TOWARDS
PATTERN - MATCHING
~~BINDING~~ & SUBSTITUTION
in
STRING DIAGRAMS

ROSS DUNCAN — UNIVERSITY OF STRATHCLYDE

Pattern matching, binding and substitution

```
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f (x :: xs) = (x+1) :: (f xs)
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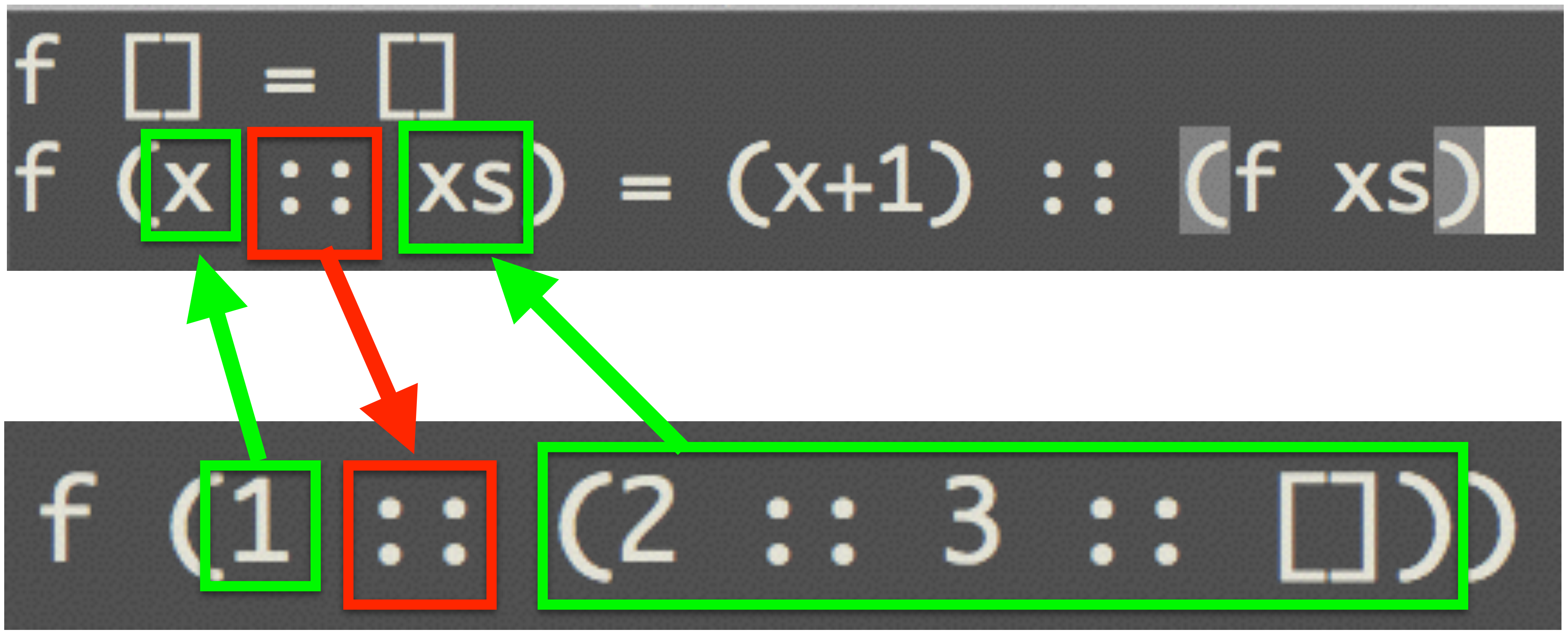

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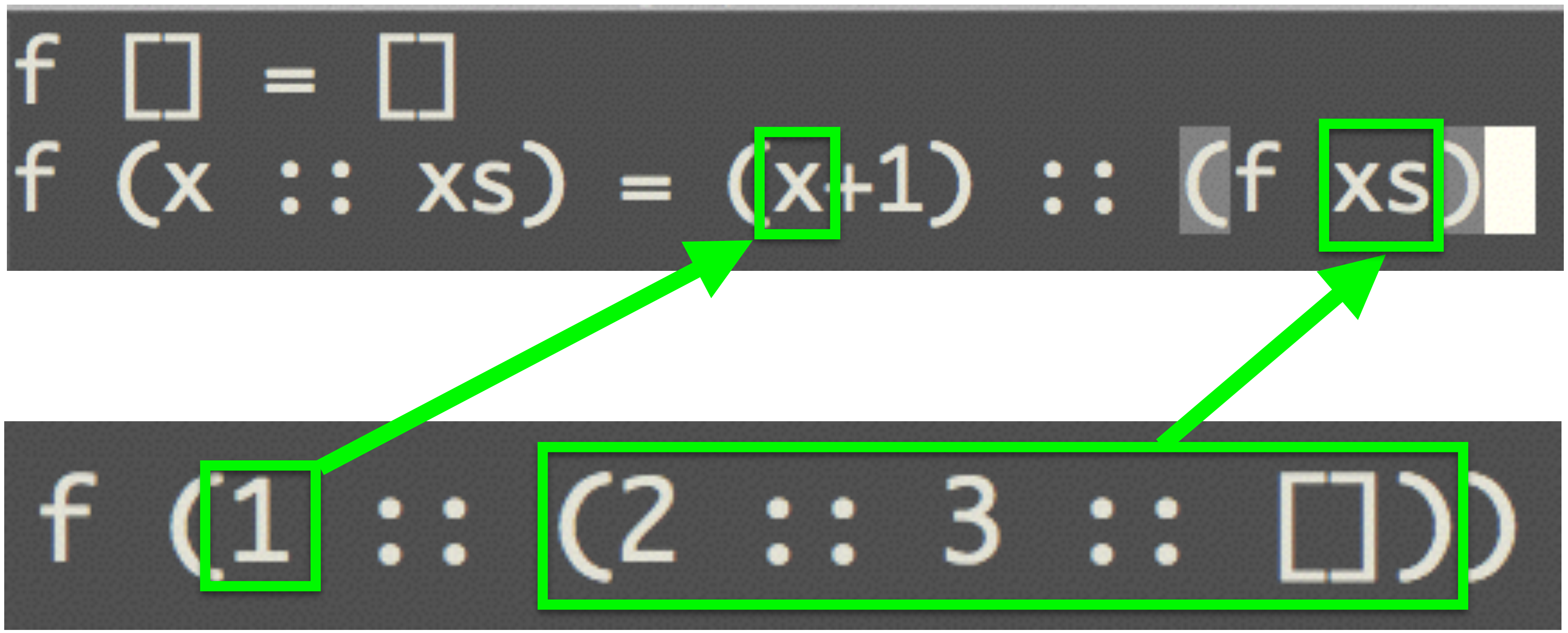

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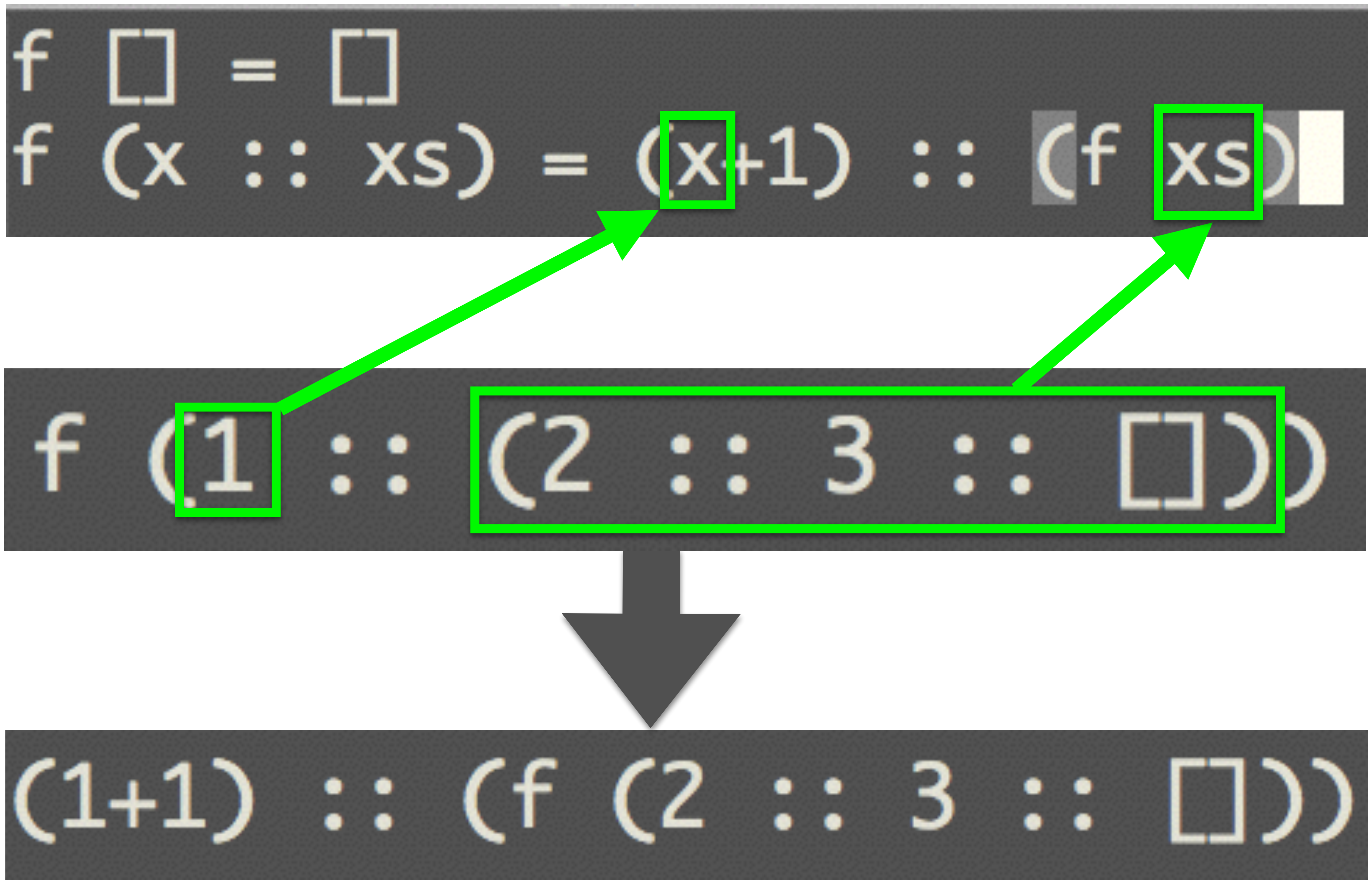


1.

Monoidal Categories

1. Symmetric Monoidal Categories

A monoidal category M is a category with a bifunctor, $\otimes$ or $\square$,

$$\square : M \times M \to M$$

written for objects a, b of M variously as a "product"

$$(a, b) \to a \square b, a \otimes b, \text{or } a\,b$$

which is associative up to a natural isomorphism

$$\alpha : a(b\,c) \cong (a\,b)c \tag{1}$$

and is equipped with an element e, which is unit up to natural isomorphisms

$$\lambda : e\,a \cong a\,, \qquad \rho : a\,e \cong e\,. \tag{2}$$

These maps must satisfy certain commutativity requirements; for α, a pentagonal diagram

$$\begin{tikzcd}
a(b(c\,d)) \arrow[r, "\alpha"] \arrow[d, "1\alpha"'] & (a\,b)(c\,d) \arrow[r, "\alpha"] & ((a\,b)c)d \arrow[d, "\alpha 1"] \\
a((b\,c)d) \arrow[rr, "\alpha"] & & (a(b\,c))d\,,
\end{tikzcd} \tag{3}$$

as in § VII.1.(5), and for λ and ρ the two commutativities

$$\begin{tikzcd}
a(e\,c) \arrow[r, "\alpha"] \arrow[d, "1\lambda"'] & (a\,e)c \arrow[d, "\rho 1"] \\
a\,c \arrow[r, phantom, "="] & a\,c\,,
\end{tikzcd} \qquad \lambda = \rho : e\,e \to e\,. \tag{4}$$

A *braiding* for a monoidal category M consists of a family of isomorphisms

$$\gamma_{a,b} : a \square b \cong b \square a \tag{5}$$

natural in a and $b \in M$, which satisfy for e the commutativity

$$\begin{tikzcd}
a \square e \arrow[r, "\gamma"] \arrow[d, "\rho"'] & e \square a \arrow[d, "\lambda"] \\
a \arrow[r, phantom, "="] & a
\end{tikzcd} \tag{6}$$

and which, with the associativity α, make both the following hexagonal diagrams commute (with the symbol $\square$ omitted):

$$\begin{tikzcd}
(a\,b)c \arrow[r, "\gamma"] \arrow[d, "\alpha^{-1}"] & c(a\,b) \arrow[d, "\alpha"] & a(b\,c) \arrow[r, "\gamma"] \arrow[d, "\alpha"] & (b\,c)a \arrow[d, "\alpha^{-1}"] \\
a(b\,c) \arrow[d, "1\cdot\gamma"'] & (c\,a)b \arrow[d, "\gamma\cdot 1"] & (a\,b)c \arrow[d, "\gamma\cdot 1"] & b(c\,a) \arrow[d, "1\cdot\gamma"] \\
a(c\,b) \arrow[r, "\alpha"] & (a\,c)b\,, & (b\,a)c \arrow[r, "\alpha^{-1}"] & b(a\,c)\,.
\end{tikzcd} \tag{7}$$

Note that the first diagram replaces each $\gamma_{ab,c}$ which has a product $a\,b$ as first index by two γ's with single indices, while the second hexagonal diagram does the same for $\gamma_{a,bc}$ with a product as second index. Note also that the first hexagon of (7) for γ implies the second diagram for γ^{-1}, and conversely. Thus, when γ is a braiding for M, then γ^{-1} is also a braiding for M.

A *symmetric monoidal category*, as already defined in § VII. 7, is a category with a braiding γ such that every diagram

$$\begin{tikzcd}
a\,b \arrow[r, "\gamma_{a,b}"] \arrow[dr] & b\,a \arrow[d, "\gamma_{b,a}"] \\
& a\,b
\end{tikzcd} \tag{8}$$

commutes. For this case, either one of the hexagons (7) implies the other.

1bis.

Monoidal Categories (Graphically)

Why Diagrams?

Why Diagrams?

$$\frac{1}{2}\left(\begin{pmatrix}1\\1\end{pmatrix}\otimes\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&e^{i\gamma}\end{pmatrix}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}\right)$$

$$\circ\Bigg(\Bigg(\bigg(\begin{pmatrix}1&0&0&0\\0&0&0&1\end{pmatrix}\otimes\begin{pmatrix}1&1\\1&-1\end{pmatrix}\bigg)\bigg)\circ\Bigg(\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}\Bigg)\Bigg)\circ$$

$$\Bigg(\Bigg(\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\Bigg)\Bigg)\Bigg)\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\Bigg)$$

$$\circ\Bigg(\Bigg(\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\circ\bigg(\bigg(\begin{pmatrix}\cos\frac{\pi}{6}\\i\sin\frac{\pi}{6}\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&e^{i\beta}\end{pmatrix}\bigg)\bigg)\Bigg)\otimes\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&e^{i\alpha}\end{pmatrix}\Bigg)$$

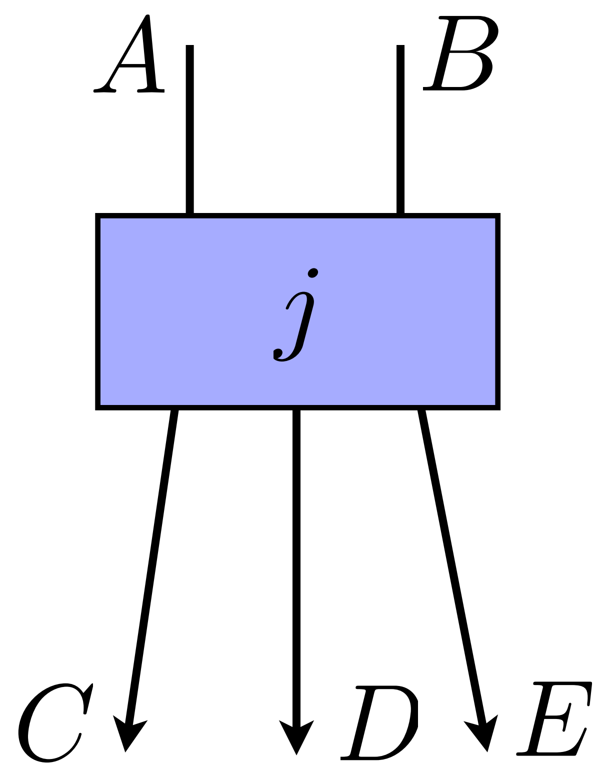
$$= ?$$

Why Diagrams?

