EXPONENTIABILITY IN DOUBLE CATEGORIES AND THE GLUEING CONSTRUCTION

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Adjunctions between double categories have been considered by Grandis and Paré in several settings. Since left adjoints are oplax and right adjoints are lax, the pair is an orthogonal adjunction in the double category $\mathbb{D}$bl, whose horizontal and vertical morphisms are lax and oplax functors, respectively. If the left adjoint is lax, then it is an adjunction in the 2-category **LxDbl**, whose morphisms are lax functors. If both adjoints are pseudo functors, then it is an adjunction in the 2-category **PsDbl**, whose morphisms are pseudo functors.

A double category $\mathbb{D}$ is *pre-cartesian*, if the diagonal $\Delta\colon \mathbb{D} \to \mathbb{D} \times \mathbb{D}$ has a right adjoint in **LxDbl**. We say $\mathbb{D}$ is *pre-cartesian closed*, if the lax functor $- \times Y\colon \mathbb{D} \to \mathbb{D}$ has a right adjoint in **LxDbl**, for every Y. If the right adjoints in question are pseudo-functors, we say $\mathbb{D}$ is *cartesian closed*. Examples of the latter include the double categories $\mathbb{R}$el and $\mathbb{S}$pan, whose objects are sets, horizontal morphisms are functions, and vertical morphisms are relations and spans, respectively.

In this talk, we consider *pre-exponentiable* objects Y in a pre-cartesian double category $\mathbb{D}$, i.e., objects for which $- \times Y$ has a right adjoint in **LxDbl**. Such an object is necessarily (classically) exponentiable in the horizontal category $\mathbb{D}_0$. Since the right adjoints we consider are merely lax, even when the left adjoints are pseudo-functors, the setting for this talk is **LxDbl**, rather than **PsDbl**.

For double categories $\mathbb{D}$ satisfying a generalization of the glueing construction from topos theory, we show that Y is pre-exponentiable in $\mathbb{D}$ if and only if it is exponentiable in $\mathbb{D}_0$. Applications include the double categories $\mathbb{C}$at, $\mathbb{P}$os, $\mathbb{S}$paces, $\mathbb{L}$oc, and $\mathbb{T}$opos, whose objects are small categories, posets, topological space, locales, and toposes, respectively. Thus, $\mathbb{C}$at and $\mathbb{P}$os are pre-cartesian closed, and the pre-exponentiable objects in $\mathbb{S}$paces, $\mathbb{L}$oc, and $\mathbb{T}$opos are precisely those we know are exponentiable in the classical sense.