SPLIT EXTENSION CLASSIFIERS IN THE CATEGORY OF COCOMMUTATIVE HOPF ALGEBRAS

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The category $Hopf_{K,coc}$ of cocommutative Hopf algebras over an arbitrary field K has been recently shown to be semi-abelian [3,5] extending the classical theorem by Takeuchi saying that the category of commutative and cocommutative Hopf algebras is abelian. The category $Hopf_{K,coc}$ is also action representable in the sense of [2], so that the categorical notions of center, centralizer and commutator can be explored in this category. In particular, we shall explain that the categorical notion of center in $Hopf_{K,coc}$ turns out to coincide with the usual notion of center from Hopf algebra theory [1]. When K is an algebraically closed field of characteristic 0 it is possible to give an explicit description of the split extension classifier of a cocommutative Hopf algebra [4]. This universal construction can be seen as the natural counterpart in $Hopf_{K,coc}$ of both the automorphism group in the category of groups and of the algebra of derivations in the category of Lie algebras.

References

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