

# Roles of Shared Relations in Induction

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## Abstract

Two experiments examined the roles of shared relations between representations in induction. Lassaline (1996) found that shared attributes contribute to the inductive strength, but shared relations do not, whereas both shared attributes and shared relations contribute to similarity judgment. A structural alignment view of induction was generalized to account for these phenomena. According to the structural alignment view proposed in this paper, (1) insufficiency of the number of shared relations caused the dissociation between shared relations and inductive strength, and (2) structural alignment during similarity judgment made shared relations so salient as to increase similarity. Experiment 1 examined the first hypothesis. Participants judged inductive strength of arguments that had a crossing number of shared attributes and shared relations. The results showed that shared relations contribute to the inductive strength if a sufficient number of relations are shared. Experiment 2 examined the second hypothesis. The participants who rated similarity between categories of arguments prior to judgment of inductive strength judged arguments having a shared relation to be stronger, whereas the participants who only judged inductive strength did not judge so. The results support the proposed structural alignment view of induction.

## Category-based Induction

People frequently make inferences and expand their knowledge in uncertainty. This type of inference is generally referred to as induction. One form of induction where the premises and the conclusion are of the form "All members of a category C have property P" is referred to as category-based induction.

In category-based induction, a categorical argument is said to be strong when the premises increase the degree of belief in the conclusion. Osherson, Smith, Wilkie, Lopez, and Shafir (1990) proposed that the strength of an inductive argument increases with (a) the degree to which the premise categories are similar to the conclusion category, and (b) the degree to which the premise categories are similar to members of the lowest-level category that includes both the premise and the conclusion categories. They implemented their idea as a mathematical model that is called similarity-coverage model. The similarity-coverage model provides a comprehensive explanation to a variety of phenomena in category-based induction.

As an alternative to the similarity-coverage model, Sloman (1993) proposed a connectionist, feature-based,

model of induction. According to the feature-based model, an argument whose conclusion claims a relation between category C (e.g., Zebras) and property P (e.g., love onions) is judged strong to the extent that the features of C have already been associated with P in the premises.

## Structural Alignment

According to the similarity-coverage model, strength of induction is based on similarities between categories. Recent studies on similarity have revealed flexible and dynamical properties of similarity (Goldstone, 1994; Goldstone, Medin, & Gentner, 1991; Markman & Gentner, 1993; Medin, Goldstone, & Gentner, 1993).

One of the most important findings is that similarity judgment involves a process of alignment of structured relational representations (Markman & Gentner, 1993). In structural alignment, the correspondences between pairs of representations are computed by seeking matches that are structurally consistent. A structurally consistent match means that each attribute or relation in one representation is placed in correspondence with, at most, one attribute or relation in the other representation.

The structural alignment view accounts for important empirical results of similarity and analogy. An important result that is relevant to induction is that attributes and relations in representations are distinguished. This result suggests the possibility that attributes and relations are also distinguished in induction.

## Roles of Attributes and Relations in Induction and Similarity

### Structural Alignment in Induction

Lassaline (1996) proposed a structural alignment view of induction based on structural alignment of similarity and analogy, and examined roles of attributes and relations in induction and similarity. She hypothesized that (1) shared attributes and shared relation between categories contribute to increasing similarity, whereas (2) nonshared binding relations that connect the target attribute (the attribute being mapped from one category to the other in an inductive judgment) to an attribute shared by the two categories contribute to increasing the strength of inductive arguments.

Inductive arguments related to Hypothesis 1 are illustrated in Figure 1. The four arguments have crossing numbers of shared attributes (2 and 3) and shared relations (0 and 1). In argument (a), two attributes “X and Z” are shared by two animals. Argument (c) is formed by adding a common causal relation to argument (a). Therefore arguments (a) and (c) have two shared attributes, and zero and one shared relation, respectively. Argument (b) and (d) are formed by adding a common attribute to (a) and (c), respectively. Therefore argument (b) and (d) have three shared attributes, and zero and one shared relation, respectively.