- Great when we have parallel and sequential composition
- Essential for talking about interacting algebraic and coalgebraic things
- Different kinds of diagram give different kinds of monoidal category

Diagrams

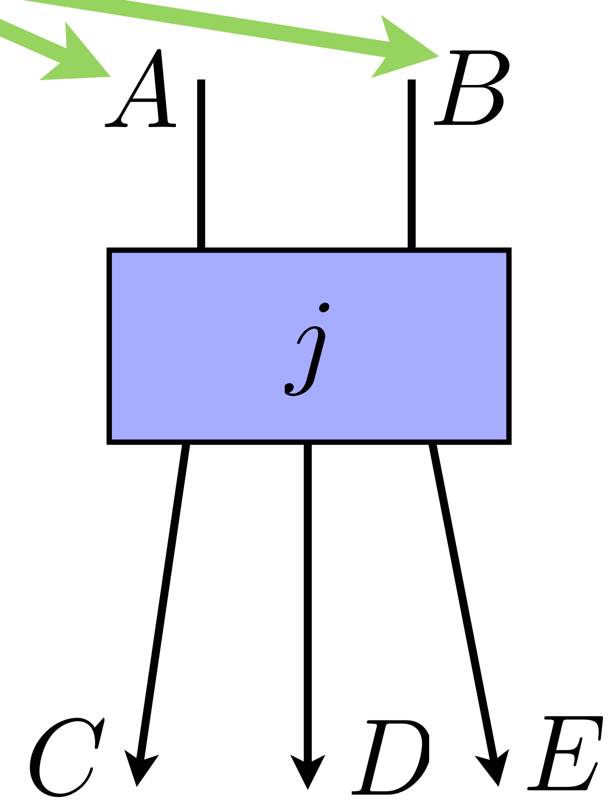
$$j : A \otimes B \to C \otimes D \otimes E$$



Diagrams

Input Systems

$$j : A \otimes B \to C \otimes D \otimes E$$



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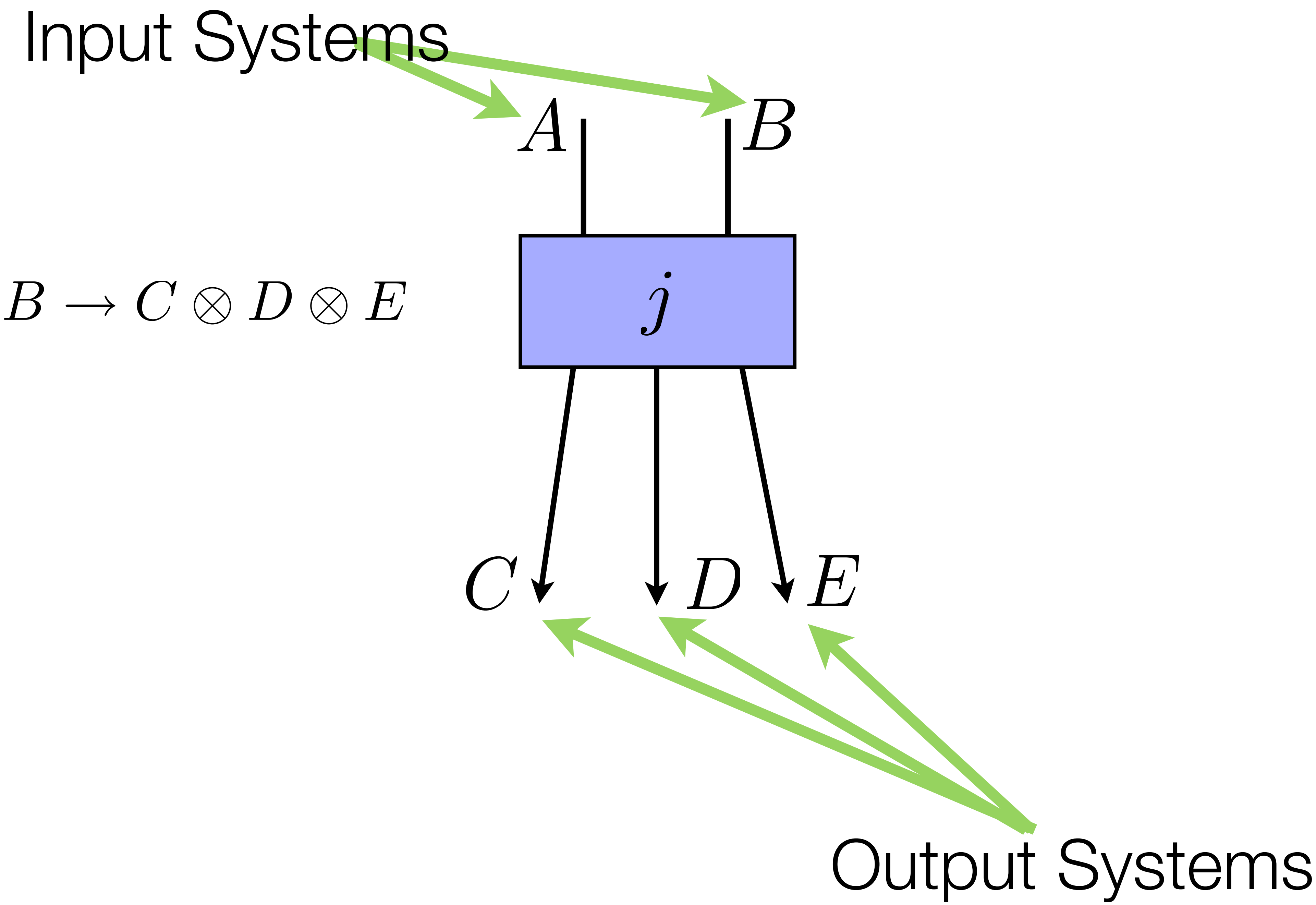

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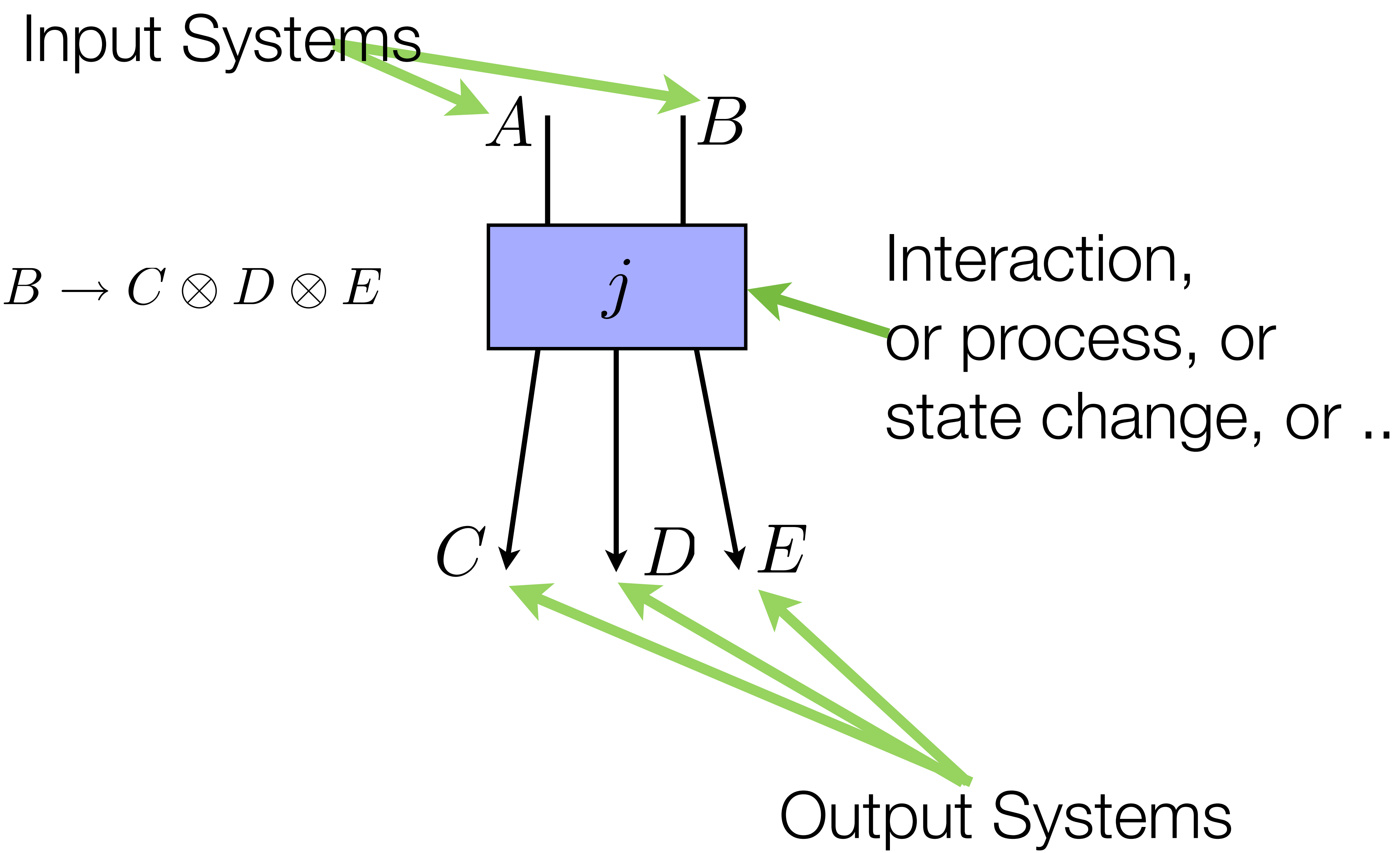
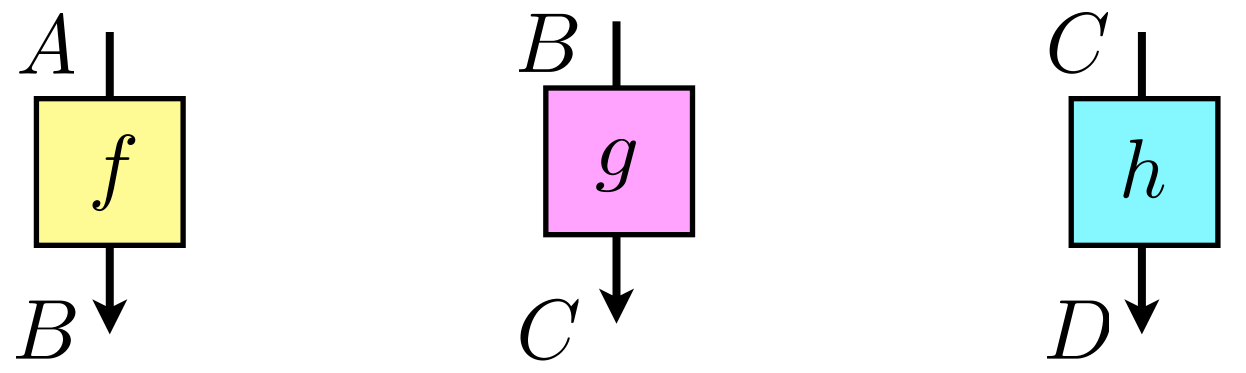


Monoidal Categories

$f : A \to B$ $\quad$ $g : B \to C$ $\quad$ $h : C \to D$



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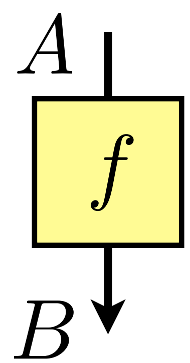
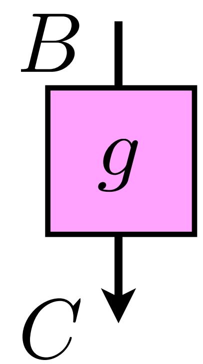
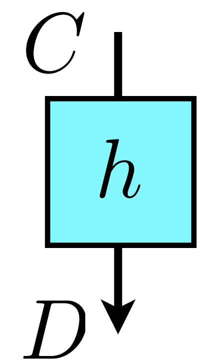
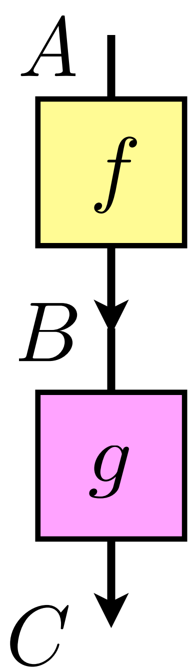




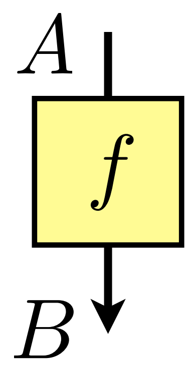


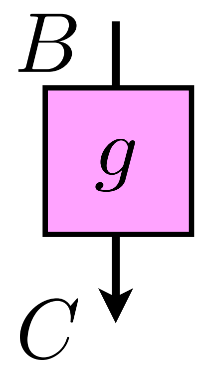
$g \circ f : A \to C$

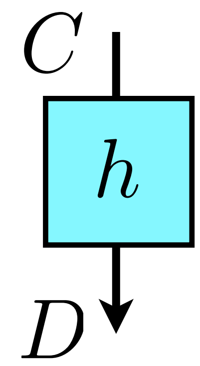


Monoidal Categories

$f : A \longrightarrow B$ $\qquad$ $g : B \longrightarrow C$ $\qquad$ $h : C \longrightarrow D$







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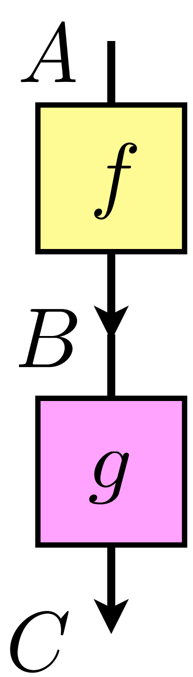
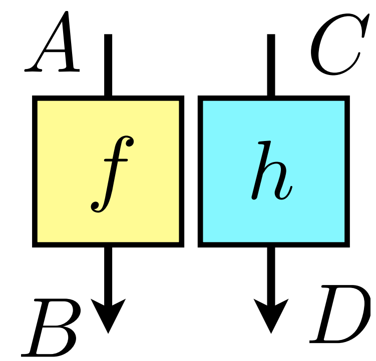


$f \otimes h : A \otimes C \longrightarrow B \otimes D$



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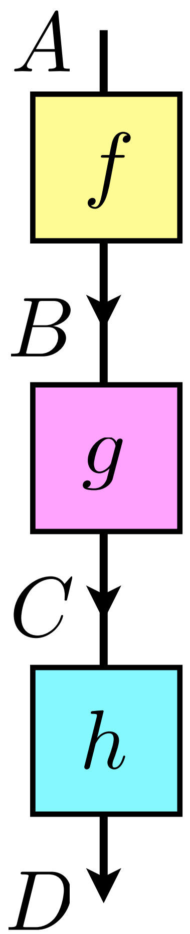
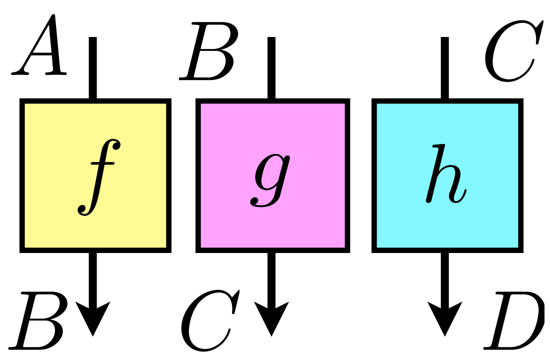
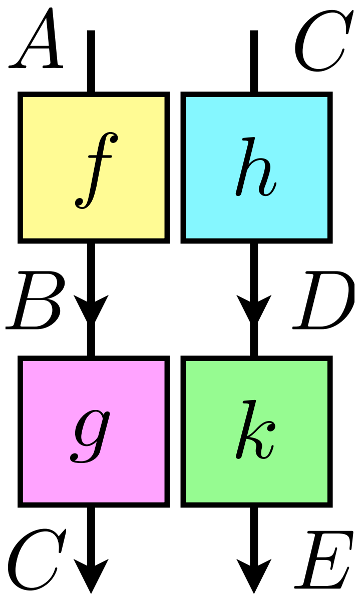






Monoidal Categories

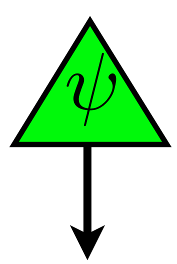
Monoidal categories have a special *unit* object called I which is a left and right identity for the tensor:

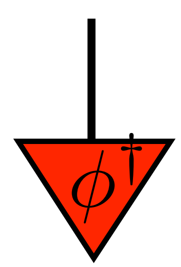
$$I \otimes A = A = A \otimes I$$

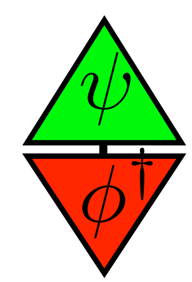
$$\mathrm{id}_I \otimes f = f = f \otimes \mathrm{id}_I$$

No lines are drawn for I in the graphical notation:

$$\psi : I \longrightarrow A \qquad \phi^\dagger : A \longrightarrow I \qquad \phi^\dagger \circ \psi : I \longrightarrow I$$

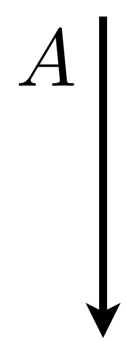






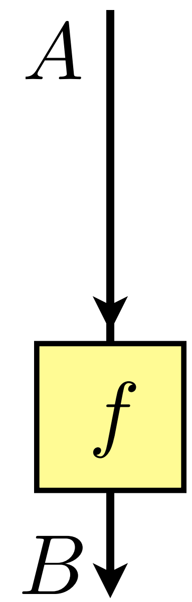
Categories

$$\mathrm{id}_A : A \to A$$



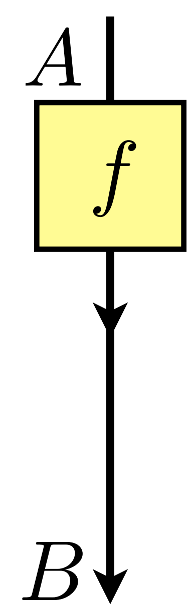
Categories

$$f \circ \mathrm{id}_A : A \to B$$



Categories

$$\mathrm{id}_B \circ f : A \to B$$



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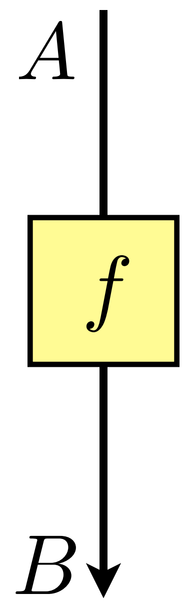
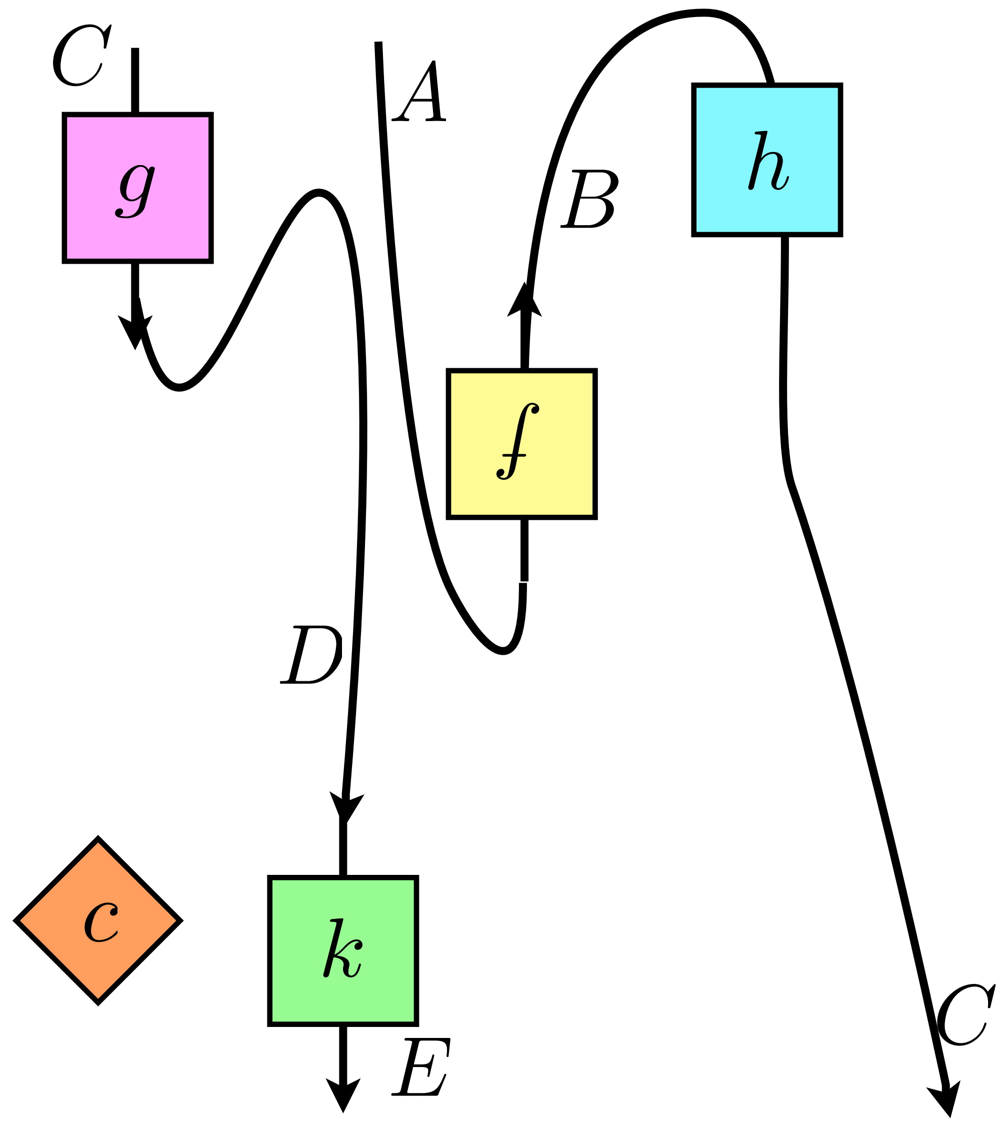


Graphical Calculus Theorem

Thm: one diagram can be deformed to another iff their denotations are equal by the structural equations of the category.



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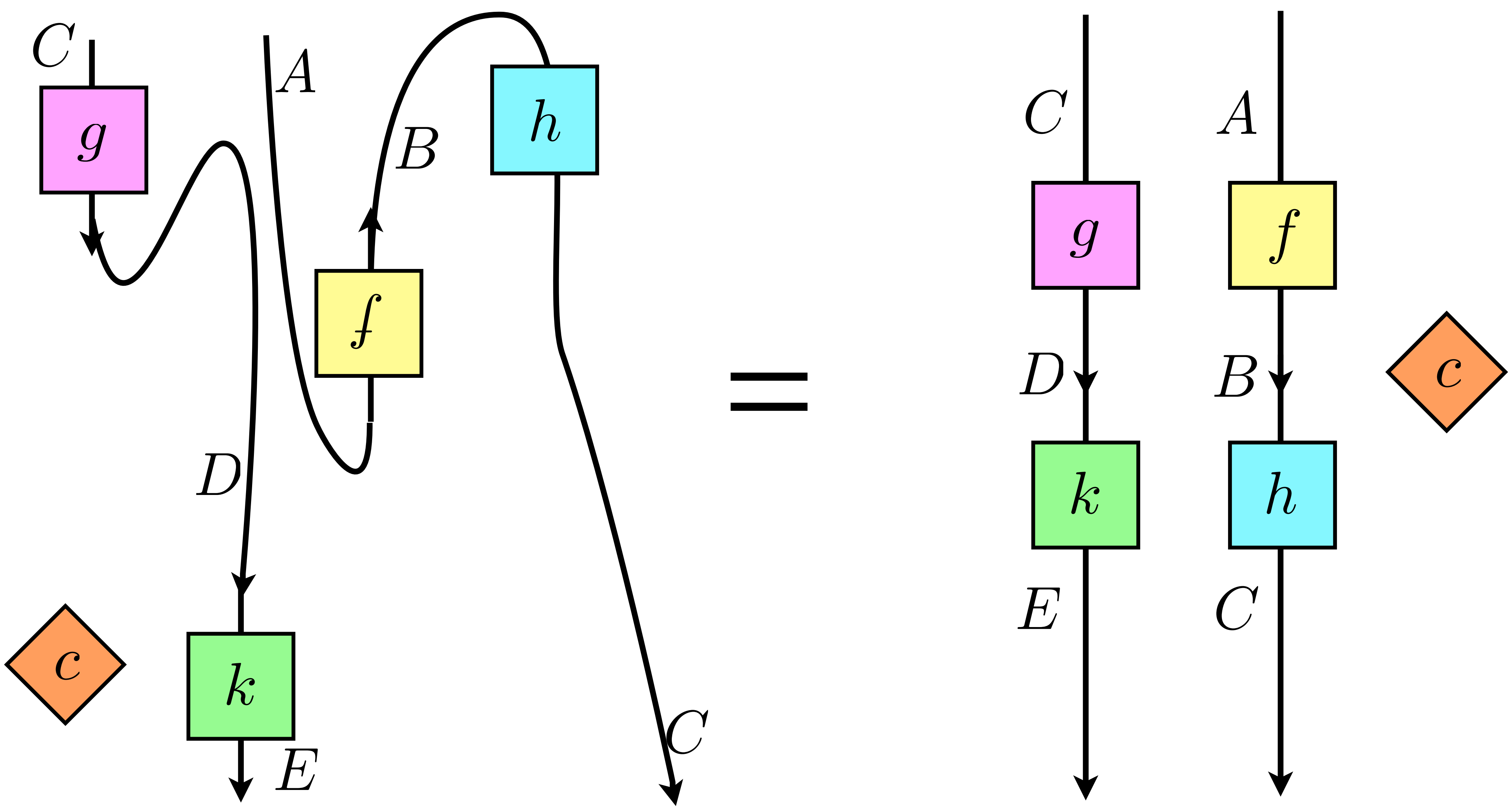

Crossing the streams

Are wires allowed to cross?

