Categorical Description of Gauge Theory

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Based on:

- arXiv:1604.01639 with Brano Jurčo and Martin Wolf

Motivation

Future progress in string theory seems to depend on more mathematical input.

String-/M-theory as it used to be

- Every 10 years a "string revolution"
- Every 2-3 years one new big fashionable topic to work on

This changed: No more revolutions or really big fashionable topics.

My explanation

We need more input from maths, in particular category theory:

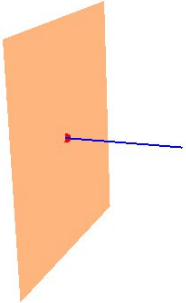
- 2-form gauge potential B-field: Gerbes or principal 2-bundles
- String Field Theory: L_∞-algebras or semistrict Lie ∞-algebras
- Double Field Theory: Courant algebroids or symplectic Lie 2-algebroids
- (2,0)-theory: parallel transport of string-like objects full non-abelian higher gauge theory

> *We will need to use some very simple notions of category theory, an esoteric subject noted for its difficulty and irrelevance.*
>
> G. Moore and N. Seiberg, 1989

What does categorification mean?
One of Jeff Harvey's questions to identify the "generation PhD>1999" at Strings 2013.

Motivation: The Dynamics of Multiple M5-Branes

To understand M-theory, an effective description of M5-branes would be very useful.

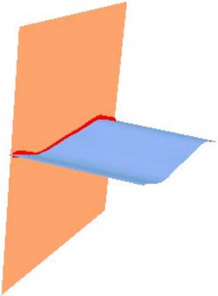


D-branes

- D-branes interact via strings.
- Effective description: theory of endpoints
- Parallel transport of these: Gauge theory
- Study string theory via gauge theory

M5-branes

- M5-branes interact via M2-branes.
- Eff. description: theory of self-dual strings
- Parallel transport: Higher gauge theory
- Holy grail: (2,0)-theory (conjectured 1995)



Lightning Review: Gauge Theory

Gauge theory describes interactions in the presence of internal symmetries.

For a physicist

- Some particles/quantum fields: posses local symmetries
- problem: $\phi(x) \to g(x)\phi(x)$, but $\frac{\partial}{\partial x^\mu}\phi(x) \nrightarrow g(x)\frac{\partial}{\partial x^\mu}\phi(x)$
- solution: gauge field $\frac{\partial}{\partial x^\mu}\phi(x) \to (\frac{\partial}{\partial x^\mu} + A_\mu(x))\phi(x)$

For a mathematician

- local symmetry: principal fibre bundle, representation
- fields are sections of associated vector bundle
- derivative becomes connection on the vector bundle

For calculations in physics: cocycles

- open cover of manifold $\sqcup_a U_a \to M$
- principal G-bundle: $g_{ab} : C^\infty(U_a \cap U_b, \mathsf{G})$ with $g_{ab}g_{bc} = g_{ac}$
- connection: $A_a : \Omega^1(U_a, \mathsf{Lie}(\mathsf{G}))$ with $A_a = g_{ab}^{-1}(\mathrm{d} + A_b)g_{ab}$

Parallel Transport of Strings is Problematic

The lack of surface ordering renders a parallel transport of strings problematic.

Parallel transport of particles in representation of gauge group G:

- holonomy functor $\mathsf{hol} : \text{path } \gamma \mapsto \mathsf{hol}(\gamma) \in \mathsf{G}$
- $\mathsf{hol}(\gamma) = P\exp(\int_\gamma A)$, P: path ordering, trivial for $\mathsf{U}(1)$.

Parallel transport of strings with gauge group $\mathsf{U}(1)$:

- map $\mathsf{hol} : \text{surface } \sigma \mapsto \mathsf{hol}(\sigma) \in \mathsf{U}(1)$
- $\mathsf{hol}(\sigma) = \exp(\int_\sigma B)$, B: connective structure on gerbe.

