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The model category of algebraically cofibrant 2-categories

As I discussed at CT2017 (see also [5]), a basic obstruction to the development of a purely **Gray**-enriched model for three-dimensional category theory is the fact that not every 2-category is cofibrant in Lack's model structure on **2-Cat** [4]. This obstruction can be overcome by the introduction of a new base for enrichment: the monoidal model category $\mathbf{2\text{-}Cat}_Q$ of *algebraically cofibrant 2-categories*, which is the subject of this talk.

This category $\mathbf{2\text{-}Cat}_Q$ can be defined as the category of coalgebras for the normal pseudofunctor classifier comonad on **2-Cat**, and is thus a non-full replete subcategory of **2-Cat** whose objects are the cofibrant 2-categories. (It can also be defined as the evident 2-categorical analogue of the category of simplicial computads studied by Riehl and Verity [6].) Using modern model category techniques [3, 2], I will show that the category $\mathbf{2\text{-}Cat}_Q$ admits an "injective" model structure, left-induced from (and Quillen equivalent to) Lack's model structure on **2-Cat** along the left-adjoint inclusion $\mathbf{2\text{-}Cat}_Q \longrightarrow \mathbf{2\text{-}Cat}$.

Remarkably, the category of bicategories and normal pseudofunctors is equivalent, via the normal strictification functor, to the full subcategory of $\mathbf{2\text{-}Cat}_Q$ consisting of the fibrant objects for the induced model structure. Moreover, like Lack's model structure on **2-Cat**, the induced model structure on $\mathbf{2\text{-}Cat}_Q$ is monoidal with respect to the (symmetric) Gray tensor product, but unlike Lack's model structure, the induced model structure is also cartesian.

Note that the word "normal" in the above definition of $\mathbf{2\text{-}Cat}_Q$ is crucial to these results: a simple argument shows that the category of coalgebras for the (non-normal) pseudofunctor classifier comonad on **2-Cat** fails the acyclicity condition, and therefore does not admit a model structure left-induced from Lack's model structure on **2-Cat**. This disproves a conjecture posed by Ching and Riehl [1].

References:

[1] Michael Ching and Emily Riehl. Coalgebraic models for combinatorial model categories. *Homology Homotopy Appl.*, 16(2):171–184, 2014.

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[5] Stephen Lack and Jiří Rosický. Homotopy locally presentable enriched categories. *Theory Appl. Categ.*, 31(25):712–752, 2016.

[6] Emily Riehl and Dominic Verity. The comprehension construction. *Higher Structures*, 2(1):116–190, 2018.