

AN ENRICHED PERSPECTIVE ON DIFFERENTIABLE STACKS

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In this talk, we will apply the theory of tangent categories to study the tangent structure of differentiable stacks. A stack is a $(2,1)$ -sheaf on a site (X, \mathcal{J}) , using the theory of locally presentable categories one may see this is equivalent to an internal groupoid in $\text{Sh}(X, \mathcal{J})$. A differentiable stack is certain kind of $(2,1)$ -sheaf on the category of smooth manifolds. The difficulty in defining a tangent bundle [1] for a differentiable stack is similar to those that arise when defining the tangent bundle in a smooth topos. When studying synthetic differential geometry [7], one restricts their attention to sheaves which satisfy a microlinearity condition, we will apply this technique to differentiable stacks. Following the work of Garner and Leung [2, 3], one may regard a tangent category [4, 5] as a kind of E -enriched category (where E is the category of microlinear presheaves on Weil algebras [6]). Then one may replace a sheaf satisfying a microlinearity condition with an enriched sheaf. In this talk we will extend this technique to stacks: we will consider a notion of strict tangent $(2,1)$ -category as a certain kind of $\text{Gpd}(E)$ -enriched category. Then we may lift a tangent category to a $(2,1)$ -tangent category and consider $\text{Gpd}(E)$ -presheaves.

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