According to Hypothesis 1, animal A and B in arguments (c) and (d) are respectively judged more similar than those in arguments (a) and (b) because arguments (c) and (d) have more shared attributes. Similarly, animal A and B in arguments (b) and (d) are respectively judged more similar than those in arguments (a) and (c) because arguments (b) and (d) have a shared relation. Note that shared relations in argument (c) and (d) do not connect the target attribute “Y” and a shared attribute. Therefore these shared relations are not binding relations. Her hypotheses do not make any specific prediction about the contribution of nonbinding shared relations to inductive strength.

<p>(a) Animal A has X and Z. Animal B has X, Z and Y. ----- Animal A also has Y? (2A-0R)</p>	<p>(b) Animal A has W, X and Z. Animal B has W, X, Z and Y. ----- Animal A also has Y? (3A-0R)</p>
<p>(c) Animal A has X and Z. Animal B has X, Z and Y. For both animals, X causes Z. ----- Animal A also has Y? (2A-1R)</p>	<p>(d) Animal A has W, X and Z. Animal B has W, X, Z and Y. For both animals, X causes Z. ----- Animal A also has Y? (3A-1R)</p>

Figure 1: Abstract structure of stimuli used in Lassaline (1996)’s Experiment 1. xA-yR indicates that the number of shared attributes is x, and the number of shared relations is y. W, X, Z, and Y represent attributes of the stimuli.

Hypothesis 2 is relevant to induction. Inductive arguments related Hypothesis 2 are illustrated in Figure 2. The four arguments have crossing numbers of shared attributes (0 and 3) and binding relation (0 and 1). Argument (e) has a shared attribute “X.” Argument (g) is formed by adding a nonshared binding causal relation to argument (e). Therefore arguments (e) and (g) have a shared attributes, and zero and one binding relation, respectively. Arguments (f) and (h) are formed by adding two common attributes to (e) and (g), respectively. Therefore argument (f) and (h) have three shared attributes, and zero and one binding relation, respectively.

<p>(e) Animal A has X and Z. Animal B has X and Y. ----- Animal A also has Y? (1A-0R)</p>	<p>(f) Animal A has W, X and Z. Animal B has W, X, Z and Y. ----- Animal A also has Y? (3A-0R)</p>
<p>(g) Animal A has X, W and Z. Animal B has X and Y. For animal B, X causes Y. ----- Animal A also has Y? (1A-1R)</p>	<p>(h) Animal A has W, X and Z. Animal B has W, X, Z and Y. For animal B, X causes Y. ----- Animal A also has Y? (3A-1R)</p>

Figure 2: Abstract structure of stimuli used in Lassaline (1996)’s Experiment 2. xA-yR indicates that the number of shared attributes is x, and the number of binding relations is y. W, X, Z, and Y represent attributes of the stimuli.

According to Hypothesis 2, arguments (g) and (h) are respectively judged as stronger inductive arguments than arguments (e) and (f) because argument (g) and (h) have a binding relation.

She examined roles of attributes and relations in induction and similarity. In her Experiment 1, the roles of shared attributes and shared relations in induction and in similarity were examined. One group of participants rated strength of inductive arguments that had crossing numbers of shared attributes and shared relations as illustrated in Figure 1. The other group of participants rated similarities of pairs of the animals described in each premise of those arguments. The results showed different pattern between inductive strength judgments and similarity judgments. Inductive strength ratings increased by adding a shared attribute, but did not increase by adding a shared relation. In contrast, similarity ratings increased by adding of the shared attribute and the shared relation.

In her Experiment 2, the roles of binding relations were examined. Participants did the same tasks as in Experiment 1 except that arguments included a binding relation as illustrated in Figure 2. The results showed inductive strength ratings increased by adding a binding relation as well as shared attributes. Similarity ratings also increased by adding a binding relation as well as shared attributes.

Both of her hypotheses that were derived from the structural alignment view were supported. Hypothesis 1 is consistent with previous research on similarity (Goldstone, 1994; Goldstone et al., 1991). Hypothesis 2 is consistent with structure mapping theory based on structural alignment of analogy. More specifically, Hypothesis 2 corresponds to systematicity principle in structure mapping theory of analogy (Gentner, 1983).

