A unified framework for notions of algebraic theory

Soichiro Fujii*

Abstract

Universal algebra uniformly captures various algebraic structures, by expressing them as equational theories or abstract clones. The ubiquity of algebraic structures in mathematics and related fields has given rise to several variants of universal algebra, such as theories of symmetric operads, non-symmetric operads, generalised operads, PROPs, PROs, and monads. These variants of universal algebra are called notions of algebraic theory. In this talk, we present a unified framework for them. The key observation is that each notion of algebraic theory can be identified with a monoidal category, in such a way that algebraic theories correspond to monoid objects therein. To incorporate semantics, we introduce a categorical structure called metamodel, which formalises a definition of models of algebraic theories. We also define morphisms between notions of algebraic theory, which are a monoidal version of profunctors. Every strong monoidal functor gives rise to an adjoint pair of such morphisms, and provides a uniform method to establish isomorphisms between categories of models in different notions of algebraic theory. A general structure-semantics adjointness result and a double categorical universal property of categories of models are also shown.

*Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606-8502, Japan. Email: s.fujii.math@gmail.com