$\mathcal{M}$-coextensivity and the strict refinement property

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Abstract

Various refinement properties exist for direct-product decompositions of universal algebras, all of which give information about the uniqueness of such direct-product decompositions. The strongest of these, the so-called *strict refinement property* [2], implies that any isomorphism between a product of irreducible structures is uniquely determined by a family of isomorphisms between the factors, in such a way that the original isomorphism is the product of these. Examples of structures which possess the strict refinement property include any unitary ring, any centerless or perfect group, and any congruence distributive algebra. Almost all geometric structures possess the dual property, which is mainly due to the fact that almost all categories of geometric structures are *extensive* in the sense of [1]. In this talk we present an analysis of the relationship between the strict refinement property and extensivity. In particular, we introduce the notion of an $\mathcal{M}$-coextensive object, which brings these two properties together. We show how it links up with regular majority categories [3], and centerless objects in an exact Mal'tsev category. When $\mathcal{M}$ is the class of all product projections in a category, then an $\mathcal{M}$-coextensive object is called *projection-coextensive*. One of the main results shows that the poset of product projections of a projection-coextensive object is a Boolean lattice. Moreover, this is a characteristic property of projection-coextensivity. This generalizes a similar result for relational structures in [2].

References

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