A LANGUAGE FOR CLOSED CARTESIAN BICATEGORIES

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A *cartesian bicategory* [CW87; Car+08] is a bicategory $\mathcal{B}$ in which all hom-categories have finite products, and the full subcategory of left adjoints has finite bicategorical products which extend to a symmetric monoidal structure on $\mathcal{B}$ in a canonical way. We call a cartesian bicategory *closed*, if all pre- and postcomposition functors have right adjoints. The canonical example of a closed cartesian bicategory is the bicategory **Prof** of small categories and profunctors, where composition is given by coends, and the closed structure by ends. I present a natural-deduction style language for cartesian bicategories which controls the 'mixed variances' appearing in calculations with (co)ends by means of a syntactic condition on judgments. Specifically, the judgments of the language are of the form

$$\langle \vec{A}_0 \rangle\, x_1 : \varphi_1 \,\langle \vec{A}_1 \rangle\ \ldots\ \langle \vec{A}_{n-1} \rangle\, x_n : \varphi_n \,\langle \vec{A}_n \rangle \vdash t : \psi$$

where the $\vec{A}_i$ are lists of variables (representing objects of categories in the interpretation in **Prof**), the φ_i and ψ are *formulas* (representing profunctors), and t is a term representing a 2-cell between a composition of the φ_i, and ψ.

Mixed variance is controlled by enforcing a syntactic criterion which says that the formula φ_i may depend on $\vec{A}_0, \ldots, \vec{A}_{i-1}$ only covariantly, and on $\vec{A}_i, \ldots, \vec{A}_n$ only contravariantly. The right-hand-side formula ψ may depend covariantly on the variables $\vec{A}_0$, contravariantly on $\vec{A}_n$, and on the other variables not at all.

References

[Car+08] A. Carboni et al. "Cartesian bicategories II". In: *Theory and Applications of Categories* 19.6 (2008), pp. 93–124.

[CW87] A. Carboni and R.F.C. Walters. "Cartesian bicategories I". In: *Journal of pure and applied algebra* 49.1-2 (1987), pp. 11–32.