Nonabelian case:

- much more involved!
- no straightforward definition of surface ordering

Naïve No-Go Theorem

Naively, there is no non-abelian parallel transport of strings.

Imagine parallel transport of string with gauge degrees in Lie(G):

$$\begin{tikzcd}[column sep=huge]
\bullet & {} \arrow[l, bend right=60, ""{name=U1, below}] \arrow[l, ""{name=M1, above}, ""{name=N1, below}] \arrow[l, bend left=60, ""{name=D1, above}] & \bullet \arrow[l, bend right=60, ""{name=U2, below}] \arrow[l, ""{name=M2, above}, ""{name=N2, below}] \arrow[l, bend left=60, ""{name=D2, above}]
\arrow[Rightarrow, from=U1, to=M1, "g_1"]
\arrow[Rightarrow, from=N1, to=D1, "g_1'"]
\arrow[Rightarrow, from=U2, to=M2, "g_2"]
\arrow[Rightarrow, from=N2, to=D2, "g_2'"]
\end{tikzcd}$$

Consistency of parallel transport requires:

$$(g_1'g_2')(g_1g_2) = (g_1'g_1)(g_2'g_2)$$

This renders group G abelian.

Eckmann and Hilton, 1962
Physicists 80'ies and 90'ies

Way out: 2-categories, Higher Gauge Theory.

Two operations $\circ$ and $\otimes$ satisfying Interchange Law:

$$(g_1' \otimes g_2') \circ (g_1 \otimes g_2) = (g_1' \circ g_1) \otimes (g_2' \circ g_2) \ .$$

We want to categorify gauge theory

Need: suitable descriptions/definitions

Gauge Theory from Parallel Transport Functors

A straightforward way to describe gauge theory is in terms of parallel transport functors.

Encode gauge theory in parallel transport functor

Mackaay, Picken, 2000

- Every manifold comes with path groupoid $\mathcal{P}M = (PM \rightrightarrows M)$

$$\begin{tikzcd} x \arrow[r, bend left, "\gamma"] & y \end{tikzcd}$$

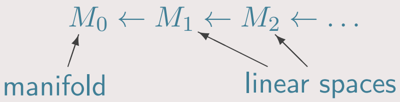
- Gauge group gives rise to delooping groupoid $\mathsf{B}\mathsf{G} = (\mathsf{G} \rightrightarrows *)$
- parallel transport functor $\mathrm{hol} : \mathcal{P}M \rightarrow \mathsf{B}\mathsf{G}$:
    - assigns to each path a group element
    - composition of paths: multiplication of group elements
- Readily categorifies:
    - use path 2-groupoid with homotopies between paths
    - use delooping of categorified group
- Problem: Need to differentiate to get to cocycles

Higher Lie Algebras from NQ-Manifolds

Semistrict Lie n-algebras are readily constructed as NQ-manifolds.

N-manifolds, NQ-manifold

- $\mathbb{N}$-graded manifold with coordinates of degree $0, 1, 2, \dots$



$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \dots$$

manifold

linear spaces

- Morphisms $\phi : M \to N$ are maps $\phi^* : C^\infty(N) \to C^\infty(M)$
- NQ-manifold: vector field Q of degree 1, $Q^2 = 0$
- Physicists: think ghost numbers, BRST charge, SFT

Examples:

- Tangent algebroid $T[1]M$, $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$, $Q = \mathrm{d}$
- Lie algebra $\mathfrak{g}[1]$, coordinates ξ^a of degree 1:

$$Q = -\tfrac{1}{2} f_{ab}^c \xi^a \xi^b \frac{\partial}{\partial \xi^c}$$

Condition $Q^2 = 0$ is equivalent to Jacobi identity for f_{ab}^c

L_∞-Algebras, Lie 2-Algebras

NQ-manifolds provide an easy definition of L_∞-algebras.

