The generalized Homotopy Hypothesis

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Abstract

The homotopy hypothesis roughly states that weak n-groupoids are algebraic models for homotopy n-types. In this talk I will introduce Grothendieck (weak) n-groupoids for $0 \leq n \leq \infty$ and describe how to obtain a homotopy theory for these objects based on a notion of homotopy groups. I will then discuss the main obstruction to a proof of the homotopy hypothesis, which essentially lies in the validity of a technical lemma about the invariance of the homotopy type of a given n-groupoid after having attached a new cell to it along its source. Next, I will introduce truncated and coskeletal models and prove several equivalences between ∞-categories of such objects, which will culminate with the result that if a left semi-model structure for n-groupoids (called the *canonical* one) exists, then the generalized homotopy hypothesis is valid. If time permits, I will spend some word on a possible strategy to tackle this problem, and the way our main result was proven.

This is joint work with S. Henry.

References

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