Effect of Choice Set on Valuation of Risky Prospects

Neil Stewart (neil.stewart@warwick.ac.uk)
Nick Chater (nick.chater@warwick.ac.uk)
Henry P. Stott (hstott@owc.com)
Department of Psychology, University of Warwick
Coventry, CV1 3GA, UK

Abstract

Current models of decision making under risk assume access to the absolute magnitudes of gamble attributes. The two experiments presented here provide evidence that decisions under risk are based, in addition, on the context of the decision. In Experiment 1 the set of options offered as certainty equivalents was shown to determine the value of simple gambles of the form "*p* chance of £*x*". Experiment 2 employed a novel procedure where the payment structure was such that it was optimal for participants to provide truthful certainty equivalents. Again, the context provided by the set of certainty equivalents influenced the choice of certainty equivalent.

Many existing theories of decision making under risk (e.g., Kahneman & Tversky, 1979; Quiggin, 1982; Tversky & Kahneman, 1992; von Neumann & Morgenstern, 1947) predict that participants use the absolute magnitudes of value and probability in making risky decisions, or that some monotonic transform of these attributes is used. For example, in expected utility theory (von Neumann & Morgenstern, 1947), the utility of an outcome is a negatively accelerated function of value, and outcomes which maximize the expected utility are preferred. The aim of the experiments described here was to investigate to what extent contextual factors also influence decision under risk.

Mellers, Ordóñez and Birnbaum (1992) measured the attractiveness ratings and buying prices of simple binary gambles presented in two different contexts. In one context, the distribution of expected values of accompanying gambles was positively skewed, and in the other context, the expected values were negatively skewed. Attractiveness ratings were influenced by context. However, for simple gambles of the form "*p* chance of £*x*" context had a minimal effect on buying price. With more complicated gambles of the form "*p* chance of £*x* otherwise £*y*", the effect was slightly larger. The effect of context on attractiveness and the lack of an effect of context on buying price is consistent with a similar demonstration by Janiszewski and Lichtenstein (1999), and consistent with a review of previous research by Poulton (1982).

However, context does affect choice of certainty equivalents (hereafter, CEs) in other conditions. (CEs are the amount of money that can be obtained for certain, participants feel is equivalent to a given gamble). Birnbaum (1992) demonstrated that skewing the distribution of CEs offered for simple gambles, whilst holding the range constant, influenced the selection of a CE. When the CE options were positively skewed (i.e., more small values) gambles were over-valued compared to the negatively skewed context, consistent with range-frequency theory (Parducci, 1965; 1974).

The aim of Experiment 1 was to demonstrate that the options offered as potential CEs influence estimates of a gamble's CE. Following a similar logic to a loudness judgment experiment by Garner (1954), participants were given a set of potential CEs for each gamble, and asked to choose the option closest to their estimate of the CE for each gamble. For each gamble, CE options were either all lower in value than the free choice CE (given by another group of participants) or all higher. If participants were not influenced by context, then their choices of CE should be highly skewed towards the mean free choice CE. If participants' responses are solely determined by context, then the distribution of responses across options should be the same for both the low and high value range of CEs. Experiment 2 introduces a new procedure to investigate these context effects in which it is optimal for participants to provide truthful CEs.

Experiment 1

The curve in Figure 1 represents a hypothetical normal distribution of CEs given under free choice conditions. If participants are not affected by the context provided by the range of CEs offered in the restricted choice conditions, it is possible to predict their distribution of responses. For an option at the lower end of the range, the probability of selecting that option is the integral of the free choice function between -∞ and a point half way between the lowest two options. Similarly for an option at the higher end of the range, the probability is the integral between a point half way between the highest two options and +∞. For an option of intermediate value, the probability is the integral between the bound half way between the intermediate option and the next lowest option, and bound half way between that intermediate option and the next highest option. In other words, it is assumed

