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Fixpoint toposes

If $F : \mathcal{E} \to \mathcal{E}$ is any functor, we can look at its category $\mathbf{Fix}(F)$ of fixpoints: objects $X \in \mathcal{E}$ endowed with an isomorphism $X \cong FX$. The first goal of this talk is to explain that, if $\mathcal{E}$ is an topos and F is a pullback-preserving endofunctor which generates a cofree comonad, then $\mathbf{Fix}(F)$ is again a topos. The proof builds on the material of [1].

Specific examples of this construction include the well-known Jonsson–Tarski topos, whose objects are sets endowed with an isomorphism $X \cong X \times X$; the generalised Jonsson–Tarski toposes of Leinster [2]; and the Kennison topos, whose objects are sets endowed with an isomorphism $X \cong X + X$. The second goal of this talk is to explain how such toposes give rise to objects of interest to algebraists, such as Cuntz–Kreiger C^*-algebras [3], Leavitt path algebras [4], and their associated étale groupoids [5].

References:

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[4] Ara, P., Moreno, M.A., & Pardo, E., Nonstable K-theory for graph algebras, *Algebras and Representation Theory* 10 (2007), 157–178.

[5] Kumjian, A., Pask, D., Raeburn, I. & Renault, J., Graphs, groupoids, and Cuntz-Krieger algebras, *Journal of Functional Analysis* 144 (1997) 505–541.