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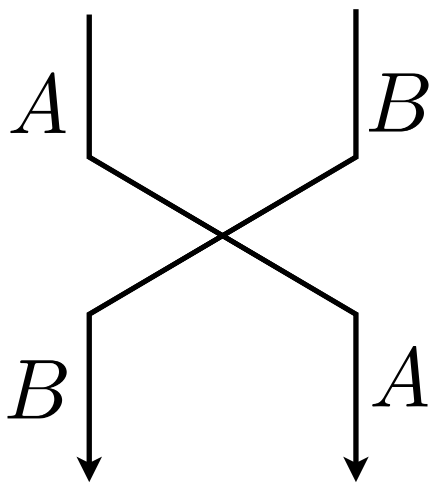

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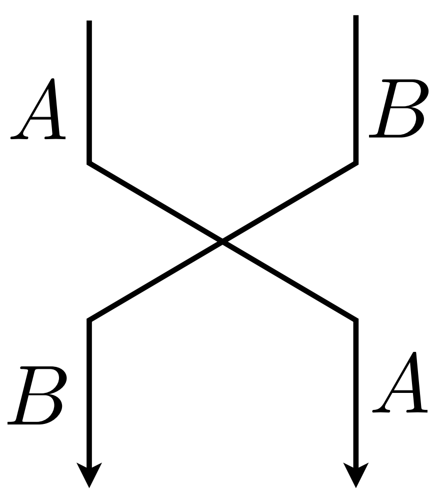
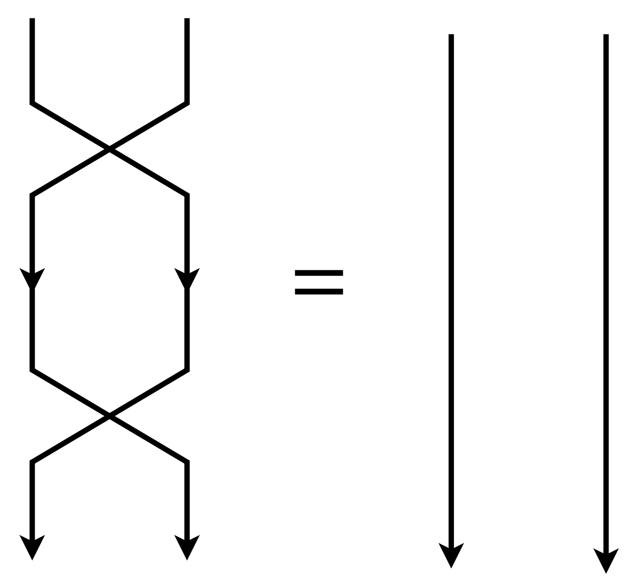
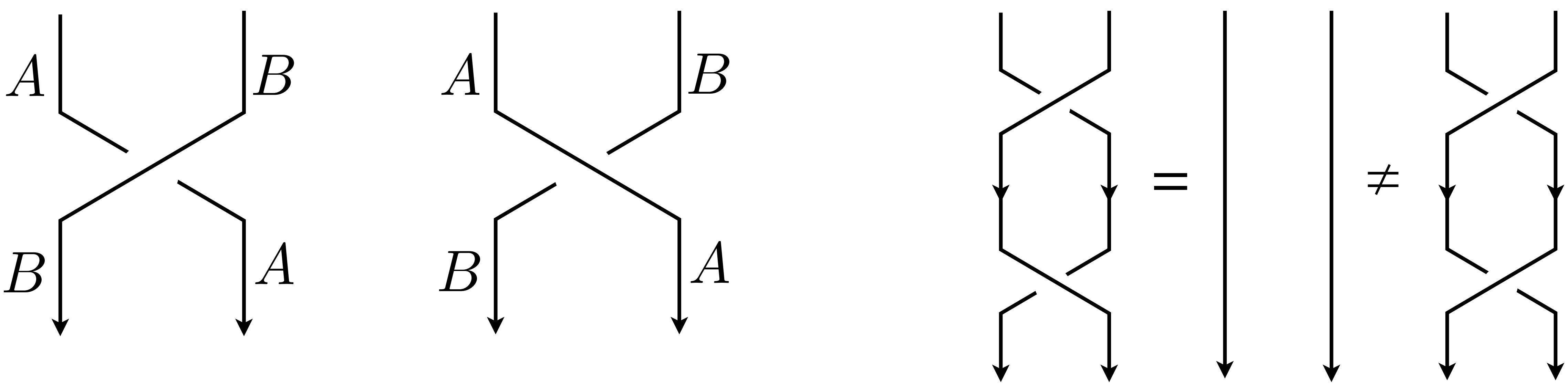




YES : symmetric monoidal — diagrams are DAGs

Crossing the streams

Are wires allowed to cross?



YES, BUT : braided monoidal

— diagrams are framed tangles

Crossing the streams

Are wires allowed to cross?

Crossing the streams

Are wires allowed to cross?

No : (planar) monoidal — diagrams are planar DAGs

Monoidal Theories

Syntactic presentation of a diagrammatic theory:

Generators
symbols with arity and coarity

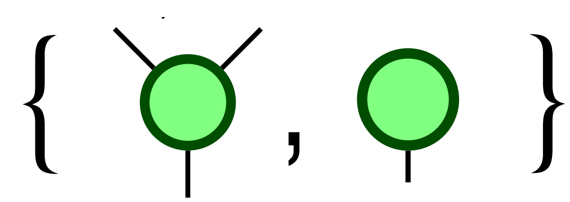
$$(\Sigma, E)$$

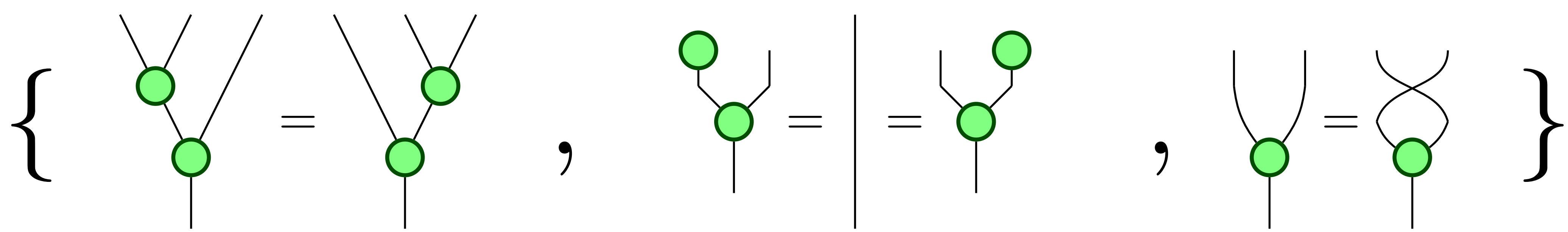
Relations
equations between terms of same type

NB : a *PRO(P)* is a (symmetric) monoidal category where the wires don't have types.

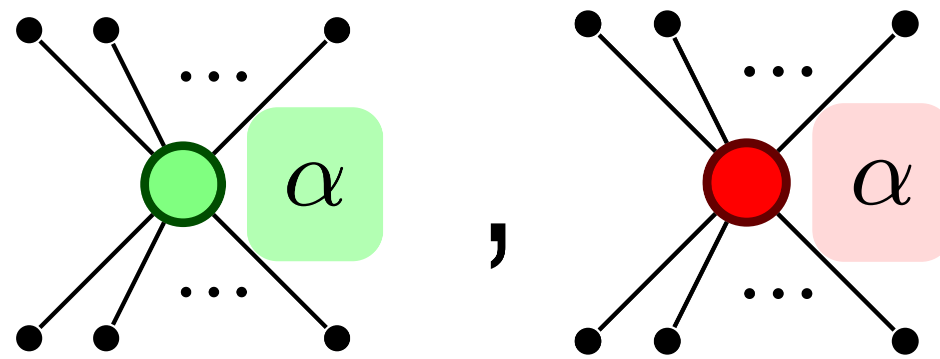
Example: commutative monoids

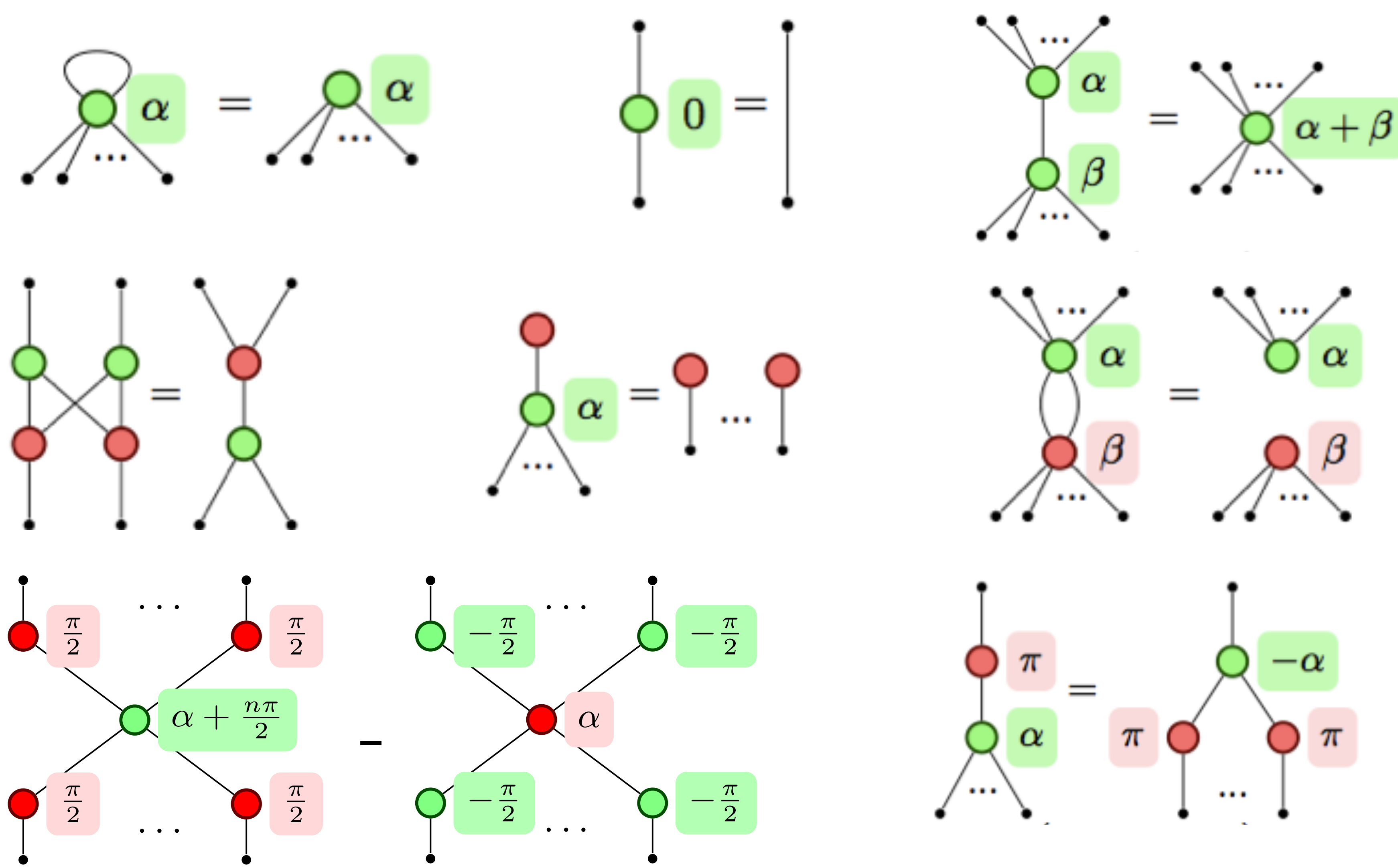
The PROP of commutative monoids $\mathbb{M}$

$\Sigma =$ 

$E =$ 

Example : the ZX-calculus

$\Sigma = \{$  $\}$

$E = \{$  $\}$

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Computing Science Group

Geometry of abstraction in quantum computation

Dusko Pavlovic
Oxford University and Kestrel Institute

CS-RR-09-13

Geometry of abstraction in quantum computation

Pavlovic (2009,2012)

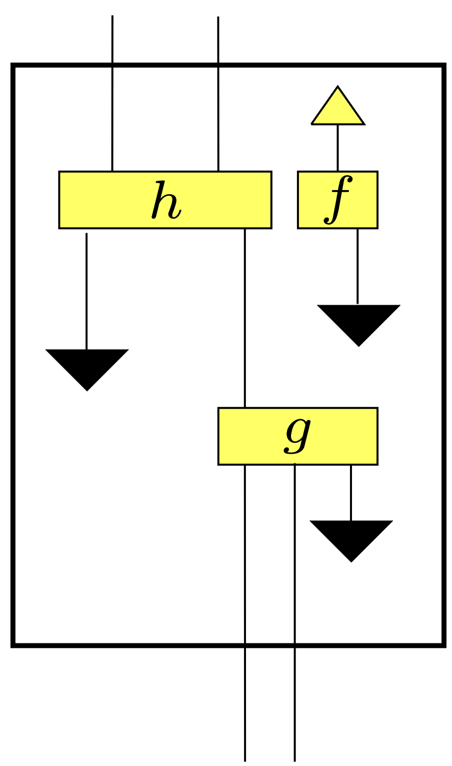
Quantum algorithms are sequences of abstract operations, performed on non-existent computers. They are in obvious need of categorical semantics.

Geometry of abstraction in quantum computation

Pavlovic (2009,2012)

monoidal category $\mathcal{C}$

polynomial monoidal category $\mathcal{C}[x : X]$



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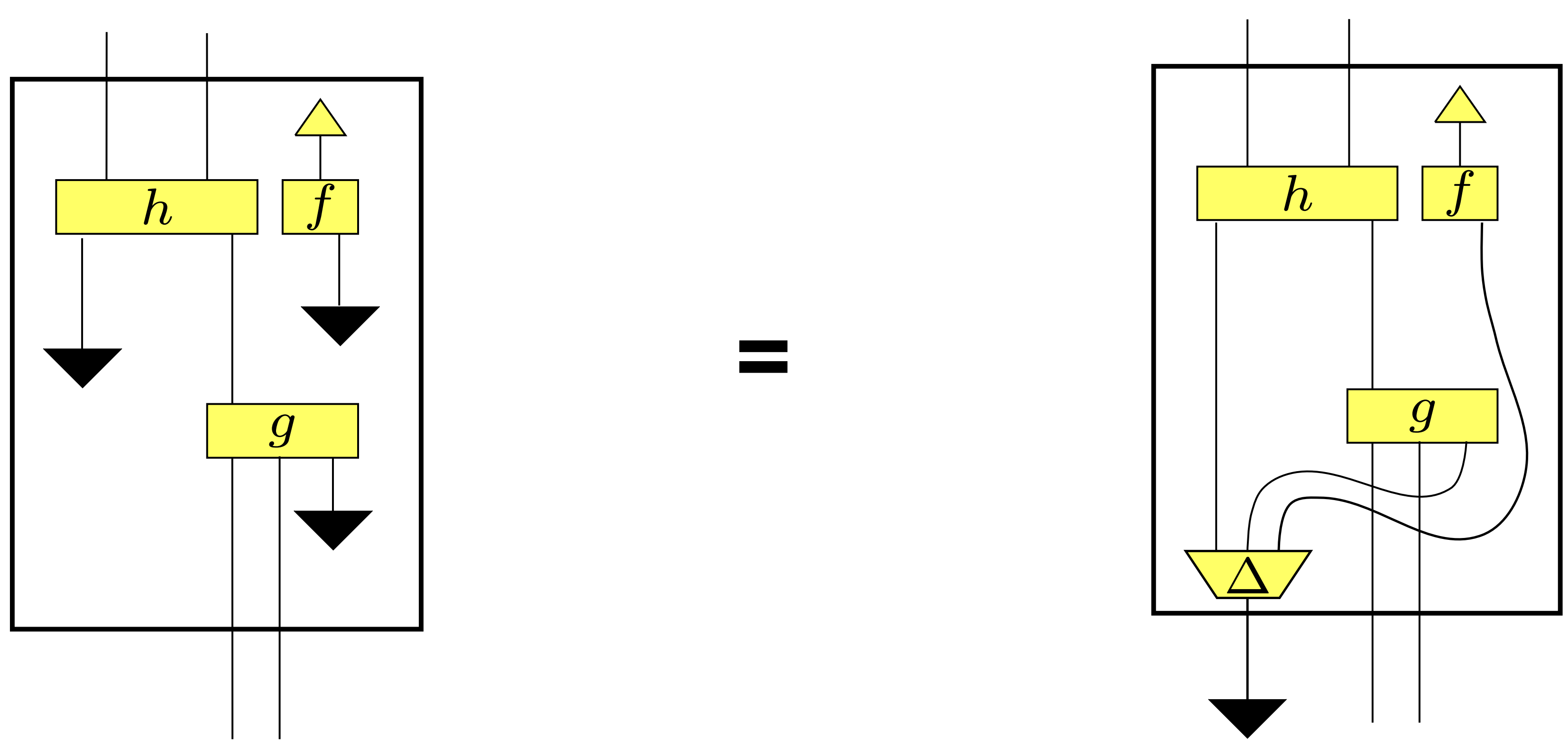

Geometry of abstraction in quantum computation

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monoidal category $\mathcal{C}$

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Theorem 3.4 *The category* $\mathrm{Abs}_{\mathrm{C}}$ *of monoidal abstractions is equivalent with the category* $\mathrm{C}_{\times}$ *of commutative comonoids in* C.

Geometry of abstraction in quantum computation

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monoidal category $\mathcal{C}$

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Theorem 3.4 *The category* $\mathrm{Abs}_{\mathrm{C}}$ *of monoidal abstractions is equivalent with the category* $\mathrm{C}_{\times}$ *of commutative comonoids in* C.

Corollary 4.5 *The category of dagger-monoidal abstractions* $\ddagger$-$\mathrm{Abs}_{\mathrm{C}}$ *is equivalent with the category* C_{Δ} *of commutative dagger-Frobenius algebras and comonoid homomorphisms in* C

TOWAR

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STRING DIAGRAM

1. OPERADs

$$A \xrightarrow{f} B \xrightarrow{g} C$$

(arrows in a category)

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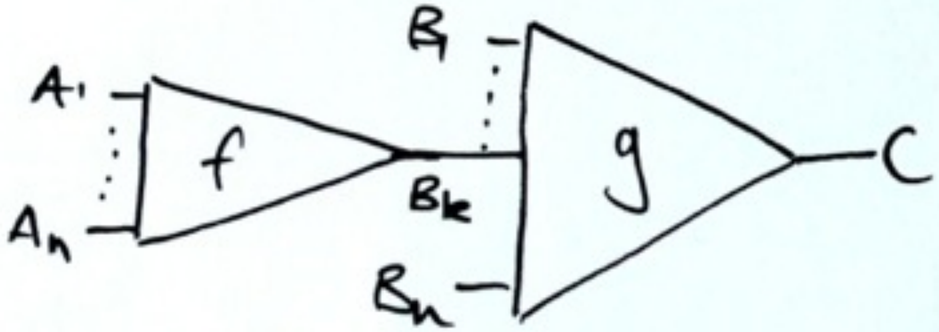
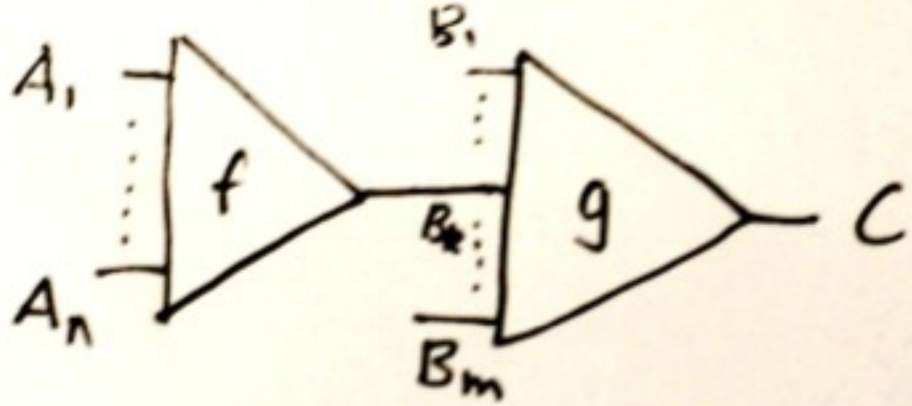


(arrows in an operad)

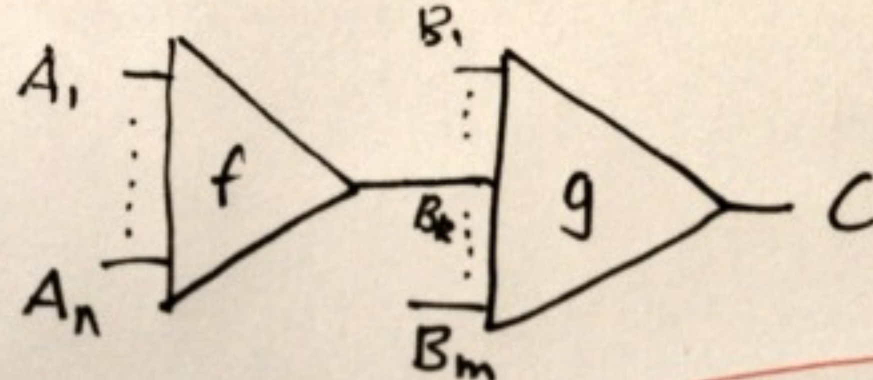
aka. multicategory.

1. OPERADS



$$\frac{x_1 : A_1, \ldots, x_n : A_n \vdash f : B_k \qquad y_1 : B_1, \ldots, y_k : B_k, \ldots, y_m : B_n \vdash g : C}{y_1 : B_1, \ldots, x_1 : A_1, \ldots, x_n : A_n, \ldots, y_m : B_m \vdash g[f/x] : C}\ \text{CUT}$$

1. OPERADS



$$\frac{x_1 : A_1, \dots, x_n : A_n \vdash f : B_k \qquad y_1 : B_1, \dots, \cancel{y_k : B_k}, \dots, y_m : B_n \vdash g : C}{y_1 : B_1, \dots, \boxed{x_1 : A_1, \dots, x_n : A_n}, \dots, y_m : B_m \vdash g[f/y_k] : C}\ \text{CUT}$$

2. Making an Operad from a PRO

- Let (Σ, E) be a presentation of a PRO.
- Adjoin "enough" new generators $x : m \to n$ for every $m, n \in \mathbb{N}$.

  variables.
- Then $(\Sigma + \mathrm{Var}, E)$ is again a PRO with (term) variables.