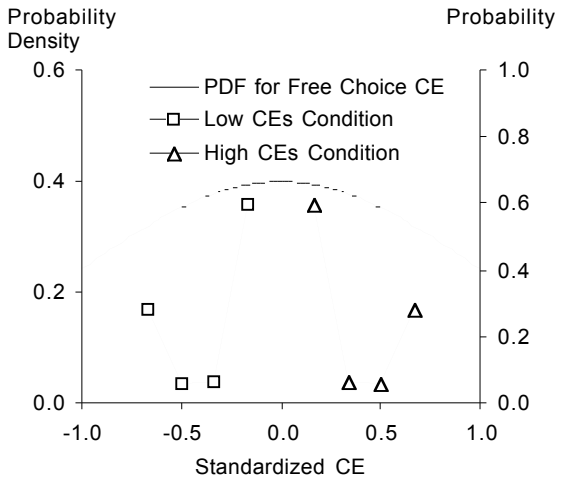


Figure 1: Predicted choices in Stage 2, under the assumption that context will not affect CE choice.

participants choose the option nearest to the CE they would have chosen under free choice conditions.

For the options used in Experiment 1 (see the Design for a description), the two lines in Figure 1 represent the expected distribution of responses. If participants are *not* influenced by the context provided by the four CE options then the key prediction is that *participants in the high CEs condition should choose the lowest option more than half of the time*, and *participants in the low CEs condition should choose the highest option more than half of the time*. This prediction holds for any symmetrical distribution of free choice CEs.

Method

Participants 30 psychology undergraduates from the University of Warwick participated for course credit. 14 took part in Stage 1 of the experiment. 16 took part in Stage 2.

Design Stage 1 was designed to measure participants' free choice CEs for a series of simple gambles. A set of 20 gambles was created by crossing the amounts £200, £400, £600, £800 and £1000 with the probabilities 0.2, 0.4, 0.6 and 0.8. In Stage 2 different participants were presented with the same gambles, and asked to select from a set of four CEs the CE that was closest to their judgment of the value of the gamble. For each gamble two sets of CEs were created. In the low CEs condition participants received options all lower than the mean free choice CE given in Stage 1. In the high CEs condition participants received CEs that were all higher. The CE sets were constructed as follows. The mean and standard deviation of the free choice CEs was calculated for each gamble. The two sets of equally spaced CEs (for the high value and low value conditions) were calculated so that their range was equal to approximately half the free choice standard deviation. One set of CEs was placed below the free choice value, and the other set above. This difference between the lowest CE in the high CEs condition (or the highest CE of the low CEs condition) and the mean free choice CEs was set to be roughly equal to the difference between CEs within a condition. Options were rounded to be familiar, easy to deal with values.

Procedure Participants were given written instructions. They were asked to imagine choosing between "£30 or a 50% chance of £100" to illustrate that gambles could have a value. They were told they would be asked to value a series of gambles, and that they should imagine they had the chance to play the gamble. For each gamble they were asked how much money for certain they thought it was fair for someone to give them for the other person to have a chance to take the gamble instead. They were also asked to consider the opposite situation, where they would be buying the gamble. It was explained that the purpose of the experiment was to investigate how much they thought the gambles were worth, and that there was no correct answer. For participants in Stage 2, it was explained that they should choose the CE option nearest the value they thought the gamble was worth.

Each gamble was presented on a separate page of a 20 page booklet. For participants in Stage 1 gambles were presented as follows:

For you, how much is the gamble
"80% chance of £600"
worth?

£____

Probabilities were always presented as percentages. For participants in Stage 2, a set of options was added. Options were always presented in numerical order, as with the following example of a low CE set:

How much is the gamble
"60% chance of £400"
worth?

Is it: £60 £80 £100 £120

Results

Participants took approximately 5 minutes to complete the task. Figure 2 plots the Stage 1 free choice CE against the gamble amount for the four gamble probabilities. As expected, the average CE increased with both probability of winning and gamble amount demonstrating that participants were sensitive to manipulations of both. The chosen CE was an approximately linear function of the independent effects of gamble amount and gamble probability. Participants were risk averse, with the mean CE being, on average,

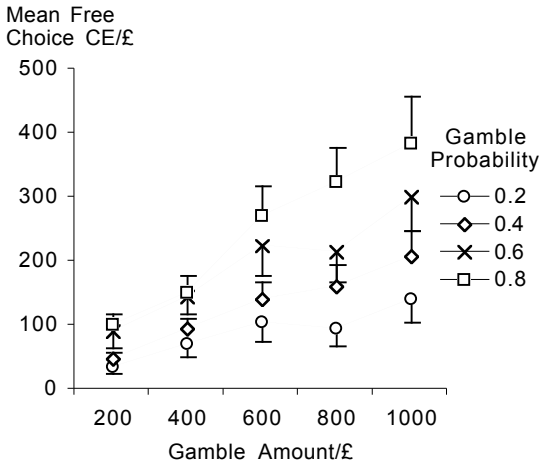


Figure 2: Mean free choice CE as a function of gamble amount for different gamble probabilities for Stage 1 of Experiment 1. (Error bars are standard error of the mean.)

