Relating the Effective Topos to HoTT

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The effective topos $\text{Eff}$ was introduced by Martin Hyland in [4] and proved to be a very useful category where to test computational properties of constructive theories, see [9]. In the talk we present a way to see $\text{Eff}$ as part of a model of Homotopy Type Theory [6].

The presentations of $\text{Eff}$ as an exact completion and of its full subcategory $\text{Asm}$ on the assemblies as a regular completion in [2] suggested that the topos might be obtained as a homotopy quotient of some appropriate category, see also [7]. This is understood in a very rough sense, based on the construction of the exact completion via the pseudo-equivalence relations of Aurelio Carboni as in [1].

By considering the category of the pseudo-equivalence relations in $\text{Asm}$ (with graph homomorphisms), we can show that $\text{Eff}$ is a full subcategory of the homotopy quotient $\text{Ho}((\mathcal{C}^{\text{op}}, \text{Asm}))$ of the category of Kan fibrant cubical assemblies, see [3, 5].

In fact, we obtain this from the stronger result that the extensional realizability topos $\text{Ext}$ of [8], into which $\text{Eff}$ embeds fully, is a full subcategory of $\text{Ho}((\mathcal{C}^{\text{op}}, \text{Asm}))$.

References


