A LANGUAGE FOR CLOSED CARTESIAN BICATEGORIES

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A cartesian bicategory [CW87; Car+08] is a bicategory \( \mathcal{B} \) in which all hom-
categories have finite products, and the full subcategory of left adjoints has finite
bicategorical products which extend to a symmetric monoidal structure on \( \mathcal{B} \) in a
canonical way. We call a cartesian bicategory closed, if all pre- and postcomposition
functors have right adjoints. The canonical example of a closed cartesian bicategory
is the bicategory \( \text{Prof} \) of small categories and profunctors, where composition is
given by coends, and the closed structure by ends. I present a natural-deduction
style language for cartesian bicategories which controls the ‘mixed variances’ ap-
pearing in calculations with (co)ends by means of a syntactic condition on judg-
ments. Specifically, the judgments of the language are of the form

\[
\langle \vec{A}_0 \rangle x_1 : \varphi_1 \langle \vec{A}_1 \rangle \ldots \langle \vec{A}_{n-1} \rangle x_n : \varphi_n \langle \vec{A}_n \rangle \vdash t : \psi
\]

where the \( \vec{A}_i \) are lists of variables (representing objects of categories in the inter-
pretation in \( \text{Prof} \)), the \( \varphi_i \) and \( \psi \) are formulas (representing profunctors), and \( t \) is
a term representing a 2-cell between a composition of the \( \varphi_i \), and \( \psi \).

Mixed variance is controlled by enforcing a syntactic criterion which says that
the formula \( \varphi_i \) may depend on \( \vec{A}_0, \ldots, \vec{A}_{i-1} \) only covariantly, and on \( \vec{A}_i, \ldots, \vec{A}_n \)
only contravariantly. The right-hand-side formula \( \psi \) may depend covariantly on
the variables \( \vec{A}_0 \), contravariantly on \( \vec{A}_n \), and on the other variables not at all.

References