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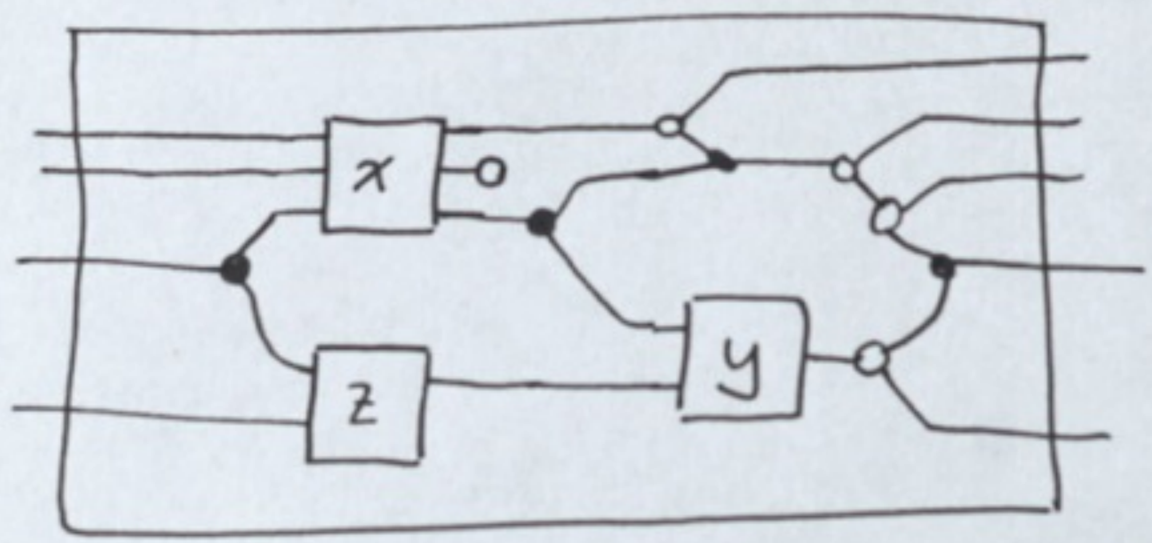
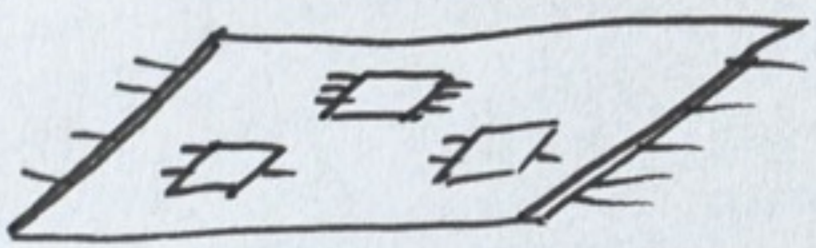
- Then $(\Sigma + Var, E)$ is again a PRO with (term) variables.

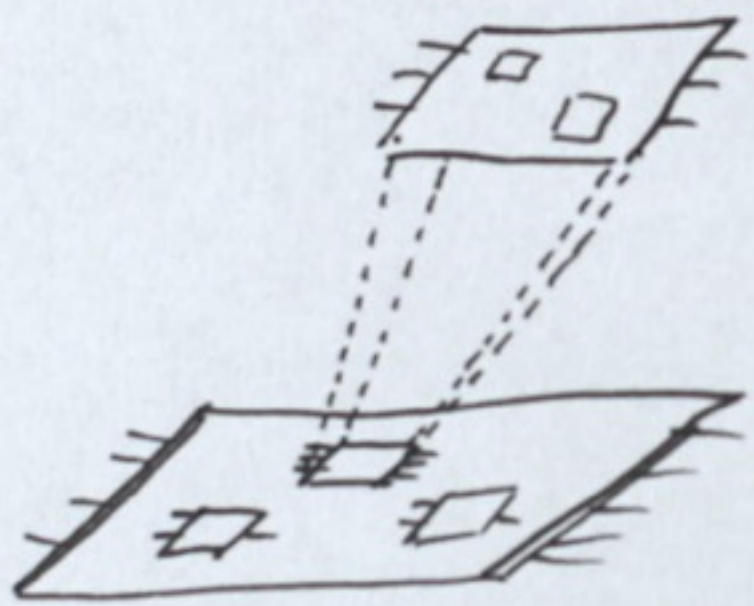


Assume variables only occur once (for now)

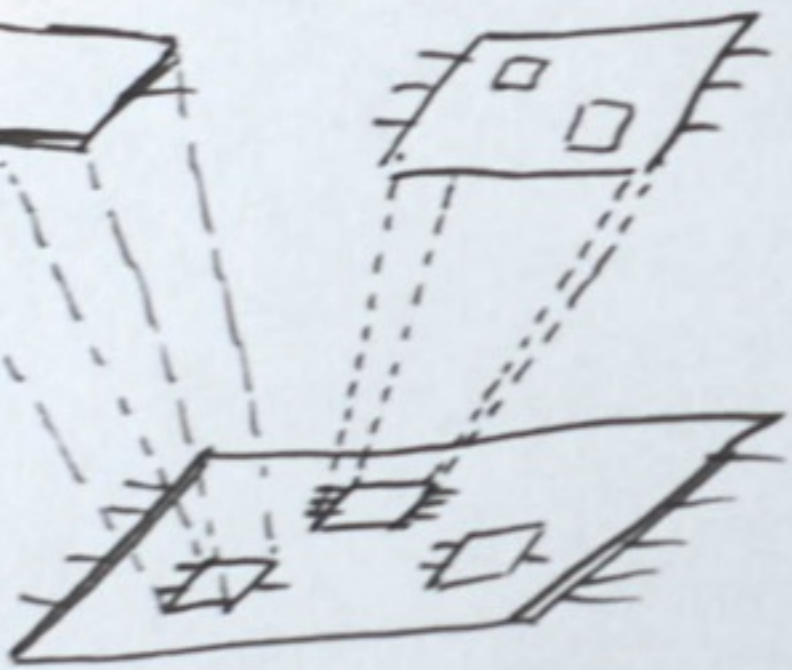
$$x:(3,3),\ y:(2,1),\ z:(2,1) \vdash f:(4,5)$$



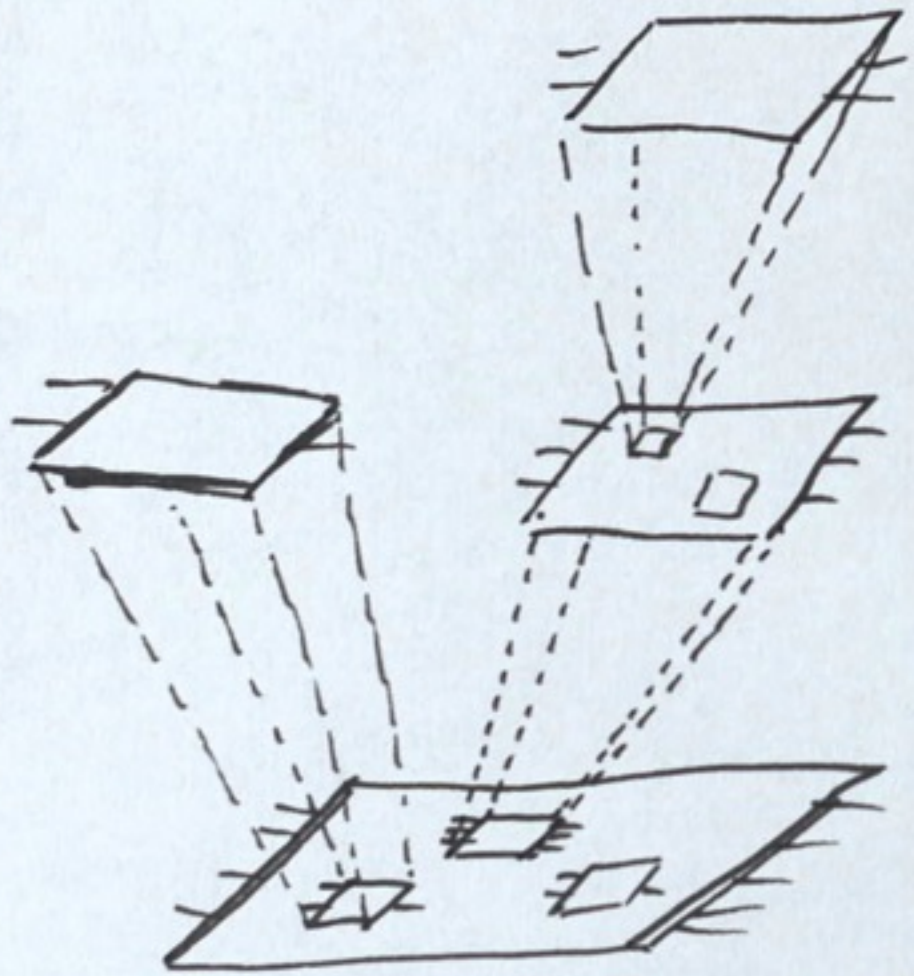
f



$f[g/x]$



$f[g/x][h/z]$



$f[g/x][h/z][k/w]$

etc.

Double Push-Out Rewriting

$$L = R$$

DOUBLE PUSH-OUT REWRITING

$$L \Rightarrow R$$

DOUBLE PUSH-OUT REWRITING

$$L \Rightarrow R$$

$$L \xleftarrow{i_1} \partial \xrightarrow{i_2} R$$

DOUBLE PUSH-OUT REWRITING

$$L \Rightarrow R$$

$$\begin{tikzcd}
L \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i_1"'] \arrow[r, hook, "i_2"] & R \\
G & &
\end{tikzcd}$$

Double Push-Out Rewriting

$$L \Rightarrow R$$

$$\begin{tikzcd}
L \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i_1"'] \arrow[r, hook, "i_2"] \arrow[d, hook] & R \\
G \arrow[ur, phantom, "\ulcorner", very near start] & G \setminus L \arrow[l, hook']
\end{tikzcd}$$

compute
push-out complement.

Double Push-Out Rewriting

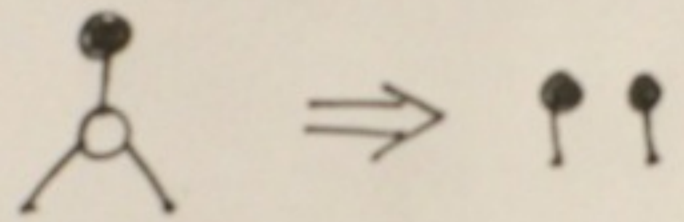
$$L \Rightarrow R$$

$$\begin{tikzcd}
L \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i_1"'] \arrow[r, hook, "i_2"] \arrow[d, hook, "j"'] \arrow[dl, phantom, "\urcorner", very near end] \arrow[dr, phantom, "\ulcorner", very near end] & R \arrow[d, hook] \\
G & G \setminus L \arrow[l, hook'] \arrow[r, hook] & G[R/L]
\end{tikzcd}$$

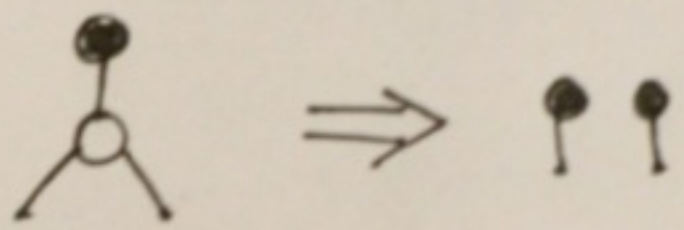
compute
push-out complement.

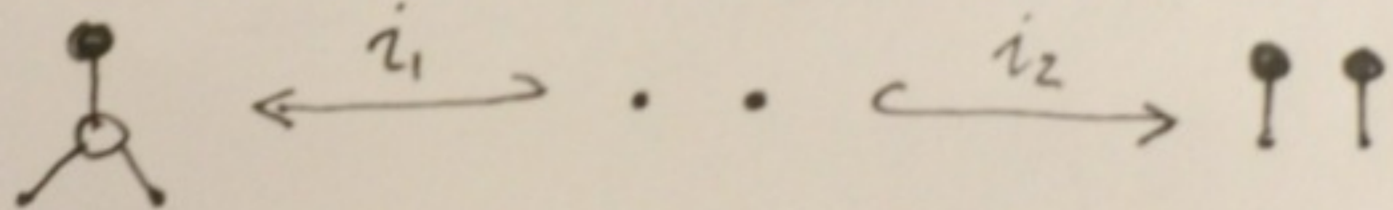
compute ~~po~~ pushout.