### Roles of Shared Relations in Induction

Lassaline’s structural alignment view does not make a specific prediction about roles of shared attributes and shared relations in induction. The results showed that

inductive strength increased by adding a shared attribute. This is consistent with two facts that shared attributes increase similarity and that similarity between categories increases strength of induction.

In contrast, the results about roles of shared relations are problematic. Inductive strength did not increase by adding a shared relation although similarity did increase. Lassaline concluded that a relation must bind the target attribute and a shared attribute to increase inductive strength. However her conclusion cannot explain the results that shared attributes increased inductive strength, but shared relations did not. A hypothesis that binding relations contribute to increasing induction corresponds to systematicity principle in analogy. As most researchers agree, systematicity principle is indeed a strong constraint of analogical mapping, but it is not a unique constraint. In fact, Gentner, Rattermann, and Forbus (1993) revealed that shared relations that are not connected to a common relational structure contribute to soundness ratings of analogy between stories as well as similarity ratings between stories. In addition, shared relations are treated as a constraint of analogy in some computer models (Holyoak & Thagard, 1989; Thagard, Holyoak, Nelson, & Gochfeld, 1990).

In this paper, the structural alignment view of induction is generalized to account for roles of shared relations in induction. Two experiments were conducted to reexamine the roles of shared relations in induction from the viewpoint of a generalized structural alignment. Experiment 1 examined the possibility that the shared relations affect inductive strength. Experiment 2 addressed dissociative roles of shared relations in induction and similarity.

### Experiment 1

Goldstone et al. (1991) found that a shared attribute has more weight on similarity than a shared relation when shared attributes are dominant, whereas a shared relation has more weight on similarity than a shared attribute when shared relations are dominant.

The arguments used in Lassaline's Experiment 1 had only zero or one shared relation and two or three shared attributes. Therefore a shared relation might have insufficient saliency on inductive strength. If this hypothesis is correct, shared relations contribute to inductive strength when the arguments have a sufficient number of shared relations.

#### Method

**Participants** Fifty-six Keio University undergraduates participated in the experiment as part of the requirements of an introductory psychology course.

**Materials and Procedure** Each participant was given a booklet that described all tasks and instructions.

Each participant was given a set of 20 pairs of "genotypes of creatures of outer space" that had crossing numbers of shared attributes and shared relations. The number of shared attributes and shared relations were varied

from 0 to 4 and from 0 to 3, respectively. Figure 3 shows examples of the pairs. In pair (i) genotypes A and B have a shared attribute and 2 shared relations because both A and B have "○" in the same place and have the same relations, "on(X, X)": two symbols of the same type are stacked, at the first and third columns. In pair (j) genotypes A and B have a shared attribute and 3 shared relations. The other pairs were constructed in the same manner.

Participants were showed pairs "genotypes of creatures of outer space," referred to as Creature A and Creature B, and were told that each description was intended to refer to a different pair of creatures. They were showed the color of Creature A as a premise. Then they were asked to rate the degree of confirmation that Creature B had the same color as Creature A by selecting a number from 1 (not confirmed) to 9 (completely confirmed) to indicate their judgment.

Four arguments were printed per page. Participants were instructed to spend about 30 seconds in rating the inductive strength of each argument. The orders of the arguments were randomized.

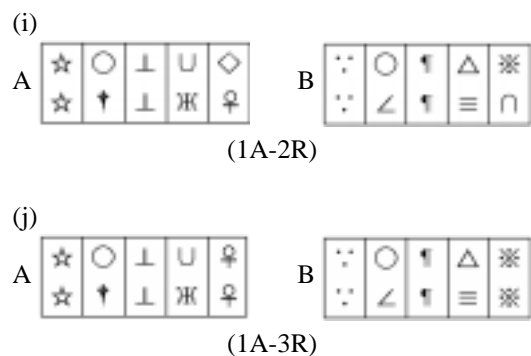


Figure 3: Example of stimuli used in Experiment 1. xA-yR indicates that the number of shared attributes is x, and the number of shared relations is y.

### Results and Discussion

Since inductive strength judgments of 7 participants included missing values, they were eliminated from later analysis. As a result, 49 participants' data sets were analyzed.