61% of the expected value of the gamble (standard error 3%).

The results of Stage 2 are of most interest. Responses are labeled A through D, with A being the lowest CE, and D being the highest CE. The proportion of times each response type was chosen is plotted in Figure 3. There is no evidence of skewing – instead the distribution of options is approximately the same for the two conditions. The highest CE in the low CEs condition was chosen less than half the time $t(7)=4.21$, $p<0.05$. The same was true of the lowest CE in the high CE condition, $t(7)=5.26$, $p<0.05$.

Discussion

Participants were asked to value a simple gamble of the form "p chance of £x". The effects of p and x on the CE were linear and independent, consistent with Birnbaum's (1992) data. Mean value judgments were below the expected value (£px) of the gamble showing participants were risk averse. Different participants were given a restricted set of CEs for each gamble, either all lower than the mean free choice CE for every gamble or all higher. CE judgments were completely determined by the range of CEs offered, and were not skewed towards the mean free choice CE. Control conditions, not reported here, rule out task demand characteristics as a potential account of these findings, as participants were happy to give highly skewed responses in these conditions. Further, only one of the 16 participants from the second stage reported they were unhappy with the restricted range of CEs offered.

Experiment 2

Experiment 2 was designed to demonstrate the same effect of restricting the range of CEs in a task where it

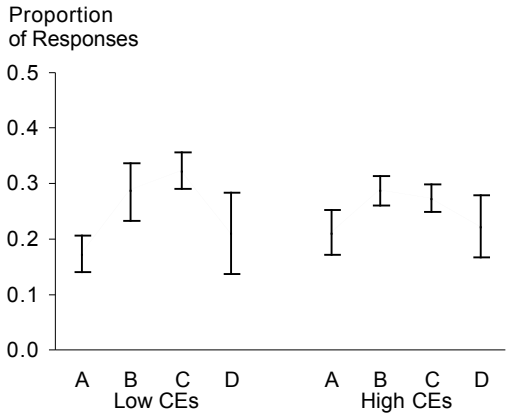


Figure 3: Proportion of responses to each CE for Stage 2 of Experiment 1. (Error bars are standard error of the mean.)

was optimal for participants to report CEs truthfully. This procedure is simpler than other methods used to elicit truthful CEs (e.g., the first price auction, or the Becker, DeGroot & Marschak, 1964, procedure). Specifically, participants divide a given amount into an amount for certain, and an amount to be won with a certain given chance. For example, they might split £1000 into a sure amount of £300 and a "60% chance of £700" gamble. Participants know that the experimenter will select either the gamble or the sure amount, taking the "better" of the two, leaving the participant with the other. Thus it is optimal for participants to split the given amount so that the resulting fixed amount has the same utility as the resulting gamble.

Method

Participants 17 participants took part in the Stage 1 of the experiment. 19 different participants took part in the Stage 2. All participants were paid £4 plus performance related winnings of up to £4.

Design In each trial in Stage 1 a participant divided a given amount of money, £x, into two smaller amounts, £y and £z, to make one fixed amount (£y) and a gamble. There is a given probability p of winning £z, otherwise nothing. Probability p is known to participants before splitting amount £x. Participants know that (if the trial is selected at random at the end of the experiment) the experimenter will take either the fixed amount or the gamble for themselves leaving the participant with the other. It is therefore optimal for the participant to split the amount £x into amounts £y and £z such that £y and a p chance of £z have equal utility for them, i.e., £y is the CE for the gamble "p chance of £z". Under the assumption that the experimenter has the same utility function as them, participants understand that the experimenter will choose the gamble with greater

utility, leaving them with less, if they do not split the amounts in this way.