DPO REWRITING

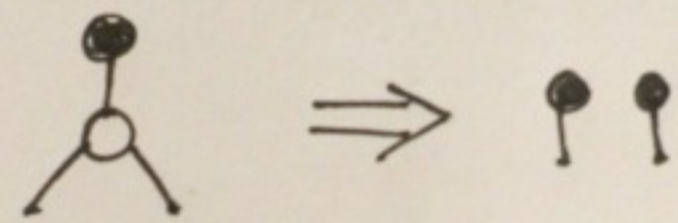


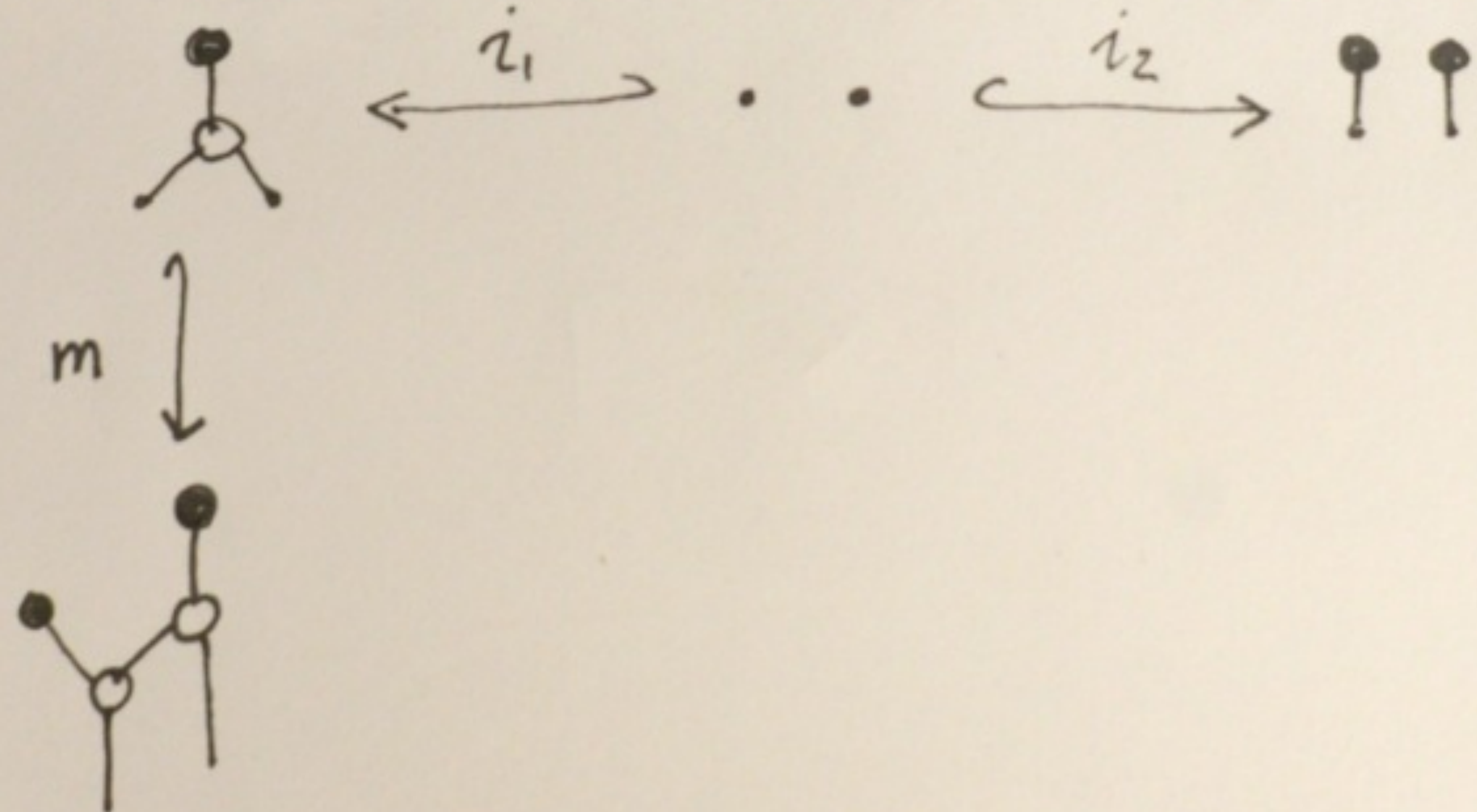
DPO REWRITING



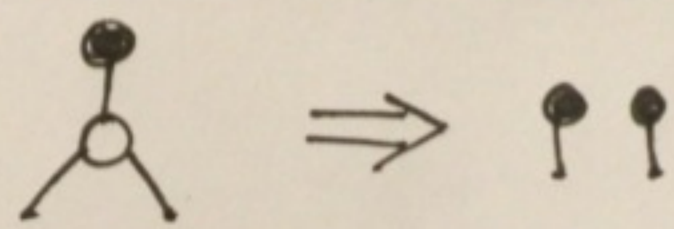


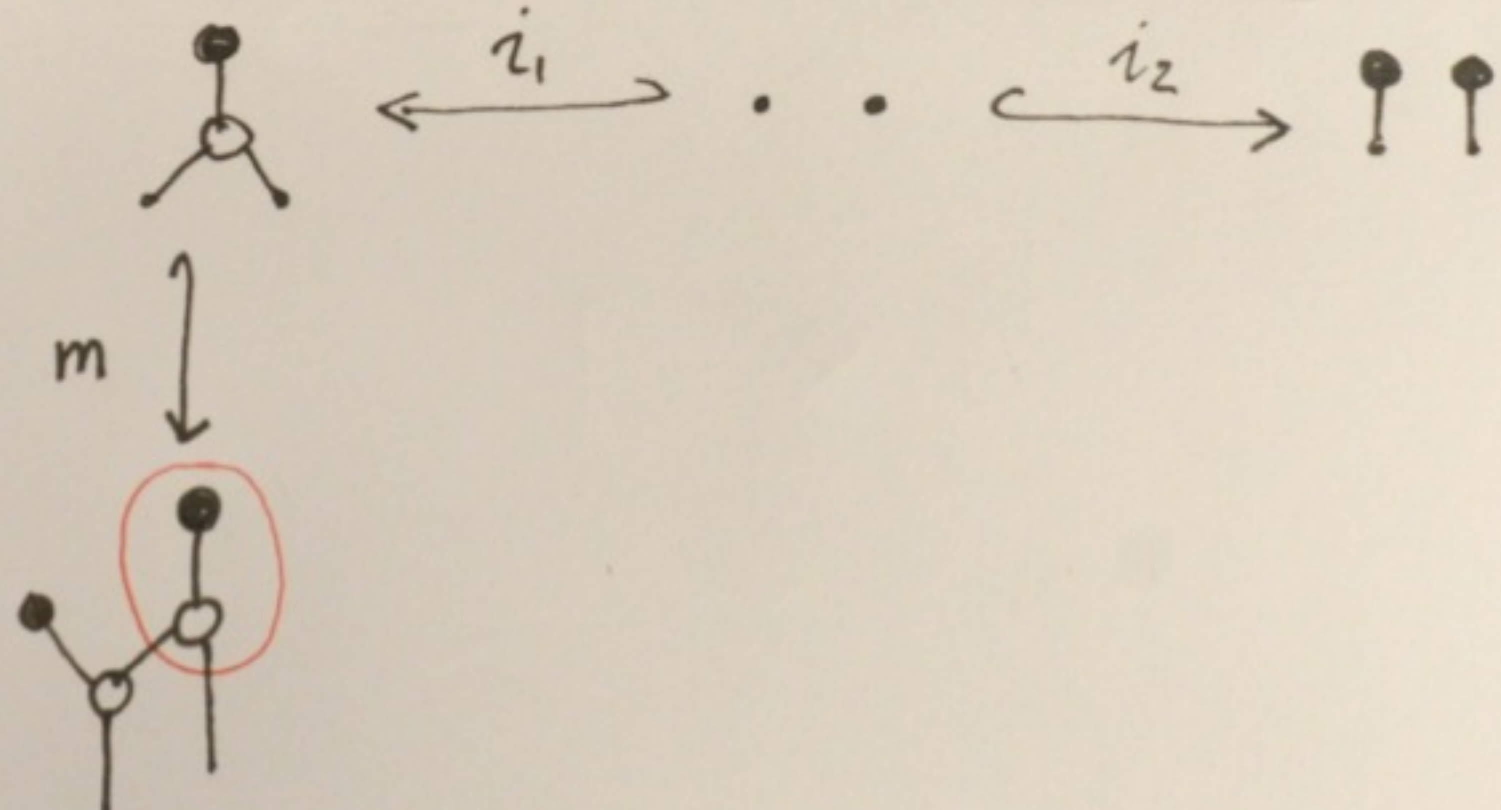
DPO REWRITING



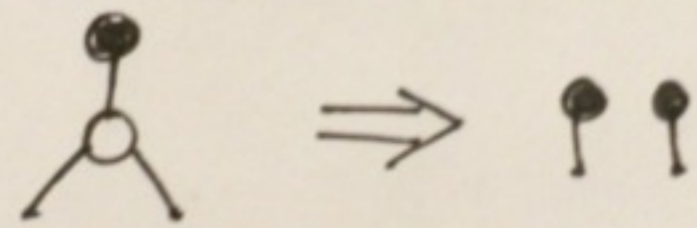


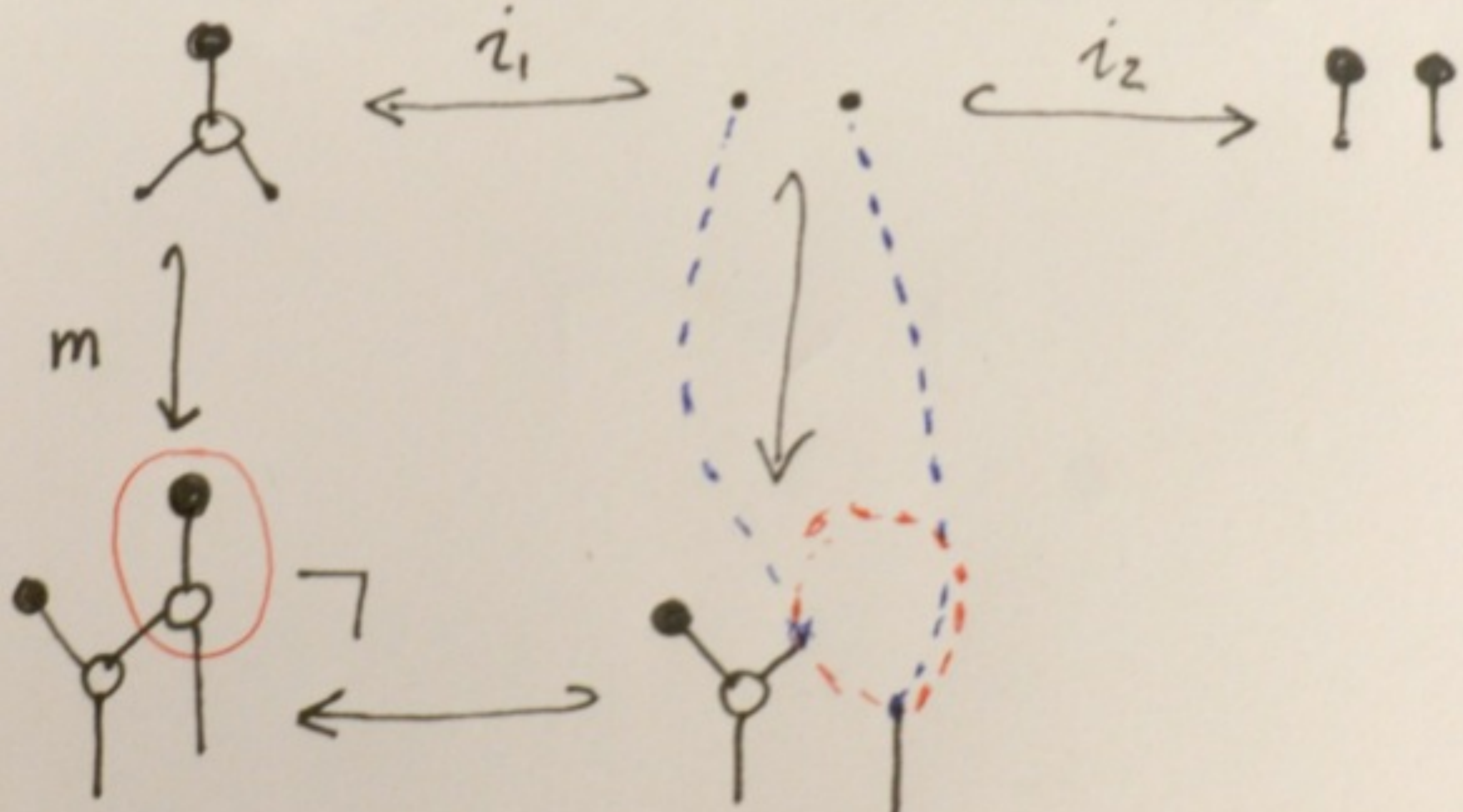
DPO REWRITING



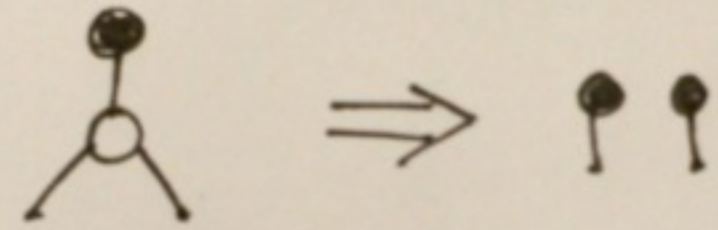


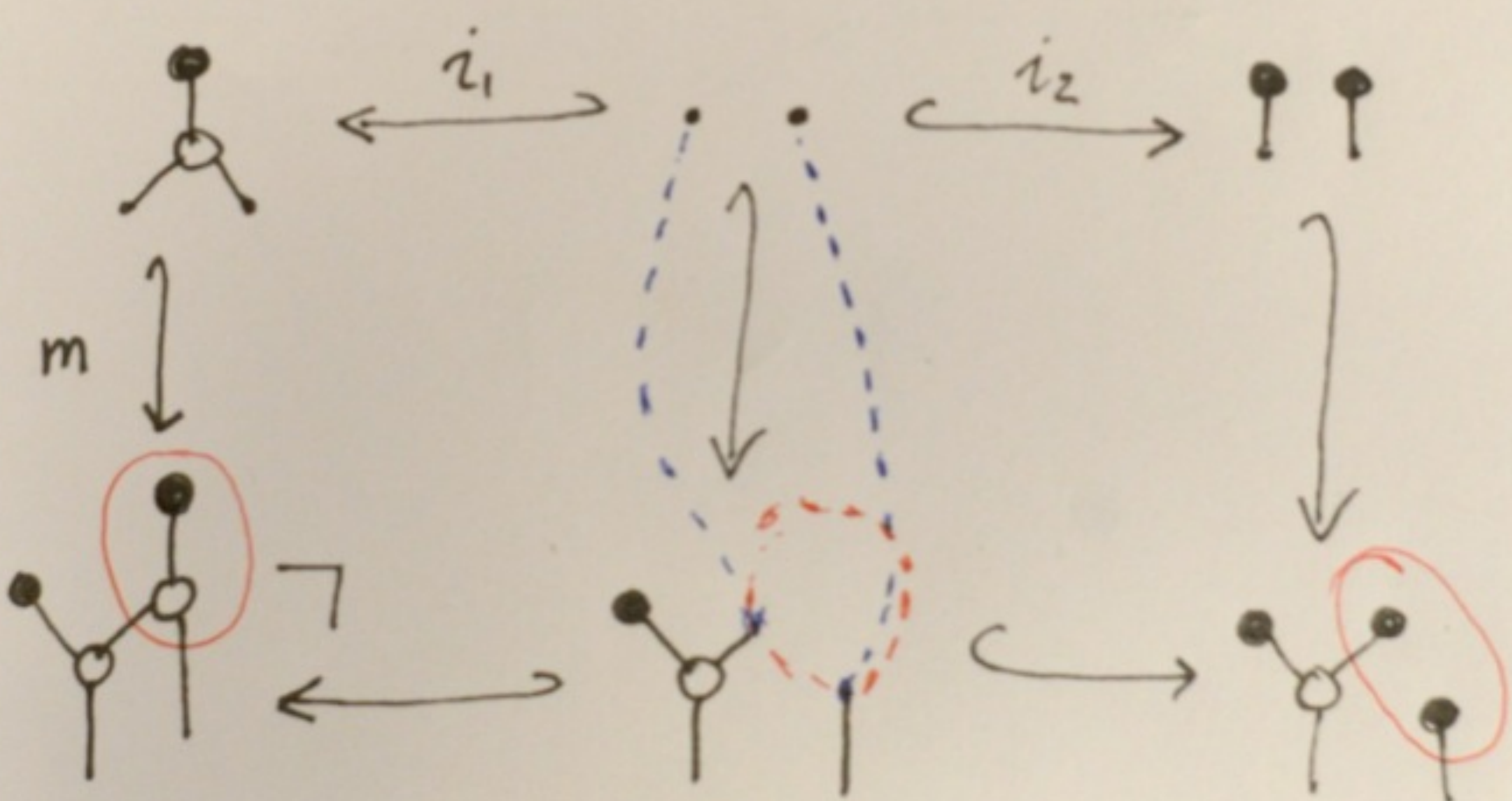
DPO REWRITING





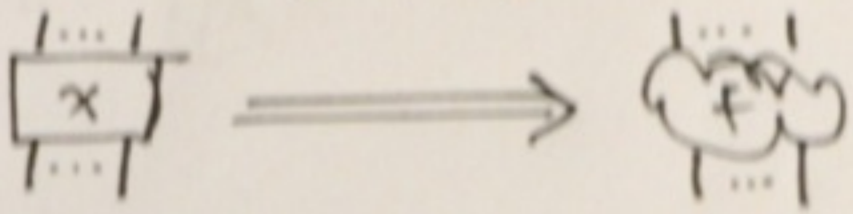
DPO REWRITING



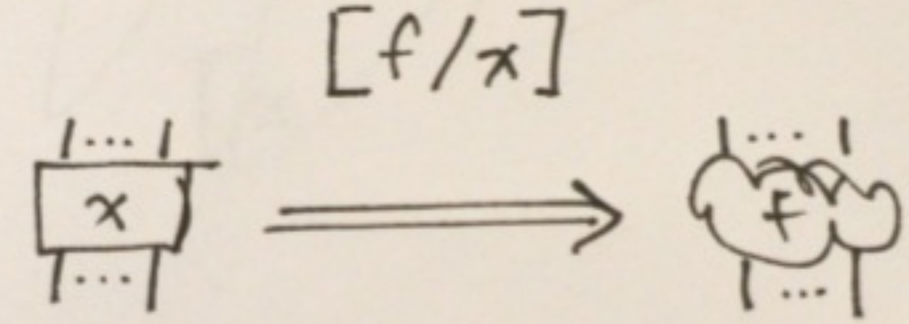


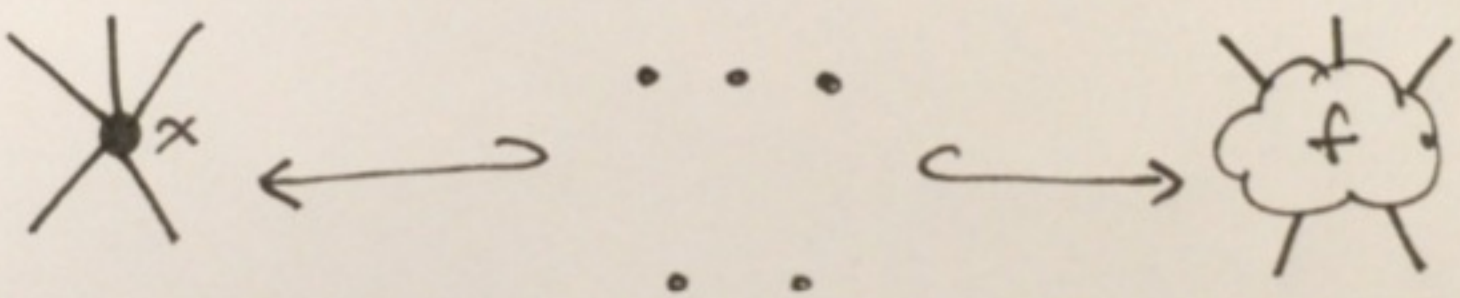
SUBSTITUTION VIA DPO

$[f/x]$



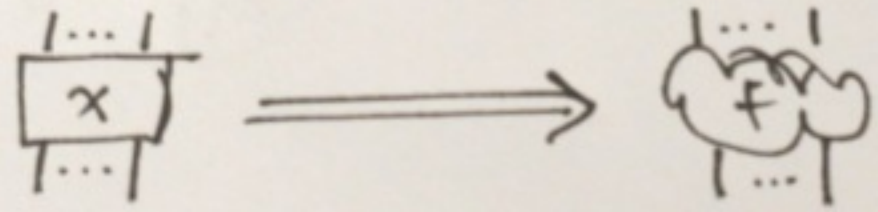
PLANE SUBSTITUTION VIA DPO

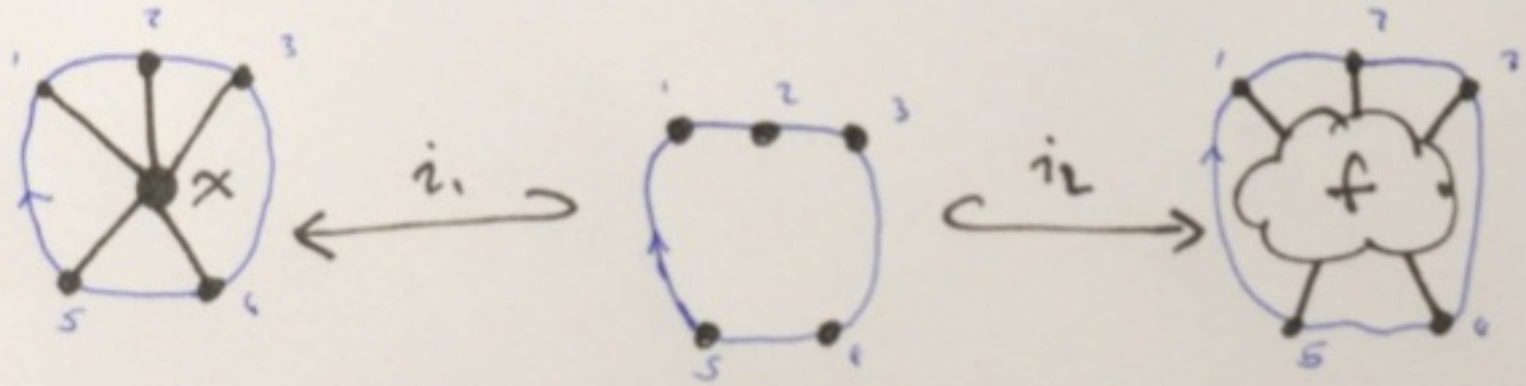




PLANE SUBSTITUTION VIA DPO

$[f/x]$

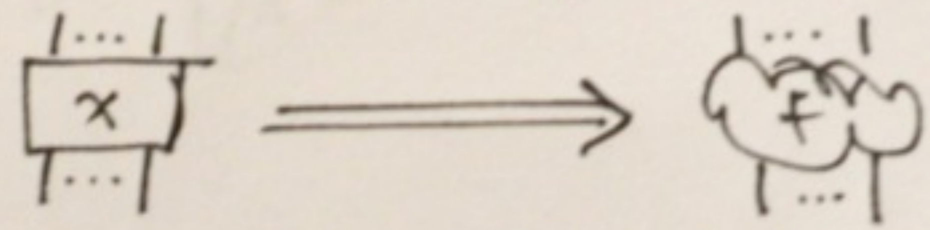


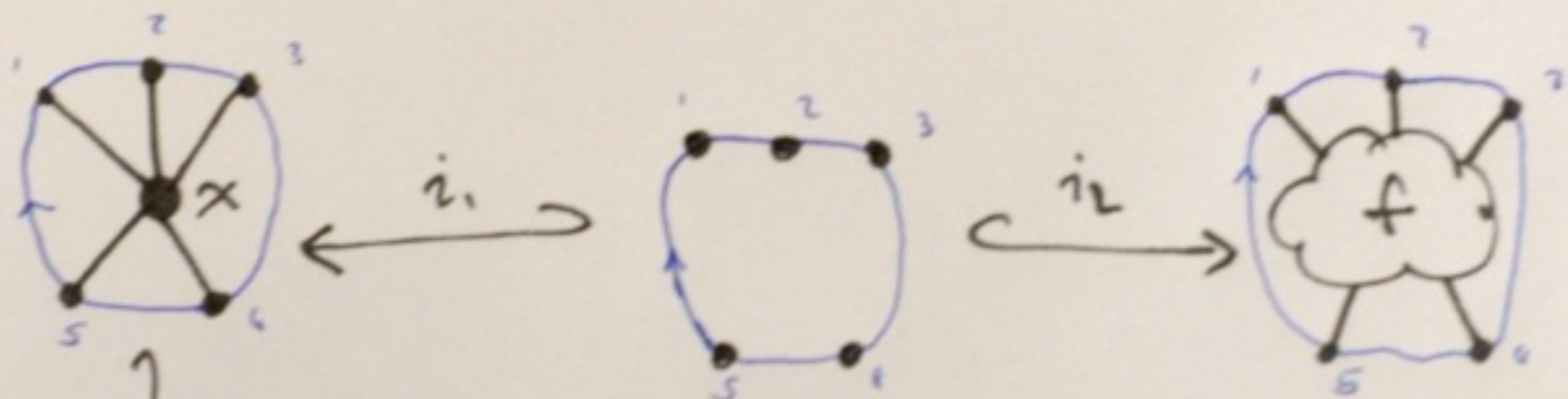


EMBEDDINGS BIJECTIVE ON THE BOUNDARY

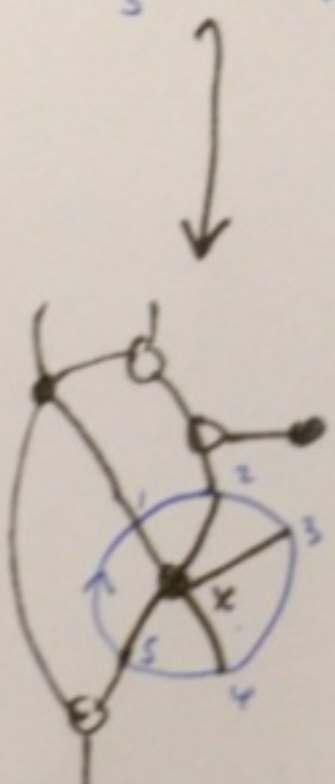
PLANE SUBSTITUTION VIA DPO

$[f/x]$



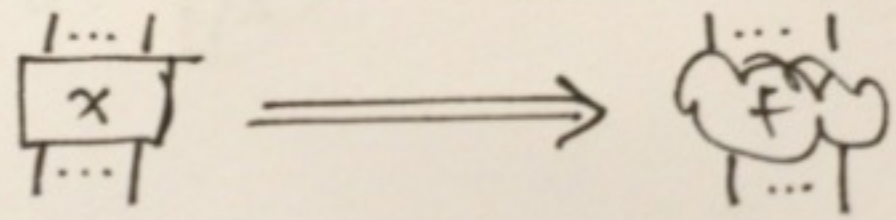


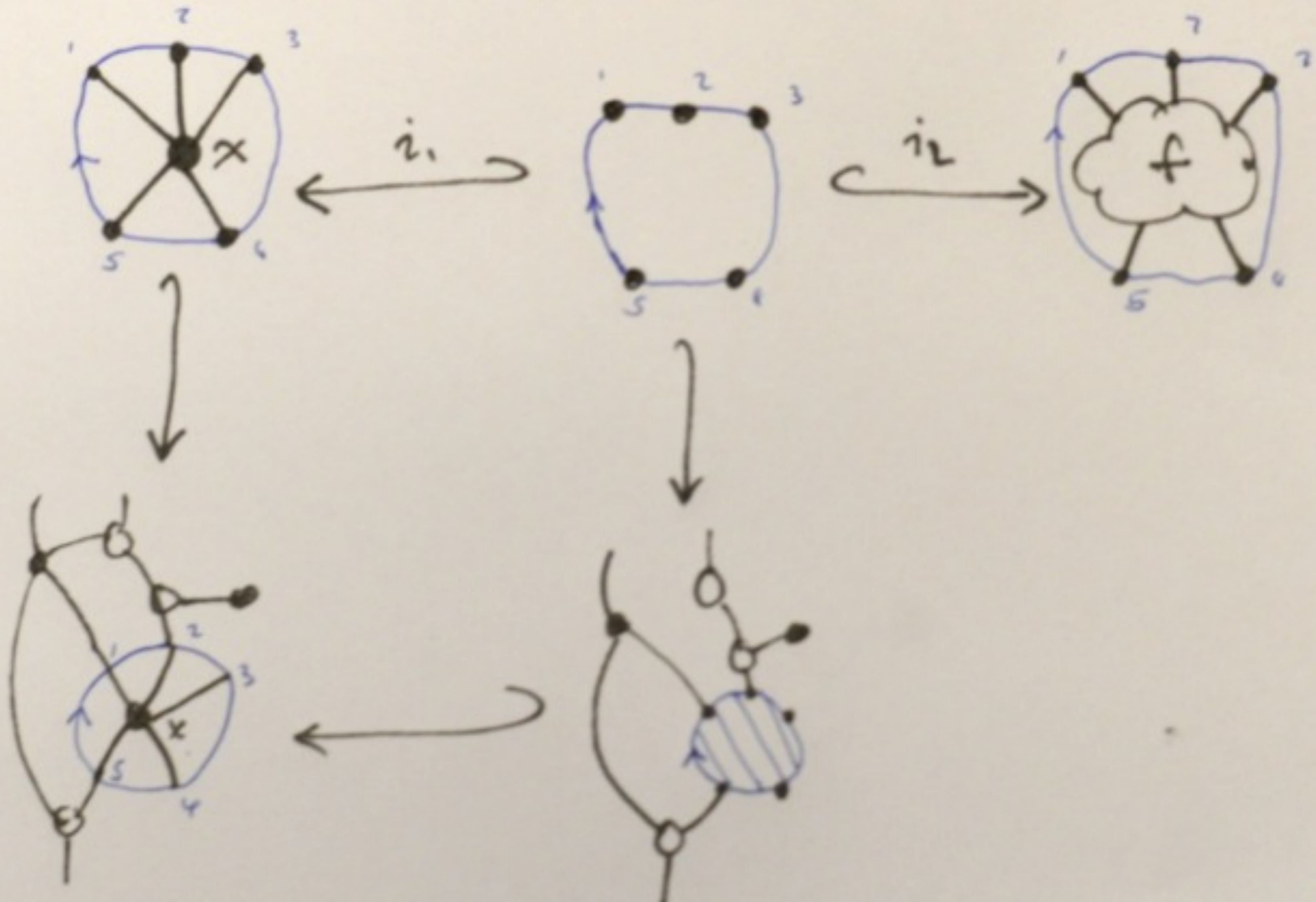
EMBEDDINGS BIJECTIVE ON THE BOUNDARY



PLANE SUBSTITUTION VIA DPO

$[f/x]$

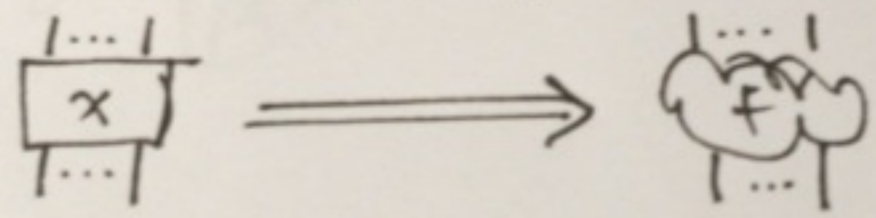


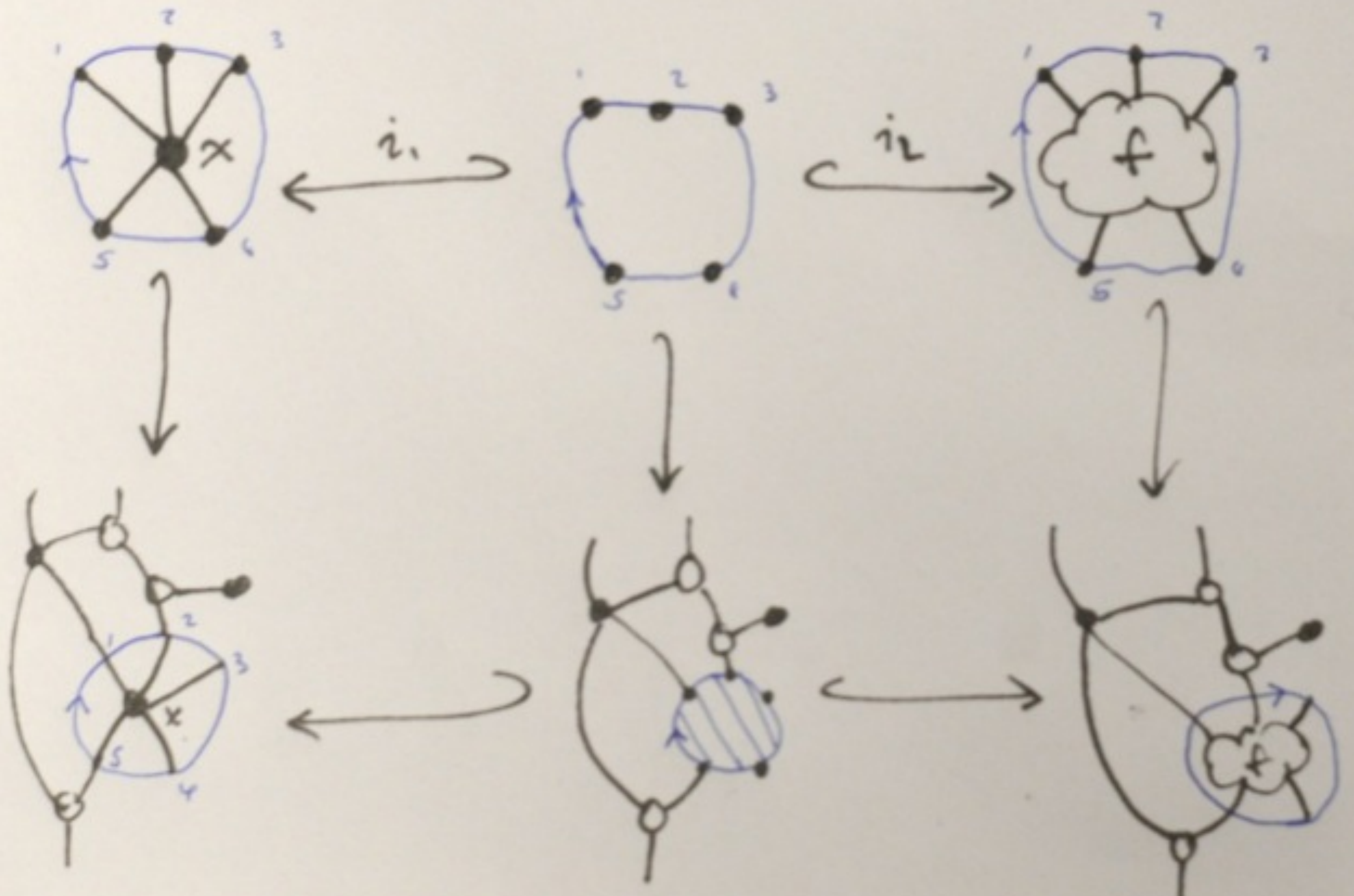


EMBEDDINGS BIJECTIVE ON THE BOUNDARY

PLANE SUBSTITUTION VIA DPO

$[f/x]$



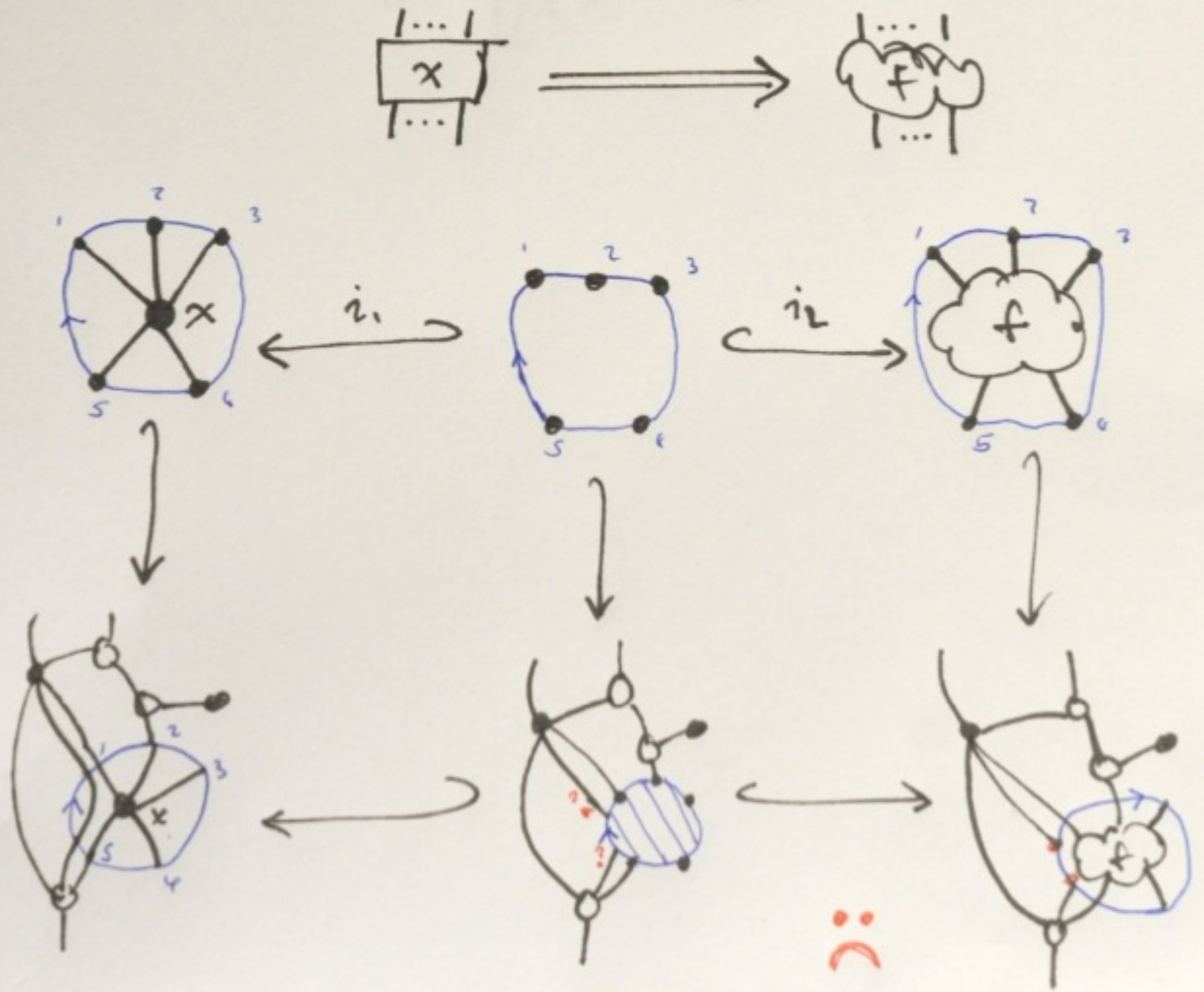


EMBEDDINGS BIJECTIVE ON THE BOUNDARY

See Slofstra

Plane Substitution via DPO

$[f/x]$



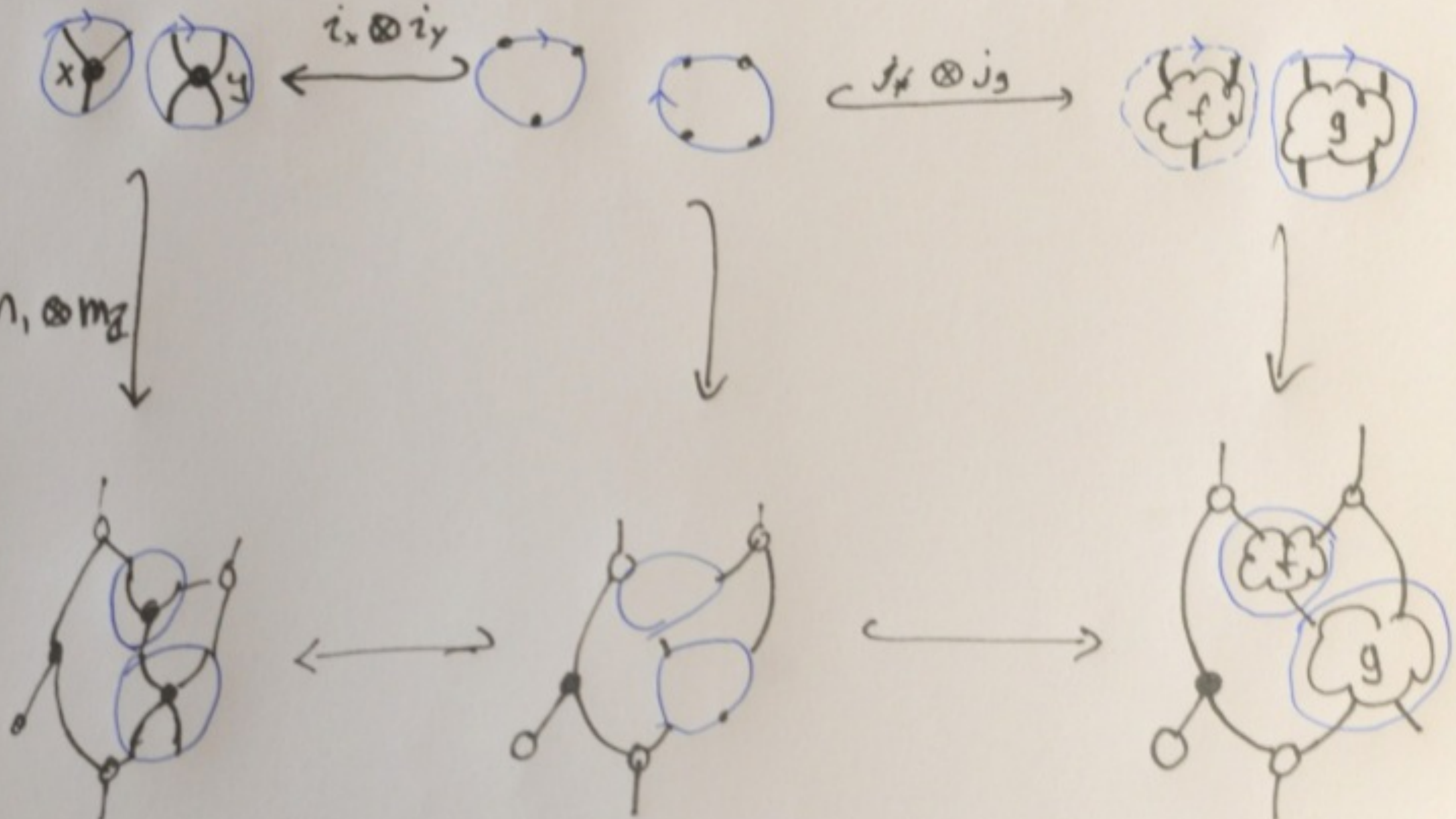
Embeddings bijective on the boundary

Bad!

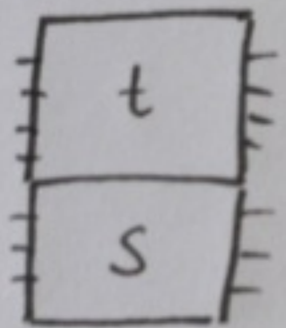
See Slofstra

PLANE SUBSTITUTION

$$[f/x,\ g/y]$$



Logical Rules

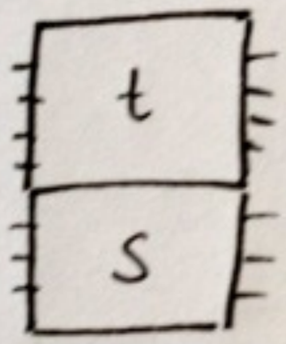


"tensor"

