COMPACT INVERSE CATEGORIES

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An *inverse category* is a category that comes with a contravariant involution $\dagger$ that acts as the identity on objects and satisfies $f = ff^\dagger f$ and $ff^\dagger gg^\dagger = gg^\dagger ff^\dagger$ on morphisms [2]. The one-object case, of *inverse monoids*, has been well-studied [4]. In particular, abelian inverse monoids obey a structure theorem [3]: any abelian inverse monoid is a semilattice of abelian groups. In the many-object case, any inverse category gives rise to a semilattice-shaped family of groupoids in a similar way, but not in a functorial way, and it is generally impossible to recover the inverse category from this family without a degree of commutativity.

From the perspective of computer science, inverse categories provide semantics for typed reversible programs. To model recursion, it would be desirable to have additional *compact closed* structure. We show that *compact inverse categories* generalise abelian inverse monoids to multiple objects, and extend the structure theorem: any compact inverse category is a semilattice of compact inverse groupoids. The latter are also known as *coherent 2-groups* or *crossed modules*, and have several characterisations [1]. This structure theorem crucially uses features inherent in compact categories such as traces and scalars.

Based on joint work with Robin Cockett.

References

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