Results are shown in Figure 4. Inductive strength increased with the addition of shared relations as well as the addition of shared attributes. Mean inductive strength when the arguments had 0, 1, 2, and 3 shared relations were 2.67, 2.95, 3.47, 4.06, respectively. The results showed that inductive strength increased with addition of shared relations. A two-way ANOVA was conducted on inductive strength, with number of shared attributes (0 to 4) and number of shared relations (0 to 3) as within-subject variables.

The ANOVA on inductive strength showed the main effects of number of shared attributes,  $F(4, 192) =$

60.12,  $p < .01$ , and number of shared relations,  $F(3, 144) = 25.71, p < .01$ .

LSD (Least Significant Difference) post-hoc multiple comparison tests showed that inductive strength increased simply with the addition a shared attribute ( $MSe = 2.77, LSD = .33, p < .05$ ), and that inductive strength increased simply with the addition a shared relation ( $MSe = 3.56, LSD = .34, p < .05$ ). As an exceptional case, there was no significant difference between the conditions where number of shared relation were 0 and 1. There was no interaction between number of shared attributes and number of shared relations,  $F(12, 576) = .578, p > .1$ .

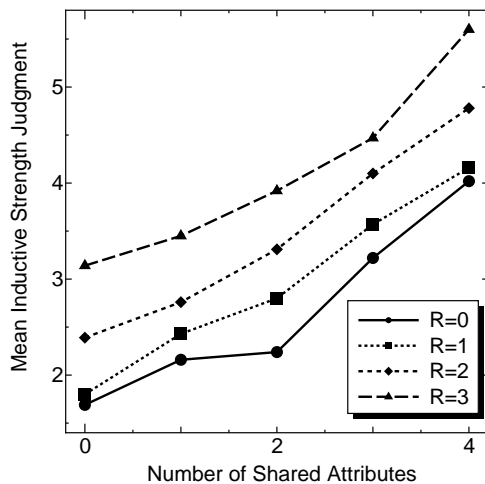


Figure 4: Mean inductive strength judgments from Experiment 1 as a function of number of shared attributes and number of shared relations.  $R = 0, 1, 2$ , and  $3$  indicate that numbers of shared relations are  $0, 1, 2$ , and  $3$ , respectively.

The results support the hypothesis that the shared relations contribute to increasing inductive strength judgments when a sufficient number of relations are shared. In addition, there was no significant difference between the conditions where number of shared relation were  $0$  and  $1$ . This is consistent with Lassaline's results and can be accounted for by insufficiency of the number of shared relations.

A structural alignment view of induction is consistently generalized through Experiment 1. However, the results of Experiment 1 cannot explain dissociative roles of shared relations between induction and similarity because Lassaline's results showed shared relations contributed to increasing similarity but did not contribute to increasing inductive strength. Experiment 2 was conducted to explain these dissociative roles of the shared relations in inductive strength and similarity.

## Experiment 2

A structural alignment view of similarity also suggests an explanation for the dissociative roles of shared re-

lations in inductive strength and similarity. Markman and Gentner (1993) proposed that similarity judgment involves a process of structural alignment. A central prediction of structural alignment is that similarity judgments lead people to attend to the matching relational structure in a pair of items. Participants were given a pair of pictures containing cross-mappings where an attribute-based mapping and a relation-based mapping compete, and were asked to select other object in one picture that went with the cross-mapped object in the other. All of these stimuli were explicitly designed so that the participants' natural tendency was to select the similar object that shared an attribute with the other. The participants who rated the similarity of the scenes prior to performing the mapping tasks more often selected the relation-based mappings than the participants who simply performed the mapping tasks without prior similarity judgments.

These results suggest the hypothesis that the shared relations contribute to increasing inductive strength if participants rate similarities prior to inductive strength judgments even when few relations are shared. If the hypothesis is supported, the dissociative roles of shared relations in inductive strength and similarity in Lassaline's results are explained as follows. In inductive strength judgments, since only a relation was shared, a shared relation was not so salient as to contribute to increasing inductive strength. In similarity judgments, a structural alignment during similarity judgment made a shared relation so salient to contribute to increasing similarity.

