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Functional distribution monads and τ-additive Borel measures

The article [1] defines a categorical framework for algebraic dualization processes that give rise to restricted double-dualization monads called *functional distribution monads*, which specialize to yield various spaces of measures, distributions, filters, closed subsets, compacta, and so forth. Functional distribution monads are canonically induced by a given enriched algebraic category $\mathscr{A}$ and a suitable object of $\mathscr{A}$ acting as 'dualizer'. These given data can be formulated equivalently as a suitable *algebraic dual adjunction* [2], comprising a given pair of algebraic $\mathscr{V}$-categories and a contravariant adjunction that captures the relevant dualization processes and the relation between an algebraic theory and its commutant [3] relative to the dualizer.

In this talk, we will show that by taking as $\mathscr{V}$ the category of convergence spaces and as $\mathscr{A}$ the category of convex spaces internal to $\mathscr{V}$, with the unit interval as dualizer, the induced functional distribution monad gives rise to the notion of *τ-additive* (or *τ-smooth*) *probability measure* on Tikhonov spaces. For locally compact Hausdorff spaces, bounded τ-additive measures coincide with bounded Radon measures, so this generalizes an earlier result of the speaker (announced in [5]). But bounded τ-additive measures on Tikhonov spaces also generalize bounded Borel measures on Polish spaces, so the resulting functional distribution monad captures a wide class of settings in topological measure theory. In proving our result, we establish a connection between τ-additive measures and continuous convergence, and we establish integral representation theorems for bounded τ-additive measures that are formulated in terms the cartesian closed structure of $\mathscr{V}$.

References:

[1] R. B. B. Lucyshyn-Wright, Functional distribution monads in functional-analytic contexts. *Advances in Mathematics* 322 (2017), 806–860.

[2] R. B. B. Lucyshyn-Wright, Algebraic duality and the abstract functional analysis of distribution monads. Invited talk at *CT 2017*, Vancouver.

[3] R. B. B. Lucyshyn-Wright, Commutants for enriched algebraic theories and monads. *Applied Categorical Structures* 26 (2018), 559-596.

[4] R. B. B. Lucyshyn-Wright, Enriched algebraic theories and monads for a system of arities. *Theory and Applications of Categories* 31 (2016) 101–137.

[5] R. B. B. Lucyshyn-Wright, Enriched algebraic theories, monads, and commutants in the foundations of categorical distribution theory. Talk at *CT 2016*, Halifax.

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