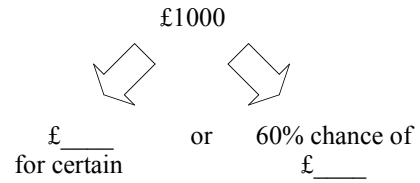
Stage 2 of the experiment differed by offering participants a choice from a set of four pre-split options, rather than giving them a completely free choice. That is, values for £*y* and £*z* were presented, and participants selected one pair which could be played at the end of the experiment. As in Stage 1, participants knew that the experimenter would choose the option from the chosen pair with the greatest utility, and therefore they should choose the pair of options closest in expected utility.

It was hypothesized that the pairs of values for *y* and *z* presented in Stage 2 would influence participants' choices, and that participants would therefore not just choose the optimal pair. To demonstrate this there was one between participants factor. The set of values for £*y* and £*z* were either selected such that £*y* was always greater than the free choice value of £*y* from Stage 1 (for equal £*x*) and £*z* smaller than the free choice value, or vice versa. The option sets were constructed as follows. The mean and standard deviation of the free choice amount was calculated for each gamble. The two sets of equally spaced options (for the high value and low value conditions) were calculated as described for Experiment 1. As in Experiment 1, if participants are not influenced by the set of choices, then the distribution of responses across the options should be skewed towards the free choice splitting.

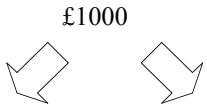
Procedure For both stages the experiment began with instructions. It was explained to participants that they were playing a gambling game, and that they should try to win as much money as possible. They were told the purpose of the experiment was to investigate how much people thought gambles were worth. The task was described. It was emphasized that it was optimal for them to split the money so they thought the amount for certain was equal in worth to a chance on the gamble. They were told that if they allocated funds so either the certain amount was worth more than the gamble, or vice versa, then the experimenter would take the better one, leaving them with less than if they had allocated the money so the gamble was worth the certain amount. They were told that although they could not be certain what the experimenter would do, they should assume the experimenter would behave like them.

Participants were given five practice trials to complete. One of the trials was chosen at random, and it was explained that if the experimenter chose the fixed amount, then the gamble would be played, and they would get the winnings. They were also told that if instead the experimenter took the gamble they would get the fixed amount. Note that this discussion was hypothetical, and participants were not actually told what the experimenter's preference would be.

After the practice the experiment began. The participant completed a booklet of gambles. Gambles were presented in a random order to each participant. An example page from a free choice condition booklet is shown below.



In the restricted choice conditions, pre-split options were presented as in the example below.



			Tick
£322 for certain	or	60% chance of £678	
£334 for certain	or	60% chance of £666	
£346 for certain	or	60% chance of £654	
£358 for certain	or	60% chance of £642	

When the experiment was completed one trial was chosen at random, and played to determine each participant's bonus using an exchange rate.

Results

Participants took between half an hour and one hour to complete the booklet. One participant was eliminated from subsequent analysis for showing a completely different pattern of results to other participants, suggesting they had misunderstood the task. The participant had decreased the value of the fixed amount, £*y*, as the chance of the gamble amount, *p*, increased (i.e., they responded as if more likely gambles were worth less to them). 14 out of the remaining 512 trials (16 participants x 32 trials) with incorrect arithmetic were deleted, and treated as missing data.

Figure 4 plots the average fixed amount £*y* as a function of the gamble chance *p* for the different total amounts (£*x*). As the total amount £*x* increased, then participants' allocation of the fixed amount £*y* increased. As the probability *p* of winning the gamble increased participants' estimates of the value of the gamble, £*y*, also increased. Thus participants' responses seem lawful and sensible. These two effects are approximately independent. The dashed lines in Figure 4 represent risk neutral responding. Data points falling above the dashed line demonstrate risk averse behavior. On average, participants were risk averse for low gamble chances (p=0.2), risk neutral for intermediate gamble chances (p=0.6) and slightly risk prone for high gamble chances (p=0.8). However, standard deviations were approximately 15% of the mean fixed amount allocated and thus for larger gamble chances