Lie n-algebroid or n-term L_∞-algebroid:

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Lie n-algebra, n-term L_∞-algebra or Lie n-algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Example: Lie 2-algebra as 2-term L_∞-algebra

- NQ-manifold: $* \leftarrow W[1] \leftarrow V[2] \leftarrow * \leftarrow \ldots$, coords. w^a, v^i
- Homological vector field:

$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \tfrac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \tfrac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i}$$

- Structure constants: higher products μ_i on $W \leftarrow V[1]$

$$\mu_1(\tau_i) = m_i^a \tau_a\ , \quad \mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c\ , \quad \ldots\ , \quad \mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$$

- $Q^2 = 0$: Higher or homotopy Jacobi identity, e.g.

$$\mu_2(w_1, \mu_2(w_2, w_3)) + \text{cycl.} = \mu_1(\mu_3(w_1, w_2, w_3))$$

Atiyah Algebroid Sequence

A straightforward way to describe gauge theory is in terms of parallel transport functors.

(Flat) connection: splitting of Atiyah algebroid sequence

$$0 \longrightarrow P \times_{\mathsf{G}} \mathsf{Lie}(\mathsf{G}) \longrightarrow TP/\mathsf{G} \longrightarrow TM \longrightarrow 0$$

Atiyah, 1957

Related approach: Kotov, Strobl, Schreiber, ...

- Gauge potential from morphism of N-manifolds:

$$a : T[1]M \to \mathfrak{g}[1] \quad \longrightarrow \quad A^a_\mu \mathrm{d}x^\mu := a^*(\xi^a)$$

- Curvature: failure of a to be morphism of NQ-manifold:

$$F^a := (\mathrm{d} \circ a^* - a^* \circ Q)(\xi^a) = \mathrm{d}A^a + \tfrac{1}{2} f^a_{bc} A^b \wedge A^c$$

- Infinitesimal gauge transformations: flat homotopies
- Readily categorifies, but integration an issue

Categorical Description of Principal Bundles

Descent data for principal bundles is encoded in a functor.

Čech groupoid of surjective submersion $Y \twoheadrightarrow M$, e.g. $Y = \sqcup_a U_a$:

$$\check{\mathscr{C}}(U) \;:\quad \bigsqcup_{a,b} U_{ab} \rightrightarrows \bigsqcup_a U_a\;, \quad U_{ab} \circ U_{bc} = U_{ac}\;.$$

Principal G-bundle

Transition functions are nothing but a functor $g : \check{\mathscr{C}}(U) \to (\mathsf{G} \rightrightarrows *)$

$$\begin{tikzcd}
\sqcup U_{ab} \arrow[r, "g_{ab}"] \arrow[d, shift left] \arrow[d, shift right] & \mathsf{G} \arrow[d, shift left] \arrow[d, shift right] \\
\sqcup U_a \arrow[r, "*"] & *
\end{tikzcd}$$

$$g_{ab} g_{bc} = g_{ac}$$

Equivalence relations: natural isomorphisms.

For categorification: want generalized spaces, higher Lie groups

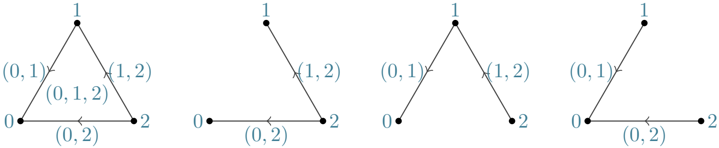
Kan simplicial sets

Kan simplicial sets form a convenient model for $(\infty, 1)$-categories.

