Tangent Categories from the Coalgebras of Differential Categories

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Differential categories [1] were introduced to provide categorical models of differential linear logic and study the algebraic foundations of differentiation. Following the pattern from linear logic, the coKleisli category of a differential category is well studied: it is a Cartesian differential category [2], whose differential structure formalize the directional derivative and provides the semantics of the differential λ-calculus. What then is the coEilenberg-Moore category of a differential category? The answer, which is the subject of this talk, is that it is a tangent category! Briefly, a tangent category [3, 4] is a category equipped with an endofunctor that formalizes the basic properties of the tangent bundle functor on the category of smooth manifolds. In this talk, we will explain how under a mild limit assumption (and thanks to an adjoint existence theorem of Butler's [5]) the coEilenberg-Moore category of a differential category is a tangent category whose tangent bundle functor is a right adjoint to a sort of infinitesimal extension on coalgebras. Key examples of such tangent categories include the opposite category of commutative rings and the opposite category of $\mathcal{C}^\infty$-rings, whose tangent category structures respectfully capture fundamental aspects of algebraic geometry and synthetic differential geometry.

This work extends on previous work by Lucyshyn-Wright [6, 7].

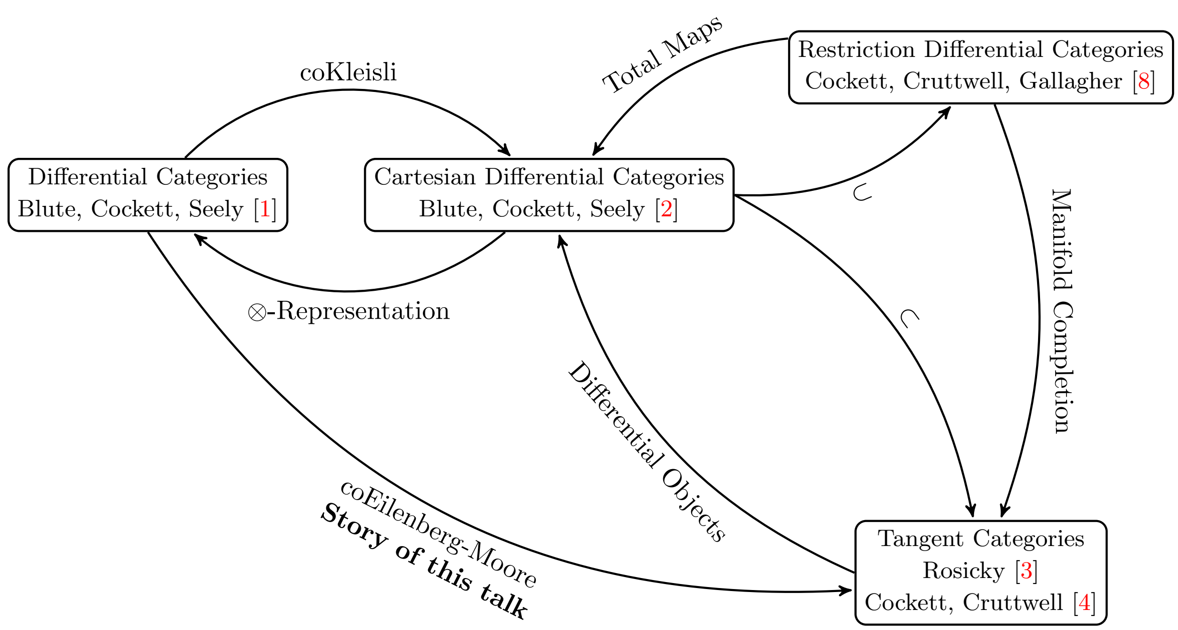

Figure 1: The world of differential categories and how it's all connected.

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