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Simplicial objects and relative monotone-light factorizations in Mal'tsev categories

In [1], Brown and Janelidze introduced a Galois structure on the category of simplicial sets, by considering the category of groupoids as a reflective full subcategory and the class of Kan fibrations as extensions, and showed that Kan complexes are admissible for this Galois structure. Later, Chikhladze introduced the notion of *relative* factorization system, and showed that this Galois structure induces a relative monotone-light factorization system for Kan fibrations [2].

On the other hand, it is known [3, 4] that every simplicial object in a regular Mal'tsev category has the Kan property; and moreover exact Mal'tsev categories form a good setting for Categorical Galois Theory, as every Birkhoff subcategory is then admissible [5]. Groupoids themselves also play an important role in the study of Mal'tsev categories [6].

It seems then natural to ask whether internal groupoids can be seen as an admissible subcategory of simplicial objects in any exact Mal'tsev category. We will prove that it is in fact a Birkhoff subcategory, and that it also admits relative monotone-light factorizations for Kan fibrations (which in this context coincide with regular epimorphisms).

References:

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