Recall: nerve of category $\mathscr{C} = (\mathscr{C}_1 \rightrightarrows \mathscr{C}_0)$ is simplicial set

$$\left\{ \cdots \substack{\longrightarrow\\\longrightarrow\\\longrightarrow\\\longrightarrow} \mathscr{C}_1 \times^{\mathsf{s,t}}_{\mathscr{C}_0} \mathscr{C}_1 \substack{\longrightarrow\\\longrightarrow\\\longrightarrow} \mathscr{C}_1 \rightrightarrows \mathscr{C}_0 \right\}$$

- faces: source/target or compositions/dropping morphisms
- degeneracies: inject identity morphisms
- Any inner horn can be filled, outer horns for groupoids:



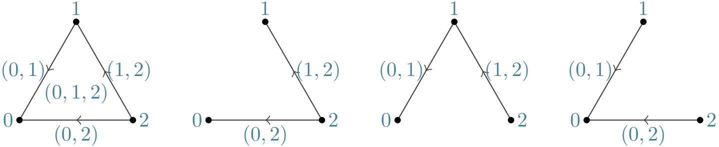
- Horn fillers are unique.
- Functors: simplicial maps
- Natural transformations: simplicial homotopies

Quasi-Categories and Quasi-Groupoids

Kan simplicial sets form a convenient model for $(\infty, 1)$-categories.

Quasi-categories, ∞-categories Boardman, Vogt, Joyal, Lurie

- Simplicial set
- Any inner horn can be filled, not necessarily uniquely



- Quasi-groupoid: all horns can be filled
- n-category/n-groupoid: k-horns with $k \geq n$ have unique fillers
- transfors much easier than in bi- or tricategories:
  - Functors: simplicial maps
  - Natural transformations: simplicial homotopies
- model for $(\infty, 1)$-categories
- readily internalize: Lie quasi-groupoids via simplicial manifolds

Higher Principal Bundles

Using Kan simplicial manifolds, we readily define higher principal bundles.

- $\check{\mathscr{C}}(\mathfrak{U} \to X)$ of open cover $\mathfrak{U}$ replaced by nerve $N(\check{\mathscr{C}}(\mathfrak{U} \to X))$:

$$\left\{ \cdots \;\substack{\longrightarrow\\[-2pt]\longrightarrow\\[-2pt]\longrightarrow\\[-2pt]\longrightarrow}\; \sqcup_{a,b,c\in A} U_a \cap U_b \cap U_c \;\substack{\longrightarrow\\[-2pt]\longrightarrow\\[-2pt]\longrightarrow}\; \sqcup_{a,b\in A} U_a \cap U_b \rightrightarrows \sqcup_{a\in A} U_a \right\}$$

- Higher Lie group: Kan simplicial manifold $\mathscr{G}$ with 1 0-simplex
- Principal bundle: simplicial map $g : N(\check{\mathscr{C}}(\mathfrak{U} \to X)) \to \mathscr{G}$
- Isomorphisms: simplicial homotopies
- Further generalization to higher spaces:
  - Motivation: orbifolds, regarded as Lie groupoids
  - Replace manifold with quasi-groupoid
  - Everything becomes bisimplicial, but works straightforwardly

Example: Ordinary Principal G-Bundle

Using Kan simplicial manifolds, we readily define higher principal bundles.

Simplicial map g from $N(\check{\mathscr{C}}(\mathfrak{U} \to X))$ to $N(\mathsf{BG})$

$$\begin{tikzcd}
\cdots & \sqcup_{a,b,c\in A} U_a \cap U_b \cap U_c \arrow[r, shift left=2] \arrow[r] \arrow[r, shift right=2] \arrow[d, "g^2_{abc}(x)"'] & \sqcup_{a,b\in A} U_a \cap U_b \arrow[r, shift left=1] \arrow[r, shift right=1] \arrow[d, "g^1_{ab}(x)"'] & \sqcup_{a\in A} U_a \arrow[d, "g^0_a(x)"'] \\
& \mathsf{G} \times \mathsf{G} \arrow[r, shift left=2] \arrow[r] \arrow[r, shift right=2] & \mathsf{G} \arrow[r, shift left=1] \arrow[r, shift right=1] & *
\end{tikzcd}$$

