INVOLUTIVE FACTORISATION SYSTEMS
AND DOLD-KAN CORRESPONDENCES

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In the late 1950’s Dold [3] and Kan [5] showed that a simplicial abelian group is completely determined by an associated chain complex and that this construction yields an equivalence of categories. This Dold-Kan correspondence is important in algebraic topology because it permits an explicit construction of topological spaces with prescribed homotopy groups (e.g. Eilenberg-MacLane spaces).

There have been several attempts to extend this kind of categorical equivalence to other contexts, most recently by Lack and Street [6]. We present here an approach based on the notion of involutive factorisation system, i.e. an orthogonal factorisation system $C = (E, M)$ equipped with a faithful, identity-on-objects functor $E^{op} \to M : e \mapsto e^*$ such that $ee^* = 1$ and three other axioms are satisfied.

We show that for each category $C$ equipped with such an involutive factorisation system $(E, M, (−)^*)$, there is an equivalence $[C^{op}, A] \simeq [\Xi(C)^{op}, A]$, where $A$ is any idempotent-complete additive category, and $\Xi(C)$ is the locally pointed category of essential $M$-maps. Since in the simplex category $\Delta$ the only essential $M$-maps are the last face operators $\epsilon_n : [n − 1] \to [n]$, we get ordinary chain complexes in $A$ on the right so that our equivalence specialises to Dold-Kan correspondence if $C = \Delta$.

Our approach recovers several known equivalences, cf. [8, 4, 2]. An interesting new family is given by Joyal’s cell categories $\Theta_n$ (cf. [1]) where our equivalence relates to $n$-th order Hochschild homology and $E_n$-homology (cf. [9, 7]).

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References