Double quandle coverings

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Quandles were introduced in D. Joyce's PhD thesis [5] and capture the algebraic theory of group conjugation with applications in geometry (as the intrinsic structure of symmetric spaces) and knot theory (as a complete invariant for oriented knots).

The category Qnd of quandles admits the category Set of sets as a subvariety. The left adjoint $\pi_0 \colon \mathsf{Qnd} \to \mathsf{Set}$ of the inclusion functor $\mathsf{Set} \to \mathsf{Qnd}$ sends a quandle to its set of connected components. This adjunction was shown to be admissible (in the sense of [4]) by V. Even in [3] where he showed that central extensions defined through Galois Theory [4] coincide with quandle coverings defined by M. Eisermann in [2]. Such coverings form a reflective subcategory of the category of surjective homomorphisms of quandles [1].

We show that this adjunction is in turn admissible, and this gives rise to a notion of double central extension of quandles for which we provide an algebraic characterisation. Both centrality conditions in dimension 1 and 2 may be expressed in terms of a new notion of commutator defined for quandle congruences.

These results provide new tools to study quandles, as well as a new context of application for higher Galois theory.

References

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