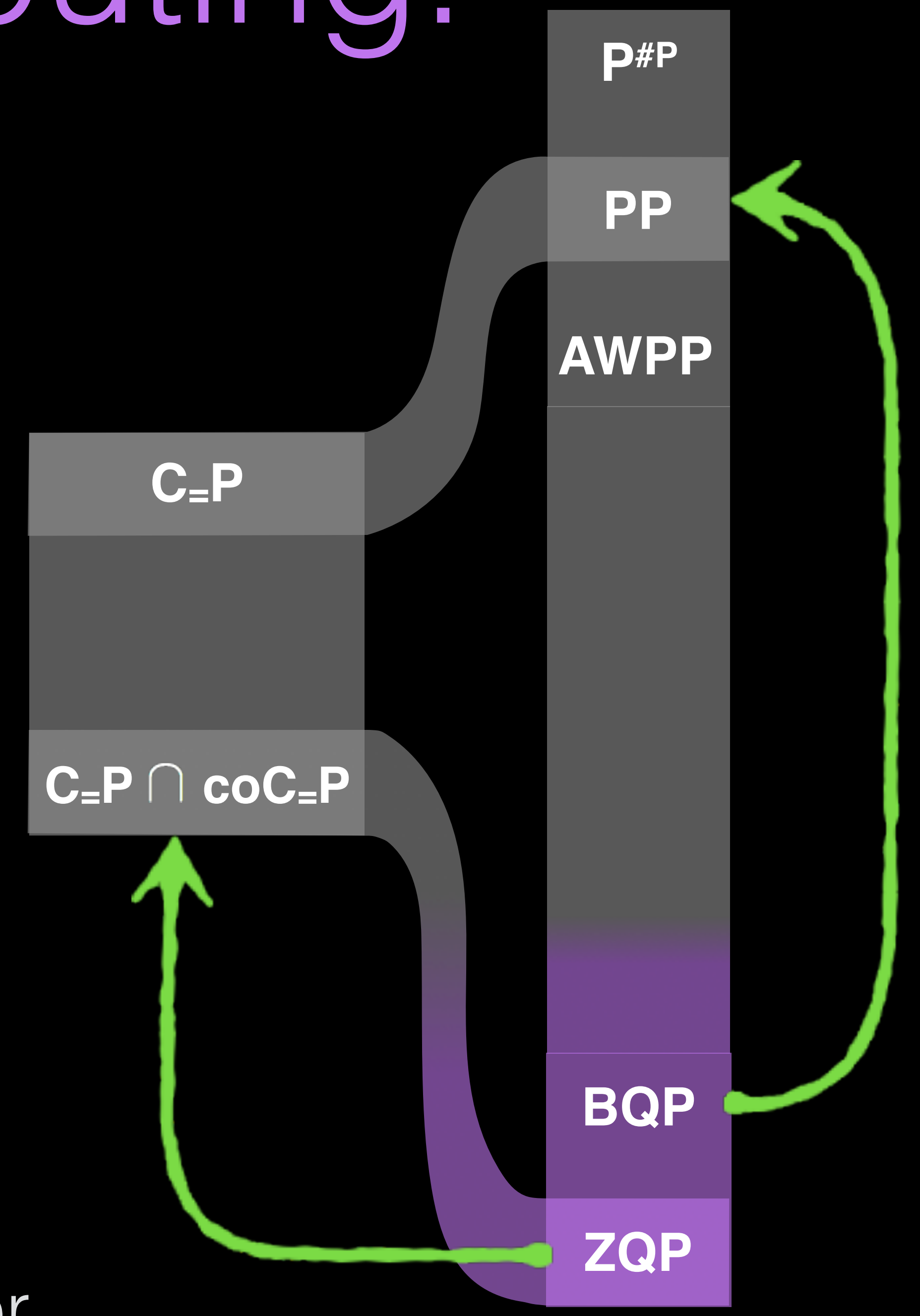
- **also:**
  Similar, but less dramatic, increases in power for *exact* (error-free and failure-free) computation with invertible gates



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- **Significance order** — $\alpha \preccurlyeq \beta$ iff $\alpha \in \beta$R (*c.f.* p-adic norm)

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- Can show that **UnitaryP**$_p$ = **Mod**$_p$**P** — a modulo-p variant of the class **NP**

Summary

the take-away
from Wonderland

What's the bottom line?

- These models of computation have in common:
  - ❖ Destructive interference is possible (like quantum computation)
  - ❖ Interference is **easier to realise** than in quantum computation
  - ❖ Bounded error / zero error computation is **very, very powerful**

Evidence of the power of destructive interference

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Intuition for quantum computation

— The class **EQP** (of problems exactly solvable by efficient quantum algorithms*) may be **much less** powerful than **BQP.**

Perhaps even **EQP** = **P** !

Thanks for listening!

Quantum Computing, Postselection, and Probabilistic Polynomial-Time
Aaronson, **[arXiv:quant-ph/0412187]**

On exact counting and quasi-quantum complexity
dB, **[arXiv:1509.07789]**

Modal quantum theory
Schumacher and Westmoreland, **[arXiv:1010.2929]**

On computation with 'probabilities' modulo k
dB, **[arXiv:1405.7381]**