Compatibility with face maps:

$$g^2_{abc,1}(x) ~=~ g^1_{ab}(x)~,~~~g^2_{abc,2}(x) ~=~ g^1_{bc}(x)~,~~~g^2_{abc,1}(x) g^2_{abc,2}(x) ~=~ g^1_{ac}(x)$$

Homotopy between g, $\tilde{g}$: $h : N(\check{\mathscr{C}}(\mathfrak{U} \to X)) \times \Delta^1 \to N(\mathsf{BG})$

$$h^0((x,a),0) ~=~ * ~=~ h^0((x,a),1)~,$$

$$g_{ab}(x) ~=~ h^1((x,a,b),(0,0)) \quad \text{and} \quad \tilde{g}_{ab}(x) ~=~ h^1((x,a,b),(1,1))~,$$

$$h_{ab,01}(x) ~:=~ h^1((x,a,b),(0,1))$$

Compatibility yields here $g^1_{ab} h^1_{bb,01} ~=~ h^1_{aa,01} \tilde{g}^1_{ab}$

Towards Connections: Differentiation

There is a differentiation procedure of quasi-groupoids due to Ševera.

Recall: Connection on principal G-bundle: Lie(G)-valued 1-forms

Lie functor as suggested by Ševera, 2006

- Functors: supermanifolds to certain principal $\mathscr{G}$-bundles

$$X \mapsto (X \times \mathbb{R}^{0|1} \twoheadrightarrow X) \mapsto \text{descent data}$$

- Moduli: Lie$(\mathscr{G})$ as an n-term complex of vector spaces
- Carries Hom$(\mathbb{R}^{0|1}, \mathbb{R}^{0|1})$-action $\rightarrow$ L_∞-algebra structure

Example: Differentiation of Lie group G.

- $g : X \times \mathbb{R}^{0|2} \rightarrow \mathsf{G}$, $g(\theta_0, \theta_1, x) g(\theta_1, \theta_2, x) = g(\theta_0, \theta_2, x)$
- implies $g(\theta_0, \theta_1, x) = g(\theta_0, 0, x)(g(\theta_1, 0, x))^{-1}$
- expand trivializ. cobdry: $g(\theta_0, 0, x) = \mathbb{1} + \alpha\theta_0$, $\alpha \in \mathsf{Lie}(\mathsf{G})[1]$
- compute $g(\theta_0, \theta_1) = \mathbb{1} + \alpha(\theta_0 - \theta_1) + \frac{1}{2}[\alpha, \alpha]\theta_0\theta_1$
- $Qg(\theta_0, \theta_1, x) := \frac{\mathrm{d}}{\mathrm{d}\varepsilon} g(\theta_0 + \varepsilon, \theta_1 + \varepsilon, x)$ with $Q\alpha = -\frac{1}{2}[\alpha, \alpha]$

Finite Gauge Transformations

The differentiation method can be extended to read off finite gauge transformations.

- First approach:
    - We have: Lie algebra element in terms of descent data g
    - Perform a coboundary transformation to $\tilde{g}$
    - Trivialize $\tilde{g}$, establish relation between moduli of g, $\tilde{g}$, e.g.

$$\tilde{\alpha} = p^{-1}\alpha p + p^{-1}Qp\ , \quad p \in C^\infty(X, \mathsf{G})$$

    - Replace Q by de Rham differential on patches.

B Jurco, CS, M Wolf, 1403.7185

- Better approach:
    - Recall: functions on $T[1]X$ yield de Rham complex
    - Replace $X \times \mathbb{R}^{0|1} \twoheadrightarrow X$ by $T[1]X \times \mathbb{R}^{0|1} \twoheadrightarrow T[1]X$
    - Replace Q by $Q + \mathrm{d}_X$
    - Yields higher connections and their finite gauge trafos

B Jurco, CS, M Wolf, 1604.01639

All this is quite powerful...

We readily define Deligne cohomology for semistrict Lie 2-group bundles.

