

# An Algebraic Combinatorial Approach to the Abstract Syntax of Opetopic Structures

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## Abstract

The starting point of the talk will be the identification of structure common to tree-like combinatorial objects, exemplifying the situation with abstract syntax trees (as used in formal languages) and with opetopes (as used in higher-dimensional algebra). The emerging mathematical structure will be then formalized in a categorical setting, unifying the algebraic aspects of the theory of abstract syntax of [2, 3] and the theory of opetopes of [5]. This realization conceptually allows one to transport viewpoints between these, now bridged, mathematical theories and I will explore it here in the direction of higher-dimensional algebra, giving an algebraic combinatorial framework for a generalisation of the slice construction of [1] for generating opetopes. The technical work will involve setting up a microcosm principle for near-semirings and subsequently exploiting it in the cartesian closed bicategory of generalised species of structures [4]. Connections to (cartesian and symmetric monoidal) equational theories, lambda calculus, and algebraic combinatorics will be mentioned in passing.

## References

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- [5] S. Szawiel and M. Zawadowski. The web monoid and opetopic sets. In *arXiv:1011.2374 [math.CT]*, 2010.