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*A type theory for cartesian closed bicategories*

I will introduce an internal language for cartesian closed bicategories [2], explaining its underlying design principles. Firstly, I will begin with a type theory for bicategories synthesised from a bicategorification of the notion of abstract clone from universal algebra. The result is a 2-dimensional type theory (in the style of Hilken [3]) with a form of explicit substitution capturing an ‘up to isomorphism’ composition operation. Next, I shall show how semantic considerations give rise to the addition of product and exponential type structures. The resulting type theory generalises the Simply-Typed Lambda Calculus and its syntactic models satisfy a suitable 2-dimensional freeness universal property, thereby lifting the Curry-Howard-Lambek correspondence to the bicategorical setting. If time permits, I will conclude by sketching a bicategorical generalisation of the categorical normalisation-by-evaluation argument of Fiore [1] to prove a conjectured coherence result for cartesian closed bicategories.

References:

- [1] M. Fiore. Semantic analysis of normalisation by evaluation for typed lambda calculus. In *Proceedings of the Fourth ACM SIGPLAN International Conference on Principles and Practice of Declarative Programming (PPDP’02)*, pp. 26–37, 2002.
- [2] M. Fiore and P. Saville. A type theory for cartesian closed bicategories. To appear in *Thirty-Fourth Annual ACM/IEEE Symposium on Logic in Computer Science (LICS’19)*, 2019.
- [3] B. P. Hilken. Towards a proof theory of rewriting: the simply typed  $2\lambda$ -calculus. *Theoretical Computer Science*, vol. 170, no. 1, pp. 407–444, 1996.

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\*Joint work with Marcelo Fiore.