Example: principal $\mathscr{G}$-bundle with $\mathscr{G}$ semistrict Lie 2-group:

Cocycle data: $(m_{ab}, n_{abc}, A_a, \Lambda_{ab}, B_a)$. Cocycle relations:

$$
\begin{aligned}
n_{abc} &: m_{ab} \otimes m_{bc} \Rightarrow m_{ac} \\
n_{acd} \circ (n_{abc} \otimes \mathrm{id}_{m_{cd}}) \circ \mathsf{a}^{-1}_{m_{ab},m_{bc},m_{cd}} &= n_{abd} \circ (\mathrm{id}_{m_{ab}} \otimes n_{bcd}) \\
\mathrm{d}A_a + A_a \otimes A_a + \mathsf{s}(B_a) &= 0 \\
\Lambda_{ab} &: A_b \otimes m_{ab} \Rightarrow m_{ab} \otimes A_a - \mathrm{d}m_{ab}
\end{aligned}
$$

$$
\begin{aligned}
&\Lambda_{cb} \circ (\mathrm{id}_{A_b} \otimes n_{bac}) \circ \mathsf{a}_{A_b,m_{ba},m_{ac}} = \\
&\quad = (n_{bac} \otimes \mathrm{id}_{A_c} - \mathrm{d}n_{bac}) \circ \left[\mathsf{a}^{-1}_{m_{ba},m_{ac},A_c} - \mathrm{id}_{\mathrm{d}(m_{ba} \otimes m_{ac})}\right] \circ \\
&\qquad \circ (\mathrm{id}_{m_{ba}} \otimes \Lambda_{ca} - \mathrm{id}_{\mathrm{d}m_{ba} \otimes m_{ac}}) \circ (\mathsf{a}_{m_{ba},A_c,m_{ac}} - \mathrm{id}_{\mathrm{d}m_{ba} \otimes m_{ac}}) \circ (\Lambda_{ab} \otimes \mathrm{id}_{m_{ac}})
\end{aligned}
$$

$$
\begin{aligned}
B_b \otimes \mathrm{id}_{m_{ab}} = \mu(A_b, A_b, m_{ab}) + \big[\mathrm{id}_{m_{ab}} \otimes B_a + \mu(m_{ab}, A_a, A_a)\big] &\circ \\
\circ \big[-\mathrm{d}\Lambda_{ab} - \Lambda_{ab} \otimes \mathrm{id}_{A_a} - \mu(A_b, m_{ab}, A_a)\big] &\circ \\
\circ \big[-\mathrm{id}_{\mathsf{s}(\mathrm{d}\Lambda_{ab})} - \mathrm{id}_{A_b} \otimes (\Lambda_{ab} + \mathrm{id}_{\mathrm{d}m_{ab}})\big]&
\end{aligned}
$$

We can now start to calculate and look for applications.

Important Problem: Fake Curvature Constraints

One additional equation of motion causes various problems.

- In gauge transformations, one encounters fake curvatures:

$$\mathcal{F} := \mathrm{d}A + \tfrac{1}{2}[A, A] - \mathsf{t}(B) = 0$$

"Lower curvatures determined by higher potentials"

✓ Renders parallel transport reparameterization-invariant

✓ In (2,0)-theory: eliminates unwanted freedom in fields

✗ Obstacle to non-trivial examples of higher bundles:

  - E.g. String 2-group, model by Schommer-Pries, 2009: Roughly: $\mathsf{U}(1) \times \mathsf{G} \rightrightarrows \mathsf{G}$, fallback to abelian case
  - The same for all obvious examples of Lie 2-groups
  - Obvious lift of 't Hooft-Polyakov monopoles: $\mathcal{F} \neq 0$

Any ideas?

Applications and Open Issues

There are many applications of our framework and important open problems.