$$\frac{\bar{x}:\Delta \vdash t:A \qquad \bar{y}:\Gamma \vdash s:B}{\bar{x}:\Delta, \bar{y}:\Gamma \vdash t\otimes s : A\otimes B}$$

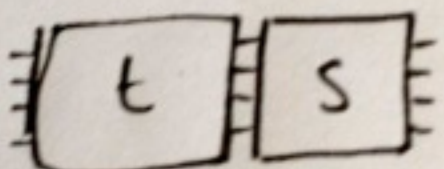
LOGICAL RULES

"tensor"



$$\frac{\bar{x}:\Delta \vdash t : A \qquad \bar{y}:\Gamma \vdash s : B}{\bar{x}:\Delta,\ \bar{y}:\Gamma \vdash t \otimes s : A \otimes B}$$

"composite"



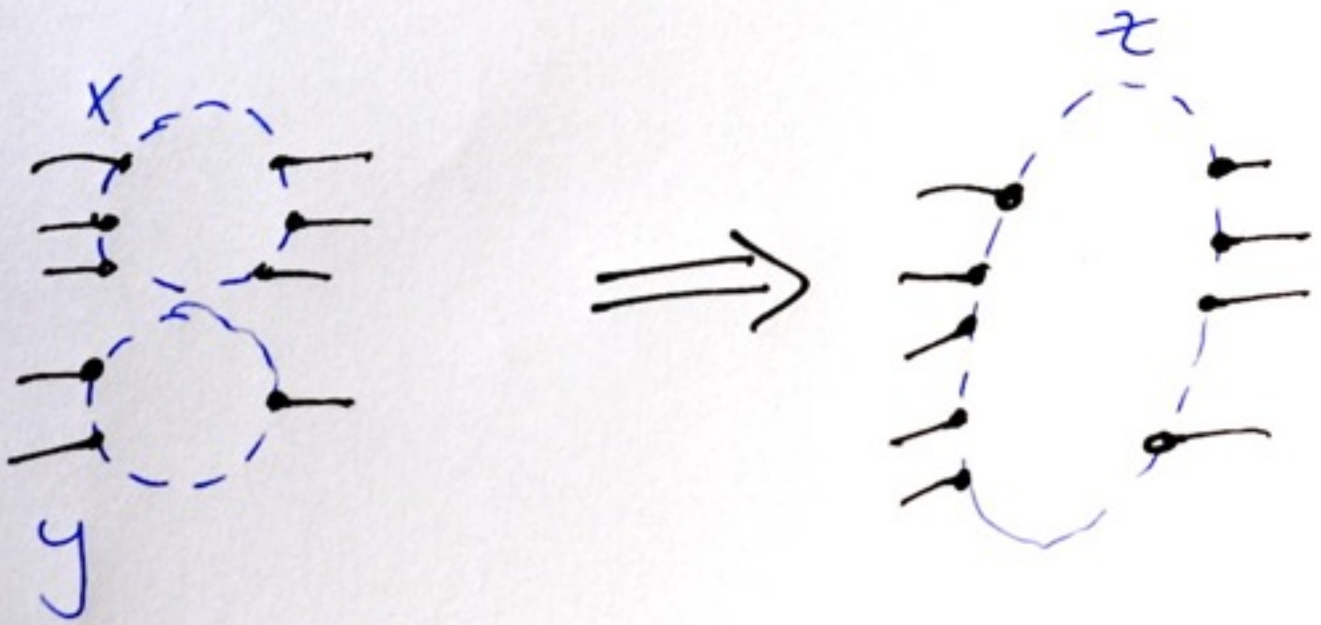
$$\frac{\bar{x}:\Delta \vdash t : (n,m) \qquad \bar{y}:\Gamma \vdash s : (m,k)}{\bar{x}:\Delta,\ \bar{y}:\Gamma \vdash s \circ t : (n,k)}$$

NOT ALLOWED

$$\frac{x : A,\ y : B \ \vdash\ t : C}{z : A \otimes B \ \vdash\ \text{let } z = x \otimes y \text{ in } t \ : C}$$

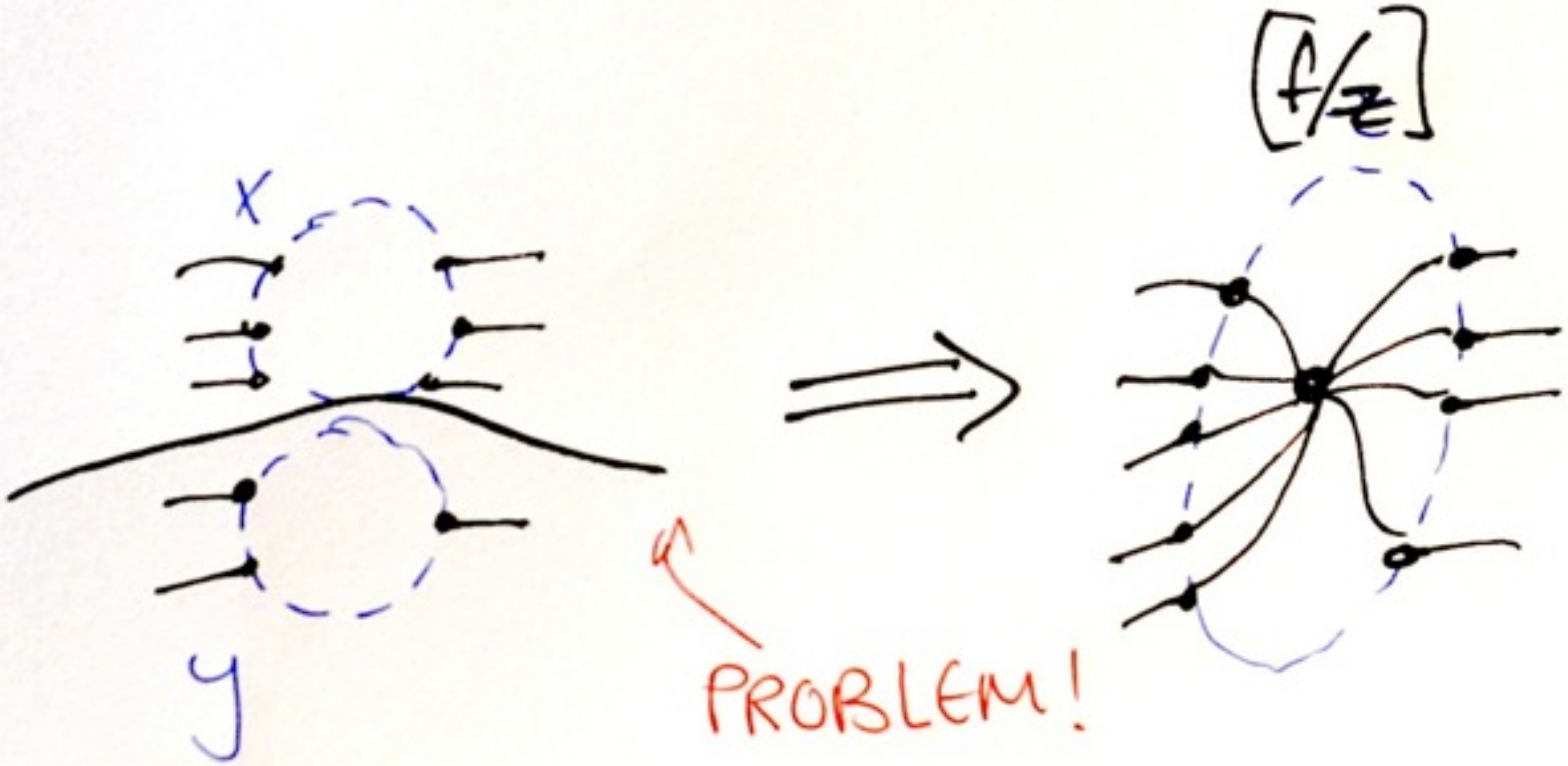
NOT ALLOWED

$$\frac{x : A,\ y : B \ \vdash\ t : C}{z : A \otimes B \ \vdash\ \text{let } z = x \otimes y \text{ in } t \ : C}$$



NOT ALLOWED

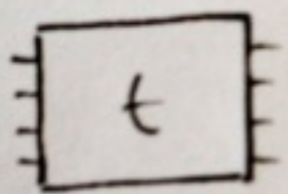
$$\frac{x : A,\ y : B \ \vdash\ t : C}{z : A \otimes B \ \vdash\ \mathrm{let}\ z = x \otimes y\ \mathrm{in}\ t \ : C}$$



DITCHING LINEARITY

LOGICAL RULES

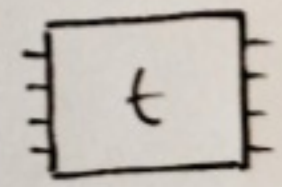
weakening



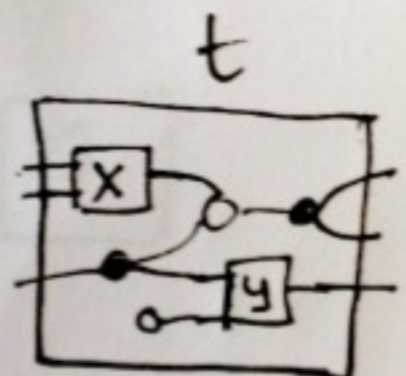
$$\frac{\bar{x} : \Delta \vdash t : A}{y : B, \bar{x} : \Delta \vdash t : A}$$

LOGICAL RULES

weakening



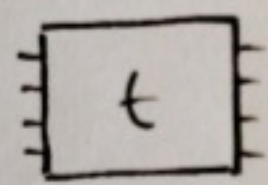
$$\frac{\bar{x} : \Delta \vdash t : A}{y : B, \bar{x} : \Delta \vdash t : A}$$



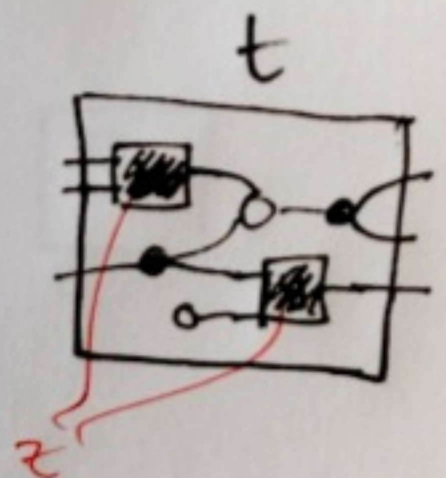
$$\frac{x : A, y : A \vdash t : B}{z : A \vdash t[z/x, z/y]}$$

LOGICAL RULES

weakening



$$\frac{\bar{x}:\Delta \vdash t:A}{y:B,\ \bar{x}:\Delta \vdash t:A}$$



$$\frac{x:A,\ y:A \vdash t:B}{z:A \quad \vdash t[z/x,\ z/y]}$$

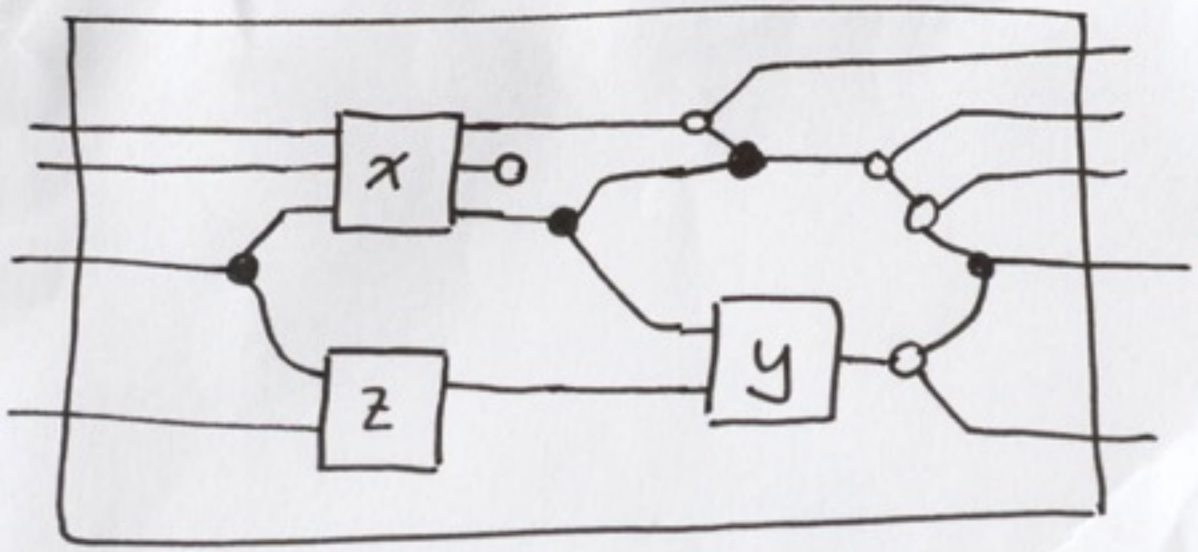
"CONTRACTION"

SUMMARY PT. 1