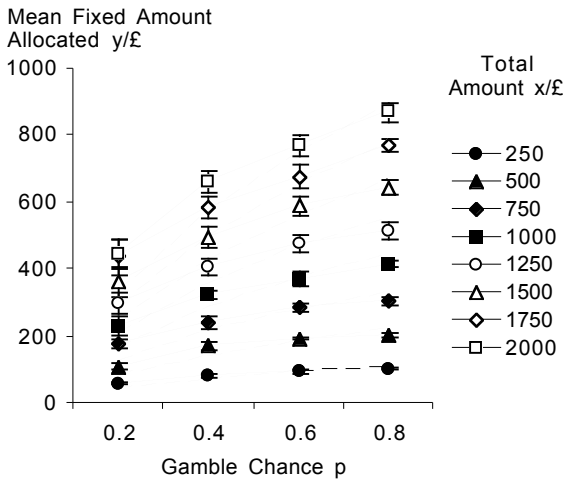


Figure 4: Mean fixed amount allocated in Stage 1 of Experiment 2 as a function of gamble chance for the different total amounts. (Error bars are standard error of the mean.)

approximately half of responses were risk prone, and half risk averse.

The choices made in Stage 2 are shown in Figure 5. Participants did prefer end options over central options in both conditions, consistent with the pattern of results expected if participants were to show no context effect. However, option D in the low *y* condition was chosen significantly less than half the time, $t(9)=3.47$, $p<0.05$. Similarly, option A in the high *y* condition was chosen significantly less than half the time, $t(8)=4.20$, $p<0.05$. This observation is consistent with participants showing some context effect. In other words, the proportion of times each option was selected differed significantly from the proportions expected under the assumption that context would *not* have an effect.

Discussion

The new procedure for eliciting CEs under free choice provides results consistent with Experiment 1. In Experiment 1 participants were, on average, risk averse. Under free choice conditions in Experiment 2 participants were only risk averse for low probability gambles, and were slightly risk prone for high probability gambles. This pattern is the opposite of Tversky and Kahneman's (1986, Problems 10 and 11). Risk averse responding for low probabilities, and risk prone responding for high probabilities is, however, consistent with over estimation of low probabilities and underestimation of high probabilities (e.g., Prelec, 1998; Tversky & Kahneman, 1992; Wu & Gonzalez, 1996). (Consider the case for $p=0.2$. If this probability is overestimated, say at 0.3, then the risk neutral strategy is to increase £*y* and decrease £*z*, as the gamble *p* chance of £*z* will be overvalued.)

The results of the restricted choice conditions in Experiment 2 replicate those shown in Experiment 1

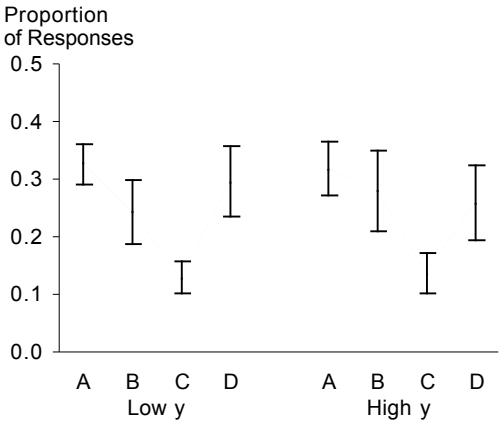


Figure 5: Proportion of each response choice for Stage 2 of Experiment 2. (Error bars are standard error of the mean.)

under a more rigorous procedure, despite participants taking at least six times longer to complete the task compared to Experiment 1. When participants were presented with a range of pre-split total amounts, so that the CE options were either always lower or always higher than the free choice value, their choice of CE was *not* skewed towards the mean free choice CE. The context provided by the pre-split options influenced their choice of CE.

General Discussion

In the experiments presented here, participants' choice of CE for simple gambles was affected by the range of option CEs offered to them, compared to CEs given by different participants under free choice conditions. For example, when the option CEs were all lower than the free choice CE, participants behaved as if their CE was lower.