Applications of very general description of higher gauge theory:

- Many existing models are higher gauge theories (HGTs)
  - Tensor hierarchy models are HGTs S Palmer&CS, 1308.2622
  - M2-brane models are HGTs S Palmer&CS, 1311.1997
- Twistor constructions of (2,0)-theories/M5-brane models:
  - Input: (Higher) spacetime, twistor space, higher gauge group
  - Output: (2,0)-model via Penrose-Ward transform
- Higher versions of monopole equations: self-dual strings

Open problems:

- Find “good” higher gauge groups (String groups?)
- Clarify meaning of fake curvature conditions
- Study more general spaces: orbifolds, ...
- Find truly non-abelian higher principal bundle + connection
- Construct a higher version of ’t Hooft-Polyakov monopole
- Find $(2,0)$-theory

Review: The 't Hooft-Polyakov Monopole

The 't Hooft-Polyakov Monopole is a non-singular solution with charge 1.

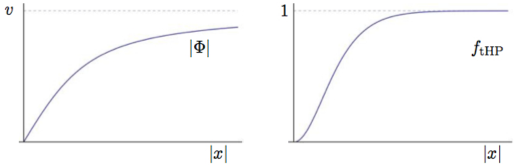
Recall 't Hooft-Polyakov monopole (e_i generate $\mathfrak{su}(2)$, $\xi = v|x|$):

$$\Phi = \frac{e_i x^i}{|x|^2}\big(\xi\coth(\xi) - 1\big)\ , \quad A = \varepsilon_{ijk}\frac{e_i x^j}{|x|^2}\left(1 - \frac{\xi}{\sinh(\xi)}\right)\ \mathrm{d}x^k$$

- At S^2_∞: $\Phi \sim g(\theta)e_3 g(\theta)^{-1}$.
  $g(\theta) : S^2_\infty \to \mathsf{SU}(2)/\mathsf{U}(1)$: winding 1
- Charge $q = 1$ with

$$2\pi q = \tfrac{1}{2}\int_{S^2_\infty} \frac{\mathrm{tr}\left(F^\dagger\Phi\right)}{||\Phi||} \quad \text{with} \quad ||\Phi|| := \sqrt{\tfrac{1}{2}\,\mathrm{tr}\left(\Phi^\dagger\Phi\right)}$$

- Higgs field non-singular:



Elementary Solutions: A Non-Abelian Self-Dual String

We can write down a non-abelian self-dual string with winding number 1.

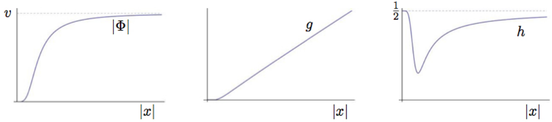
Self-Dual String (Lie 2-algebra $\mathfrak{su}(2)\times\mathfrak{su}(2)\xleftarrow{\mu_1}\mathbb{R}^4$, $\xi = v|x|^2$):

$$\Phi = \frac{e_\mu x^\mu}{|x|^3}\, f(\xi)\ ,\quad B_{\mu\nu} = \varepsilon_{\mu\nu\kappa\lambda}\frac{e_\kappa x^\lambda}{|x|^3}\, g(\xi)\ ,\quad A_\mu = \varepsilon_{\mu\nu\kappa\lambda} D(e_\nu, e_\kappa)\ \frac{x^\lambda}{|x|^2}\ h(\xi)$$

- Solves indeed $H = \star\nabla\Phi$ for right $f(\xi)$, $g(\xi)$, $h(\xi)$
- At S^∞_3: $\Phi \sim g(\theta) \rhd e_4$. $g(\theta) : S^3_\infty \to \mathsf{SU}(2)$ has winding 1.
- Charge $q = 1$:

$$(2\pi)^3 q = \tfrac{1}{2}\int_{S^3_\infty}\frac{(H,\Phi)}{||\Phi||}\quad \text{with}\quad ||\Phi|| := \sqrt{\tfrac{1}{2}(\Phi,\Phi)}\ ,$$

- Higgs field non-singular:



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Categories, Logic, and Physics, Scotland, 14.4.2016