COALGEBRAS FOR ENRICHED HAUSDORFF (AND VIETORIS) FUNCTORS

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Starting with early studies in the nineties [11] until the introduction of uniform notions of *behavioural metric* in the last decade [3], several works investigate coalgebras over metric-like spaces and their respective limits. Existing work on coalgebras over metric spaces focus on four specific areas: (1) liftings of functors from the category Set to categories of metric spaces [3, 4, 12] (as a way of lifting state-based transition systems into transitions systems over categories of metric spaces); (2) results on the existence of final coalgebras and their computation [11, 3] (as a way of calculating the behavioural distance of two given states of a transition system); (3) the introduction of behavioural metrics with corresponding up-to techniques [3, 4] (as a way of easing the calculation of behavioural distances); (4) the development of coalgebraic logical foundations over metric spaces [2] (so that one can reason about transition systems in a quantitative way).

Our work aims at contributing to these lines of research. In continuation of our study [6], we investigate completeness properties of categories of coalgebras for "powerset-like" functors; moving now from ordered structures to quantale-enriched ones. As a starting point, we show that, for an extension of a Set-functor to a topological category X over Set which commutes with the forgetful functor, the corresponding category of coalgebras over X is topological over the category of coalgebras over Set and therefore cannot be "more complete". Secondly, based on a Cantor-like argument [5], we observe that Hausdorff functors [1, 9] on categories of quantale-enriched categories do not admit a final coalgebra. Motivated by these "negative" results, in this talk we combine quantale enriched categories and topology *à la* Nachbin [8, 10]. Besides studying some basic properties of these categories, we investigate functors which simultaneously encode the classical Hausdorff metric and Vietoris topology and, moreover, lead now to complete categories of coalgebras. Seemingly unrelated, we use these constructions to improve some of the duality results of [7].

This talk is based in joint work with Renato Neves (Minho University) and Pedro Nora (University of Aveiro).

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