1. SUBSTITUTION AND OPERATIONS IN UNDERLYING PRO form a "monoidal++" operad.

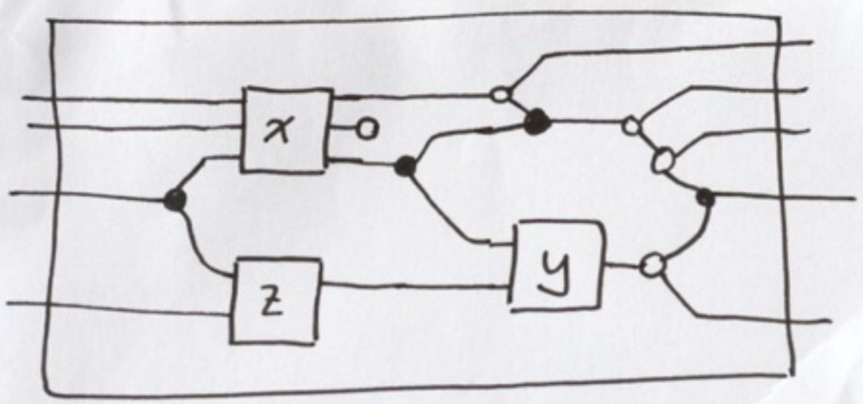
2. Variable manipulations give a cocommutative comonoid. But don't allow $\otimes$.

3. PATTERN-MATCHING.

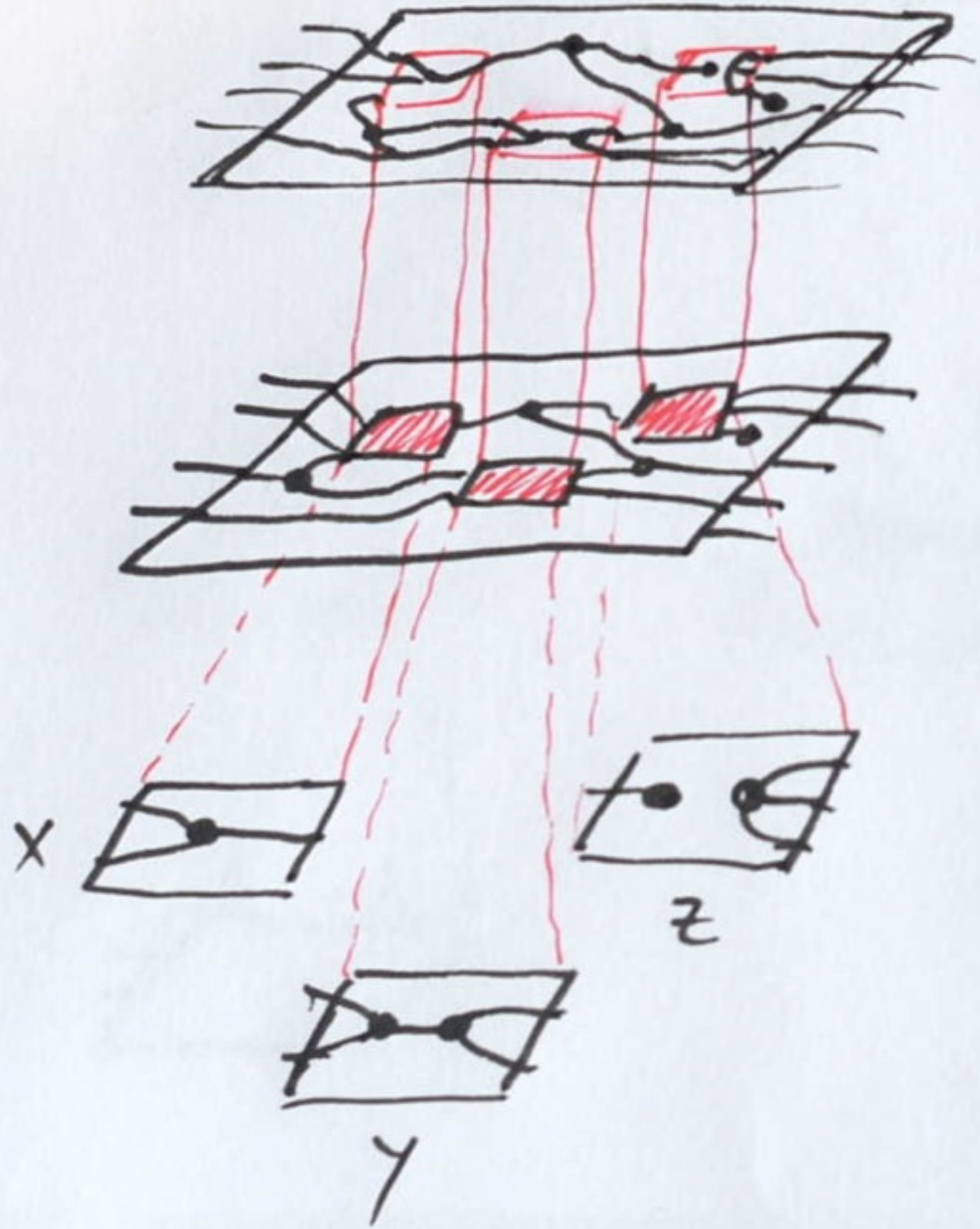


$x:(3,3),\ y:(2,1),\ z:(2,1) \vdash f:(4,5)$

3. PATTERN-MATCHING



$x:(3,3),\ y:(2,1),\ z:(2,1) \vdash f:(4,5)$

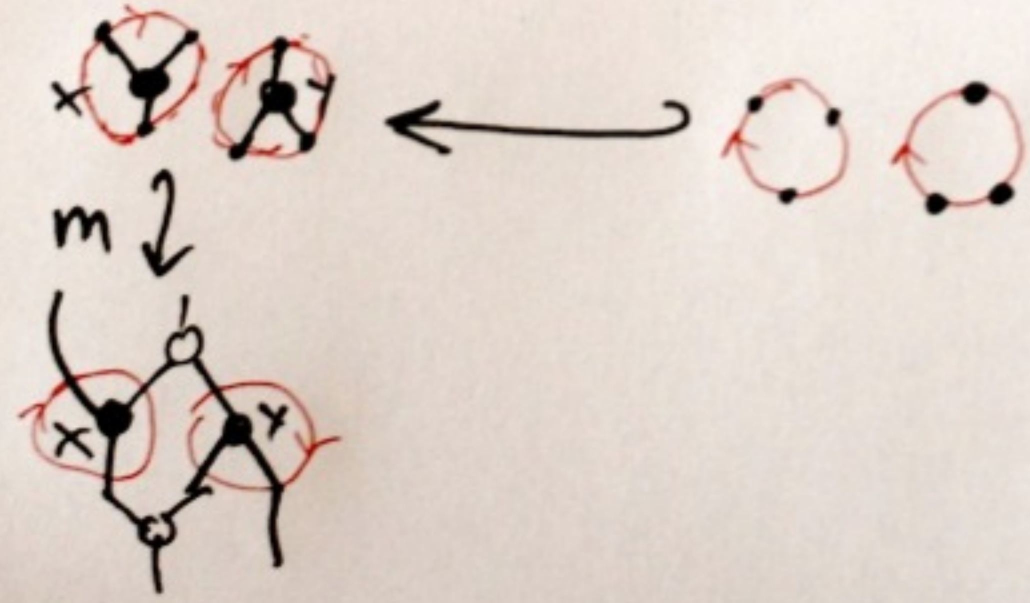


3. PATTERN MATCHING

$$\begin{tikzcd} V \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i"'] \\ G & \end{tikzcd}$$

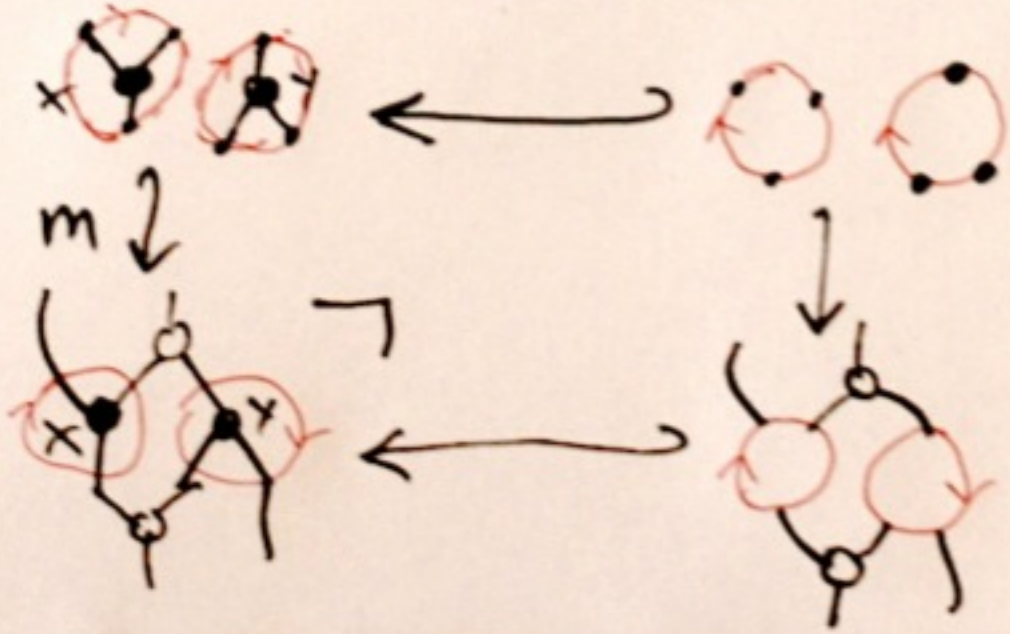
3. Pattern Matching

$$\begin{tikzcd} V \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i"'] \\ G \end{tikzcd}$$



3. Pattern Matching

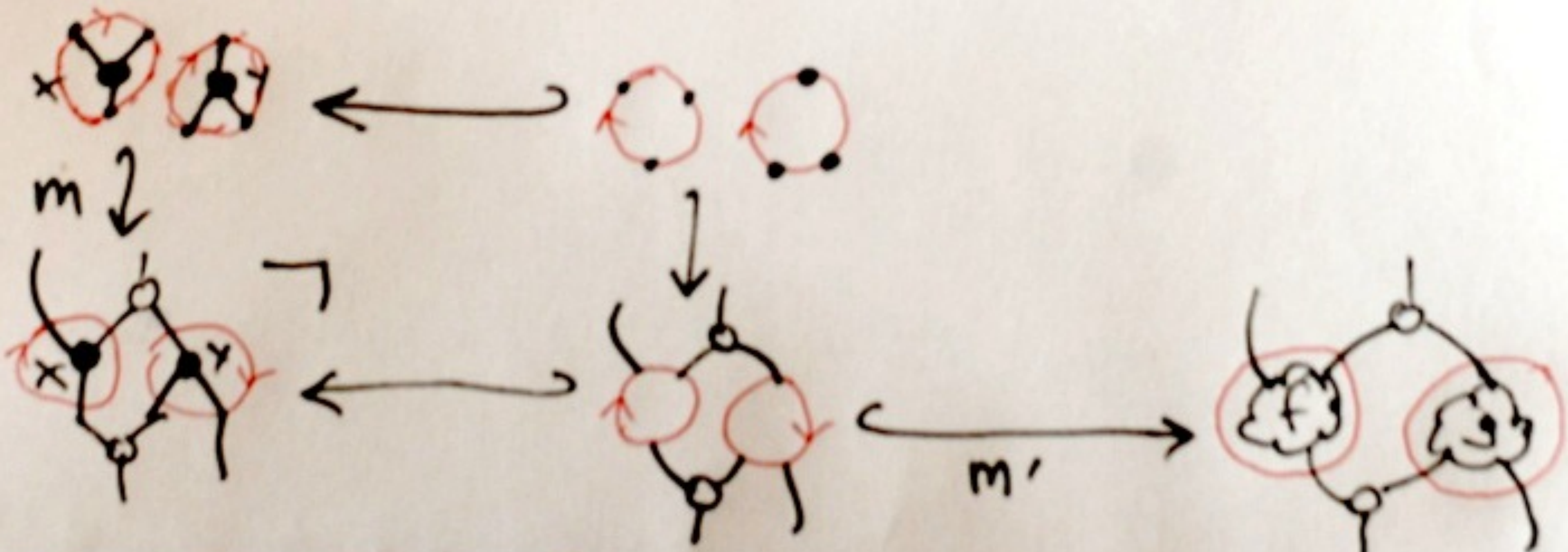
$$\begin{tikzcd}
V \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i"'] \arrow[d, hook] \\
G \arrow[ur, phantom, "\urcorner", very near start] & G \setminus V \arrow[l, hook']
\end{tikzcd}$$



3. Pattern Matching

$$\begin{tikzcd}
V \arrow[d, "m"'] & \partial \arrow[l, hook', "i"'] \arrow[d, hook] & \\
G \arrow[ur, phantom, very near start, "\urcorner"] & G \setminus V \arrow[l, hook'] \arrow[r, hook, "m'"'] & H
\end{tikzcd}$$

suppose this exists.

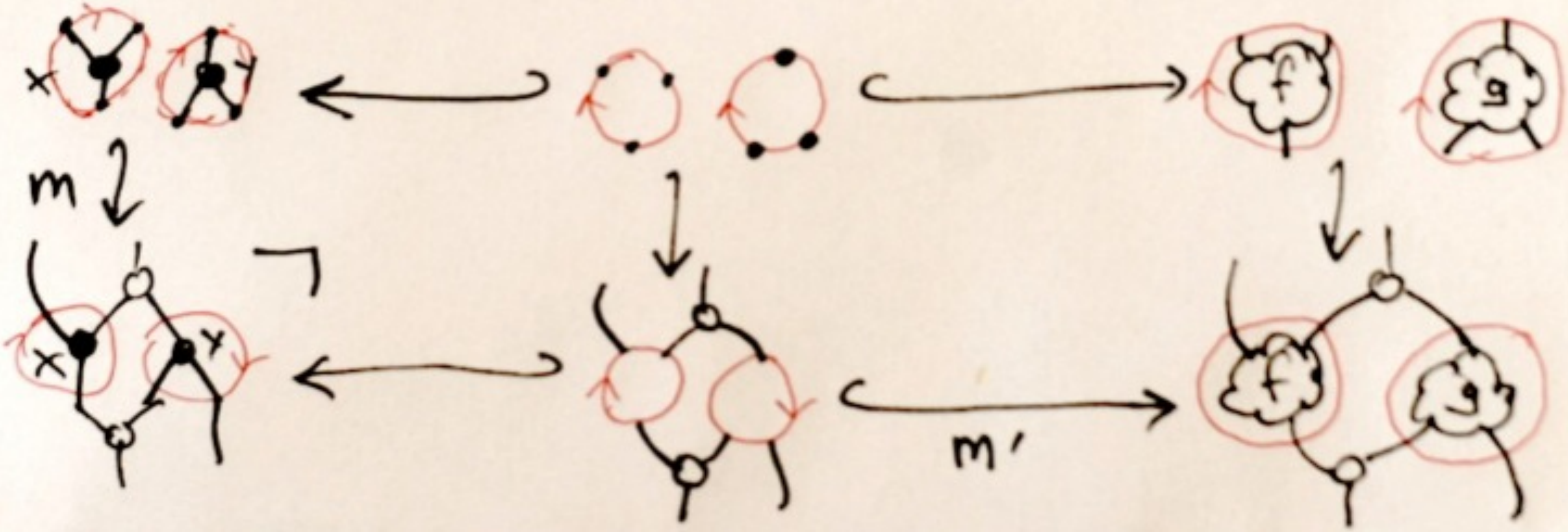


3. Pattern Matching

compute this pushout complement.

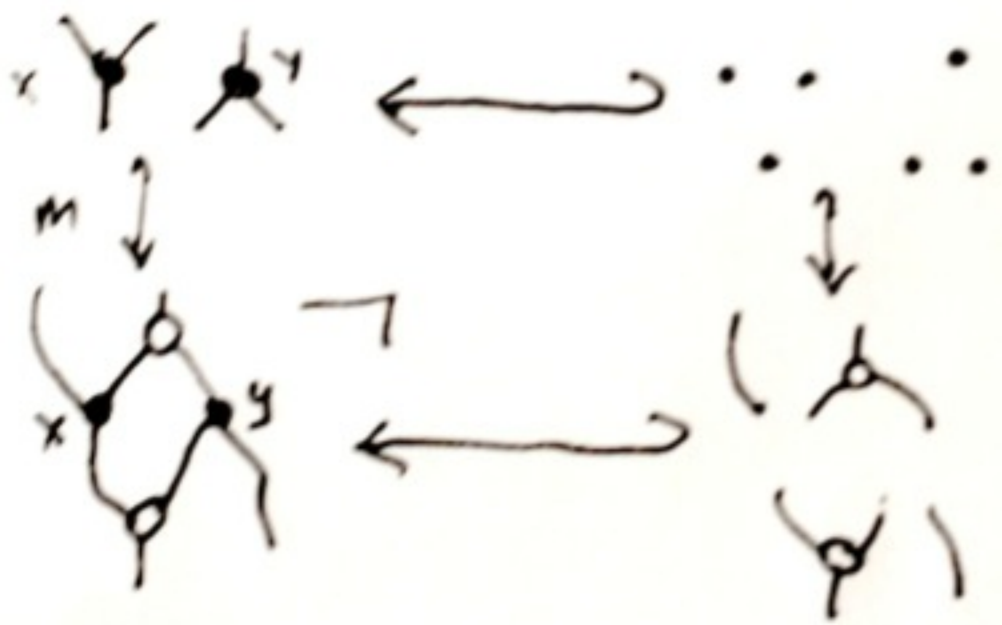
$$
\begin{tikzcd}
V \arrow[d, hook, "m"'] & \partial \arrow[l, hook', "i"'] \arrow[r, hook, "i'"] \arrow[d, hook] & H \setminus (G \setminus V) \arrow[d, hook] \\
G \arrow[ur, phantom, "\lrcorner", very near start] & G \setminus V \arrow[l, hook'] \arrow[r, hook, "m'"'] & H \arrow[ul, phantom, "\ulcorner", very near start]
\end{tikzcd}
$$

suppose this exists.