Judged and Choice Certainty Equivalents

Careful discussion by Luce (2000) highlights the a difference between judged CEs, where participants provide a single judgement of the value of a gamble, and choice CEs, derived from a series of choices between gambles and fixed amounts. For example, for the kinds of gambles used here, with large amounts and moderate probabilities, judged CEs are overvalued compared to choice CEs (e.g., Bostic, Herrnstein & Luce, 1990). Luce (2000) advocates developing separate theories to for judged and choice certainty equivalents. Participants in these experiments were instructed to complete the restricted CE conditions by judging CEs. However, in Experiment 2, the design should certainly have encouraged imagining choices between gambles and fixed amounts. The degree to which this was the case may explain the 'u' shaped

pattern of results in Experiment 2, rather than the 'n' shaped pattern in Experiment 1.

Other Context Effects

Here the context *provided by a set of certainty equivalent options* has been found to influence the CE. In other experiments, the context *provided by a set of gambles* has been shown to influence preferences amongst those gambles. For example, Simonson and Tversky (1992) demonstrated that there was a tendency to prefer a given option when there are other options in the choice set that are unfavorable when compared to the given option. Specifically, in making risky choices between three three-outcome gambles, a gamble was preferred if it dominated another gamble in the choice set.

Implications

Existing models of decision making under risk typically assume that only the attributes of the gamble need be considered when reaching a CE decision. The context or anchoring effects demonstrated here show that the context also needs to be considered. The extent to which context can cause deviation from 'rational choice' has implications for other domains, such as economics and political science, where 'rational choice' models of the individual are applied (e.g., expected utility theory and game theory).

Relation to Perception

The demonstration of context effects in risky decision making suggests that the representation of the utility dimension is similar to that for perceptual psychological dimensions where context effects have also been demonstrated. For example, Garner (1954) showed that participants were completely unable to determine which of six tones was more or less than half as loud as a reference loudness. Instead, participants' judgments were entirely influenced by the range of the six tones. (Laming, 1997, provides an extensive discussion of other similar findings.) Further research is underway in this laboratory to investigate to what extent the context provided by simultaneously presented gambles (see also Mellers et al., 1992) and the context provided by recently considered gambles affects the utility of gambles. This research should help to establish whether utility really is like perceptual dimensions.

Acknowledgments

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References

Becker, G. M., DeGroot, M. H., & Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral Science, 9,* 226-232.

Birnbaum, M. H. (1992). Violations of monotonicity and contextual effects in choice-based certainty equivalents. *Psychological Science, 3,* 310-314.

Bostic, R., Herrnstein, R. J., & Luce, R. D. (1990). The effect of preference-reversal phenomenon of using choice indifferences. *Journal of Economic Behavior and Organization, 13,* 193-212.

Garner, W. R. (1954). Context effects and the validity of loudness scales. *Journal of Experimental Psychology, 48,* 218-224.

Janiszewski, C., & Lichtenstein, D. R. (1999). A range theory account of price perception. *Journal of Consumer Research, 25,* 353-368.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47,* 263-291.

Laming, D. (1997). *The measurement of sensation.* London: Oxford University Press.

Luce, R. D. (2000). *Utility of gains and losses: Measurement-theoretical and experimental approaches*. Mahwah, NJ: Erlbaum.

Mellers, B. A., Ordóñez, L. D., & Birnbaum, M. H. (1992). A change-of-process theory for contextual effects and preference reversals in risky decision making. *Organizational Behavior and Human Decision Processes, 52,* 311-369.

Parducci, A. (1965). Category judgment: A range-frequency theory. *Psychological Review, 72,* 407-418.

Parducci, A. (1974). Contextual effects: A range-frequency analysis. In L. Carterette and M. P. Friedman (Eds.), *Handbook of Perception* (Vol. II). New York: Academic Press.

Poulton, E. C. (1989). *Bias in quantifying judgments*. Hillsdale, NJ: Erlbaum.

Prelec, D. (1998). The probability weighting function. *Econometrica, 66,* 497-527.

Quiggin, J. (1993). *Generalized expected utility theory: The rank-dependent model.* Boston: Kluwer Academic.

Simonson, I., & Tversky, A. (1992). Choice in context: Tradeoff contrast and extremeness aversion. *Journal of Marketing Research, 29,* 281-295.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5,* 204-217.

von Neumann, M., & Morgenstern, O. (1947). *Theory of games and economic behavior.* (2nd ed.). Princeton, NJ: Princeton University Press.

Wu, G., & Gonzelez, R. (1996). Curvature of the probability weighting function. *Management Science, 42, 1676-1690.*