Experiment 2 examined whether contribution of shared relations to inductive strength increases because of similarity judgments prior to inductive strength judgments.

Participants were assigned to one of the three inductive task conditions. Participants in the "Induction-only" condition performed inductive strength judgment in the same manner as in Experiment 1. Participants in the "Similarity-first" condition first performed similarity rating of categories in the premise and then performed inductive strength judgment. Participants in the "Nonaligning-first" condition first performed the non-aligning task and then performed inductive strength judgment. The Nonaligning-first condition was added in order to rule out the possibility that the difference of inductive strength between Induction-only and Similarity-first conditions was reduced to the difference in the time that participants looked at stimuli.

According to the hypothesis, the inductive strength judged by the participants in the Similarity-first condition is more affected by the shared relation compared with its strength judged by the participants in the two other conditions.

## Method

**Participants** One hundred and sixty Keio University undergraduates participated in the experiment as part of the requirements of an introductory psychology course. They were randomly assigned to one of the three

between-subject inductive task conditions.

**Materials and Procedure** Each participant was given a booklet that described all tasks and instructions in the same manner as in Experiment 1.

Participants in the Induction-only condition performed the same tasks as those in Experiment 1 except for the presented arguments. Each participant was given a set of 8 pairs of “genotypes of creatures of outer space.” The number of shared attributes and shared relations were varied from 0 to 3 and 0 to 1, respectively. Figure 5 shows examples of the pairs.

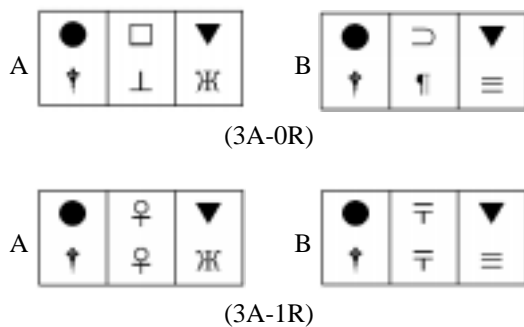


Figure 5: Example of stimuli used in Experiment 2. xAyR indicates that the number of shared attributes is x, and the number of shared relations is y.

Participants in the Similarity-first condition first rated similarities of “genotypes” in the arguments by selecting a number 1 (not similar at all) to 9 (very similar) to indicate their judgment. They then performed inductive strength ratings in the same manner as the participants in the Induction-only condition.

Participants in the Nonaligning-first condition first judged whether each creature was an animal or a plant. They then performed inductive strength ratings in the same manner as the participants in the Induction-only condition.

Participants were instructed to spend about 30 seconds in each judgment task.

## Results and Discussion

Since judgments of 8 participants included missing values, they were eliminated from later analysis. As result, 51, 51, and 50 participants’ data sets in the Induction-only, the Similarity-first, and the Nonaligning-first conditions were analyzed, respectively.

Results are shown in Figure 6. Inductive strength ratings increased by 1.13 points with the addition of a shared relation in the Similarity-first condition whereas inductive strength ratings increased by 0.43 and 0.30 points with the addition of a shared relation in the Induction-only and Nonaligning-first conditions, respectively.

A three-way ANOVA was conducted on inductive strength with inductive task conditions (Induction-only, Similarity-first, Nonaligning-first) as a between-subject

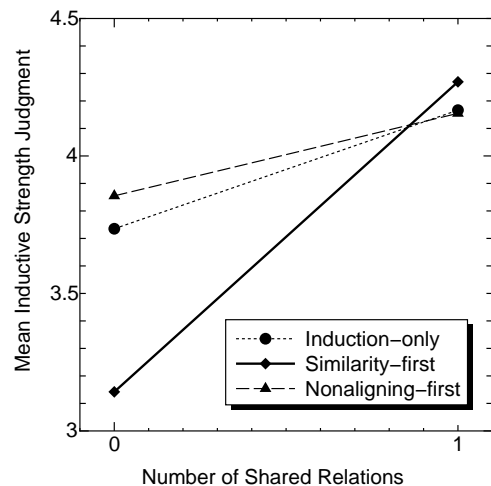


Figure 6: Mean inductive strength judgments from Experiment 2 as a function of number of shared relations.

variable, and with number of shared attributes (0 to 3) and number of shared relation (0, 1) as within-subject variables.

