Duality, definability and conceptual completeness for κ-pretoposes

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The κ-geometric toposes introduced in [2], for regular κ such that $\kappa^{<\kappa} = \kappa$ (or any regular κ if the Generalized Continuum Hypothesis holds), are associated to κ-geometric logic, an extension of geometric logic in which conjunction of less than κ many formulas as well as quantification of less than κ many variables are possible. They in fact occur as κ-classifying toposes, i.e., toposes with the obvious universal property applied to κ-geometric morphisms (those whose inverse image preserve all κ-small limits). When these toposes are in addition κ-separable, they turn out to have enough κ-points. We prove here that this completeness theorem has the surprising consequence that for $\lambda > \kappa$, the λ-classifying topos of any theory with at most κ axioms, expressible in κ-geometric logic, is in fact the topos of presheaves over the category of λ-presentable models of the theory.

As applications we get positive results on definability theorems for infinitary logic, conceptual completeness for κ-pretoposes, infinitary versions of Joyal's completeness theorem for infinitary intuitionistic logic, a Stone type duality in the form of a biequivalence arising from a syntax-semantics adjunction, the descent theorem for κ-pretoposes, and a characterization of categoricity for models of infinitary sentences. Time permitting we will show how this latter result provides a topos-theoretic approach to Shelah's eventual categoricity conjecture.

1 References

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