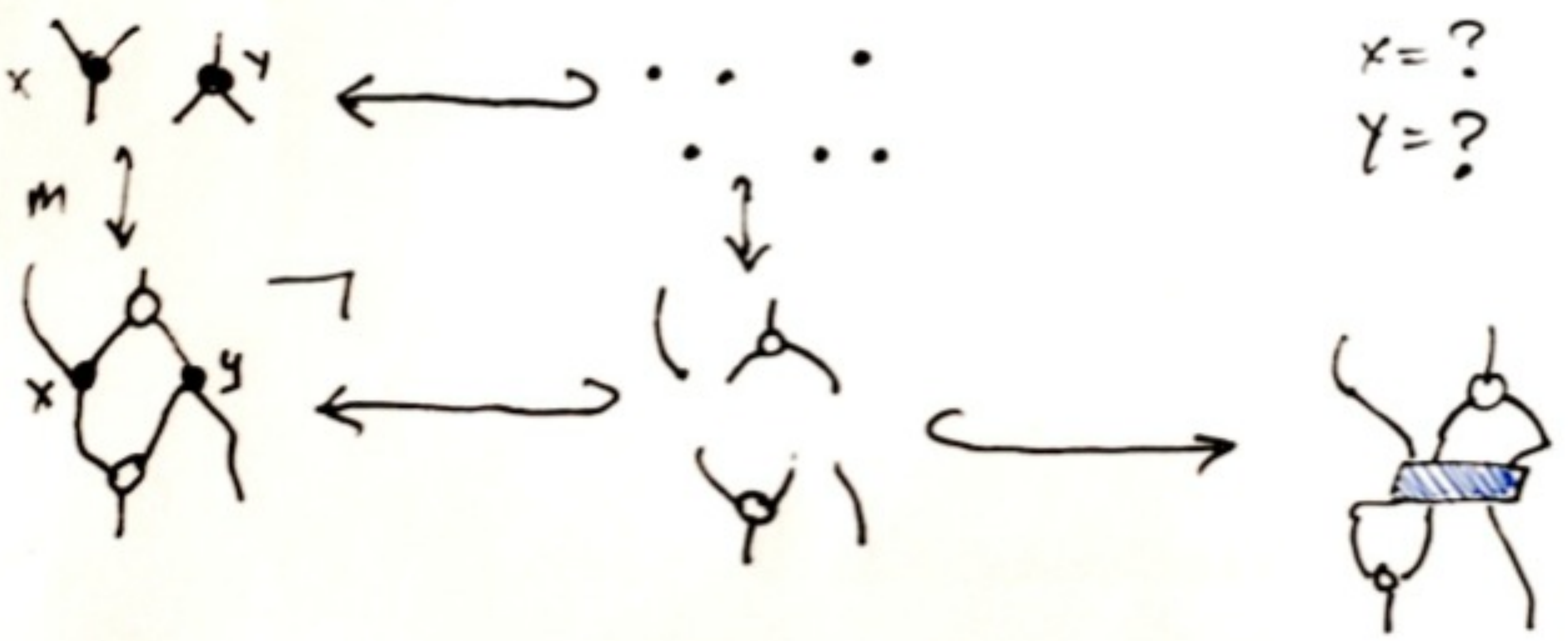
3. PATTERN MATCHING

NOTE PRESERVATION OF BOUNDARY CURVE IS ESSENTIAL:



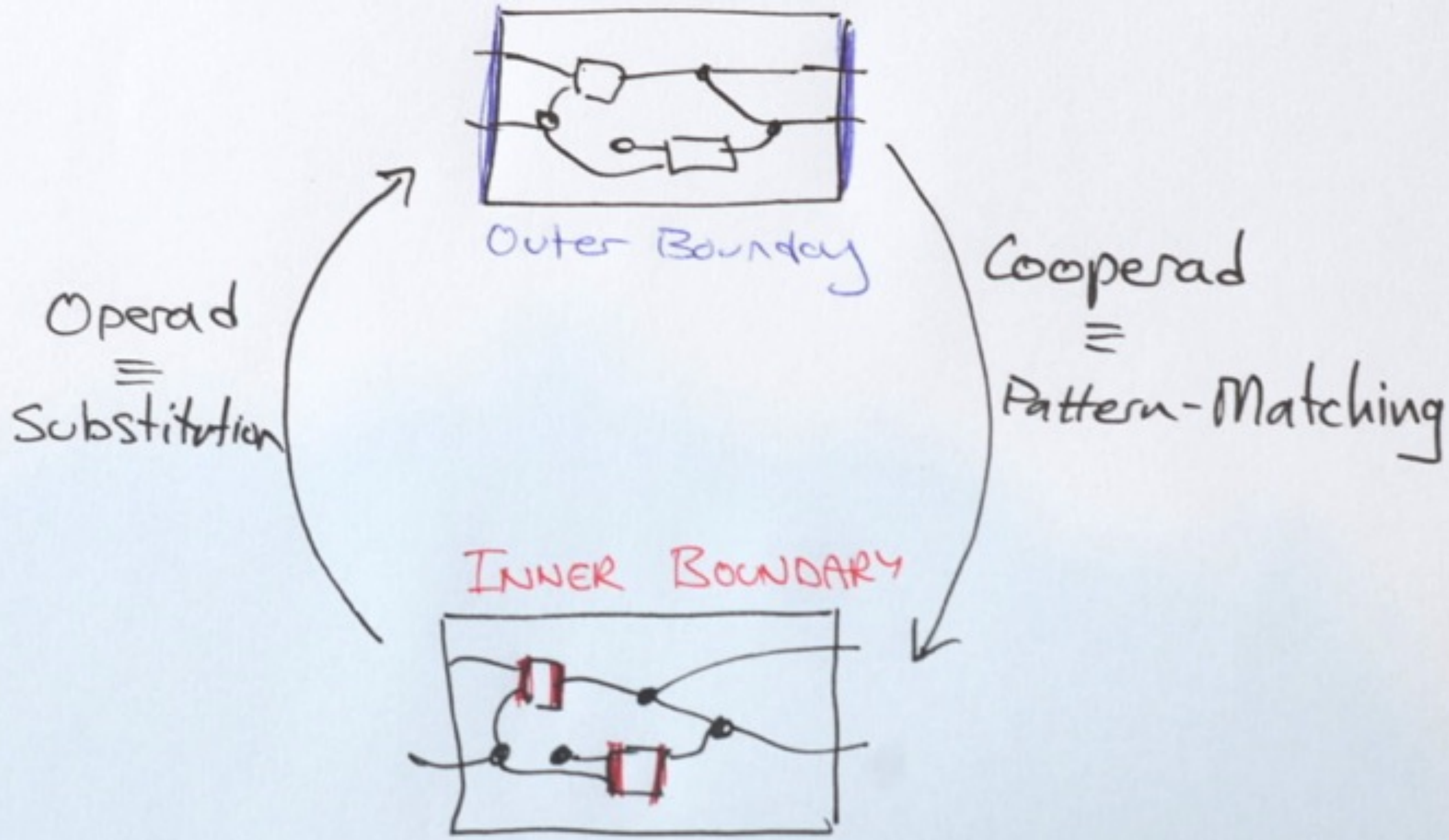
3. PATTERN MATCHING

NOTE PRESERVATION OF BOUNDARY <u>CURVE</u> IS ESSENTIAL:



PUTTING IT TOGETHER

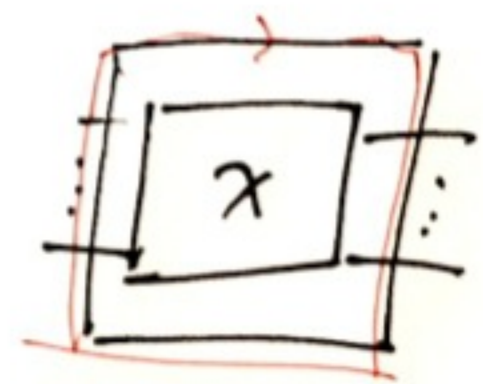
STRING DIAGRAMS w/ VARIABLES



Putting it together

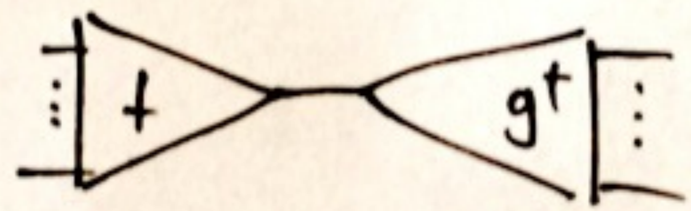
Identities are same in operad and cooperad

$$\frac{}{x : A \vdash x : A}$$



PUTTING IT TOGETHER

Composing like this

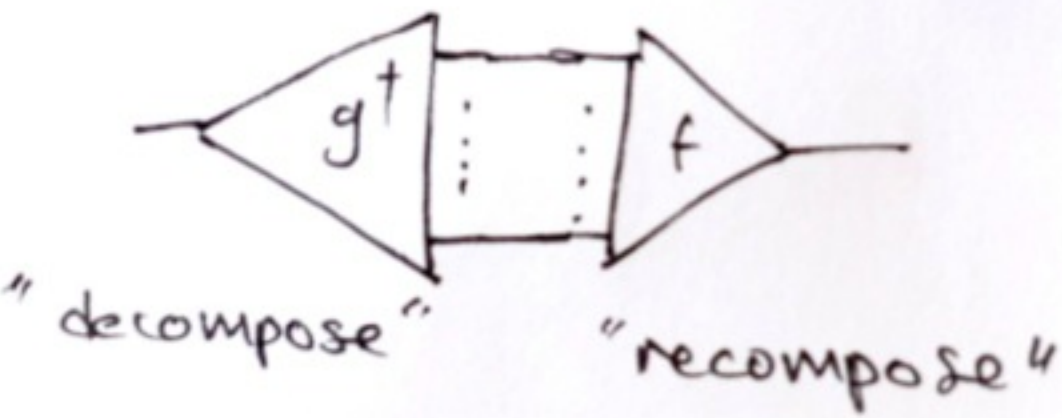


"construct" "deconstruct"

makes sense.

PUTTING IT TOGETHER

Composing like this



makes sense.

PUTTING IT TOGETHER

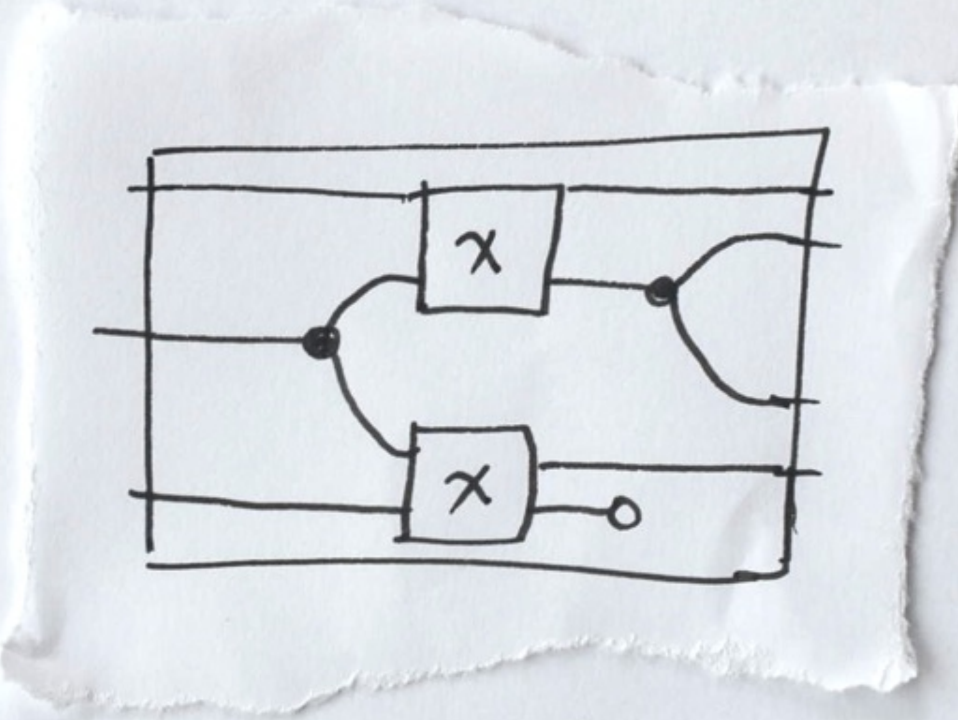
STRING DIAGRAMS WITH VARIABLES
FORM A { COMPUTAD / POLYCATEGORY }

OF MANY-TO-MANY, DIAGRAM TRANSFORMATIONS
PARTIAL.

(WITH THE MIX RULE)

4. Ditching Linearity

$$\frac{\Delta \vdash t : A,\ t' : A}{\Delta \vdash t'' : A}\ \text{Contraction}$$

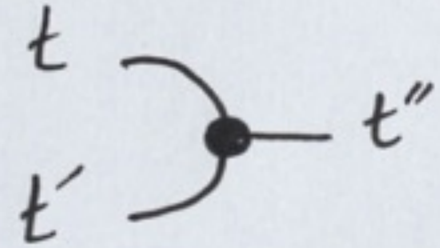


$$\frac{\Delta \vdash t : A}{\Delta \vdash t : A,\ t'' : B}\ \text{Weakening}$$

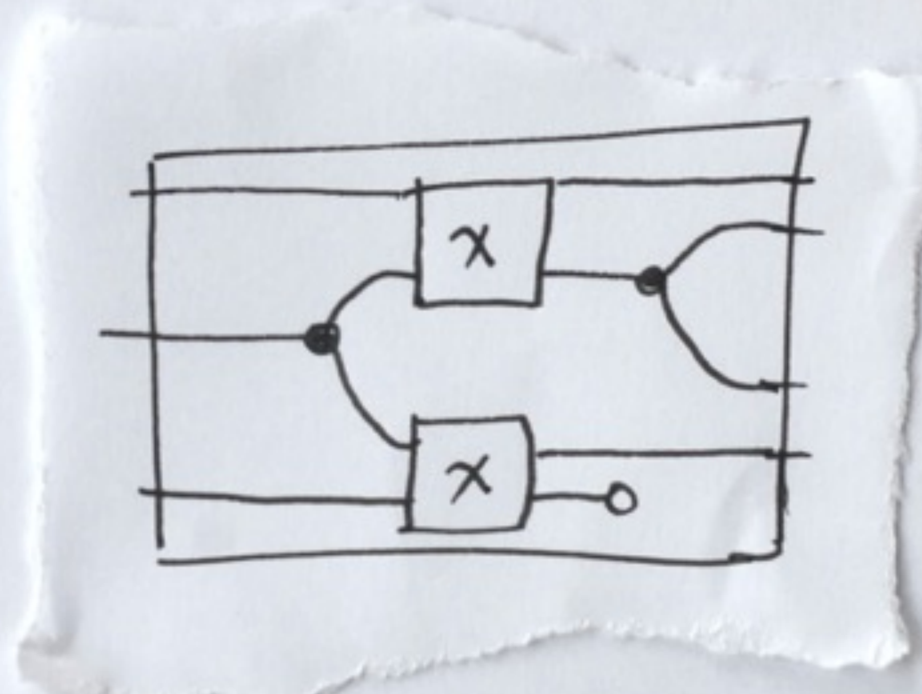
4. Ditching Linearity

$$\frac{\Delta \vdash t : A,\ t' : A}{\Delta \vdash t'' : A}\ \text{Contraction}$$

with $t = t' = t''$



$$\frac{\Delta \vdash t : A}{\Delta \vdash t : A,\ t'' : B}\ \text{Weakening}$$



4. Ditching Linearity

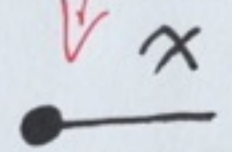
$$\frac{\Delta \vdash t : A,\ t' : A}{\Delta \vdash t'' : A}\ \text{Contraction}$$

$$\frac{\Delta \vdash t : A}{\Delta \vdash t : A,\ t'' : B}\ \text{Weakening}$$

with $\cancel{t = t' = t''}$

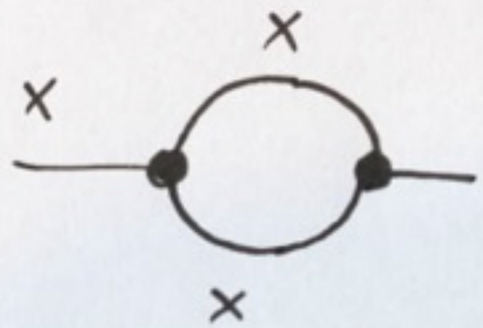
$t'' = \text{M.C.U.}(t, t')$

Commutative Monoid!*



where x is a fresh variable.

4. DITCHING LINEARITY.

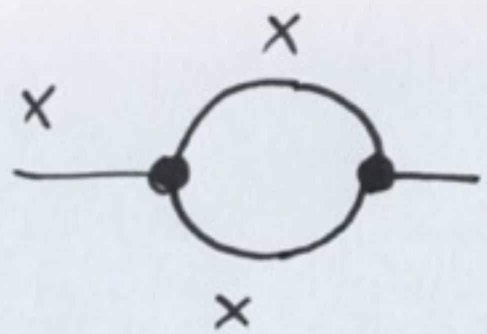
SPECIAL! 

MGU(x, x) = x

= 

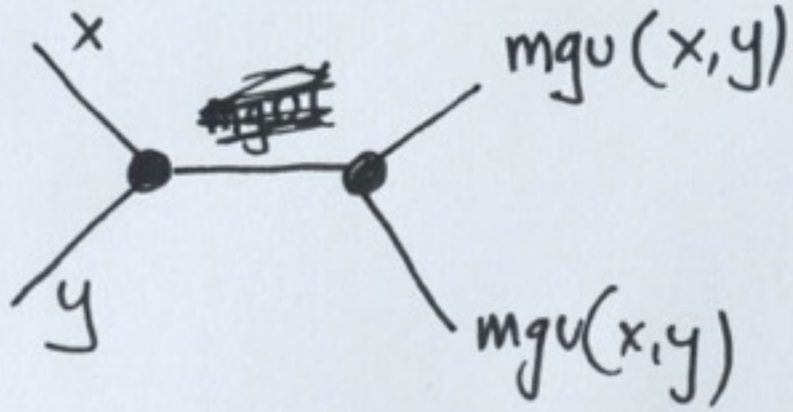
4. DITCHING LINEARITY.

SPECIAL!

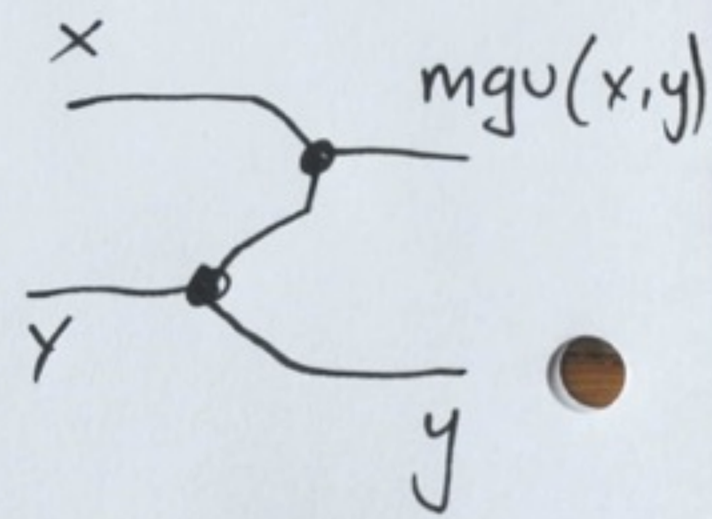


MGU(x, x) = x

=



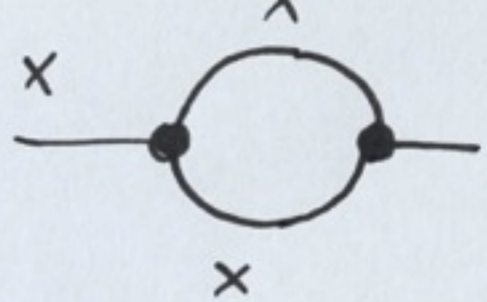
$\neq$



NOT FROB :(

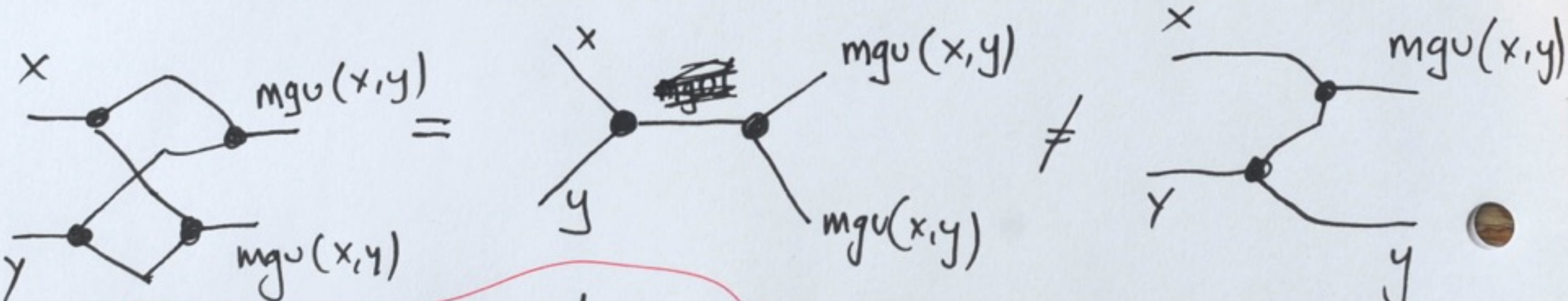
4. Ditching Linearity.

SPECIAL!



$\mathrm{MGU}(x, x) = x$

=



IS BIALGEBRA! :)

NOT FROB :(

Open Problems

- How to compute MGU for two diagrams?
    - Trickier than expected because the category does not ave many push-outs!
- Cut-elimination for the whole computad?
- Can we we express the separation condition for combinatorial planar graphs?


THANKS!