The ANOVA on inductive strength showed the main effects of number of shared relations,  $F(1, 149) = 38.46, p < .01$ , and number of shared attributes,  $F(3, 447) = 242.91, p < .01$ . There was no effect of inductive task condition,  $F(2, 149) = .69, p > .1$ .

There was a significant interaction between inductive task conditions and number of shared relation,  $F(2, 149) = 6.60, p < .01$ . There was also a significant interaction between number of shared attributes and number of shared relation,  $F(3, 447) = 2.84, p < .05$ . There was no interaction between inductive task conditions and number of shared attributes, nor a three-way interaction ( $F(6, 447) = 1.49, p > .1$  and  $F(6, 447) = .51, p > .1$ , respectively).

A significant interaction between inductive task condition and number of shared relation supports the hypothesis that similarity judgments prior to inductive strength increase contribution of shared relation to inductive strength. The results cannot be reduced to the difference of the time that participants looked at stimuli because nonaligning tasks did not increased contribution of shared relation to inductive strength judgments.

There were simple main effects of shared relation in the Induction-only and the Nonaligning-first conditions as well as in the Similarity-first condition ( $F(1, 149) = 6.21, p < .05$ ,  $F(1, 149) = 3.01, p < .1$ , and  $F(1, 149) = 42.45, p < .01$ , respectively). The most likely explanation for these effects is that pictorial representations of stimuli made participants sensitive to shared relation.

The mean ratings of inductive strength in the Similarity-first condition was lower than two other conditions although there was no effect of inductive task condition. A possible interpretation is that maximal inductive strength was restrained because no pair of stimuli

was not so similar, and the absence of a shared relation decreased inductive strength.

A significant interaction between number of shared attributes and number of shared relation does not have specific interpretation. This interaction was caused by that effects of shared relation were smaller when the shared attribute was zero. The ANOVA was conducted again, this time eliminating the conditions where number of shared attributes was zero. The results showed no interaction between number of shared attributes and number of shared relation. The other interactions and main effects did not change.

The results support the hypothesis that the shared relations contribute to increasing inductive strength if participants rate similarities prior to inductive strength judgments even when few shared relations are shared. The results also confirm the explanation for the dissociative roles of shared relations in inductive strength and similarity in Lassaline's results. In her results, a shared relation did not contribute to increasing inductive strength because the shared relation was not salient in this case, whereas a shared relation contributed to increasing similarity because structural alignment during similarity judgment made the shared relation salient.

### General Discussion

The structural alignment view proposed by Lassaline (1996) can be consistently generalized as follows. First, attributes and relations are distinguished in induction as well as in similarity judgment. Second, a relation binding the target attribute and shared attributes is a strong constraint in induction. Third, in addition, shared attributes and shared relations are also constraints in induction if they are sufficiently salient. If shared relations are salient, participants easily align relations as well as attributes.

According to the structural alignment view proposed here, shared relations contribute to increasing inductive strength if they are sufficiently salient. In Experiment 1, a sufficient number of shared relations increased to make relations salient. The results showed that shared relations contributed to increasing inductive strength when a sufficient number of relations were shared.

In Experiment 2, participants rated similarity prior to inductive strength judgment to make a shared relation salient. According to the structural alignment view of similarity, similarity judgment involves a structural alignment that leads participants to attend to the matching relational structure. Therefore participants who rated similarity prior to inductive strength judgment were expected to be able to easily align a shared relation. The results showed that a shared relation contributes to increasing inductive strength if participants rated similarity prior to inductive strength judgment.

The results of Experiment 1 and 2 are consistent with the proposed structural alignment view, and are also consistent with the fact that shared relations contribute to soundness ratings of analogy. The proposed structural alignment view of induction more consistently corre-

sponds to a structural alignment view of similarity and analogy than does Lassaline's.

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