COHERENCE FOR TRICATEGORIES VIA WEAK VERTICAL COMPOSITION

EUGENIA CHENG

It is well known that every strict 2-category is equivalent to a weak one, but that the analogous result for 3-categories does not hold. Rather, coherence for weak 3-categories (tricategories) needs more nuance. One way of viewing this is that we need to take account of possible braidings that arise and cannot be strictified into symmetries. The original coherence result of Gordon–Power–Street says, essentially, that every tricategory is equivalent to one in which everything is strict except interchange. The intuition is that "braidings arise from weak interchange". However, from close observation of how the Eckmann-Hilton argument works, Simpson conjectured that weak units would be enough, and this result was proved for the case n=3 by Joyal and Kock. Their result involves a weak unit I in an otherwise completely strict monoidal 2-category. They showed that End(I) is naturally a braided monoidal category, and that every braided monoidal category is equivalent to a End(I) for some monoidal 2-category. Regarding this as a degenerate 3-category, this would mean that everything is strict except horizontal units. In this talk we will address a third case, in which everything is strict except vertical composition; this amounts to considering categories strictly enriched in the category of bicategories with strict functors, with respect to Cartesian product. We will show that every doubly degenerate such tricategory is naturally a braided monoidal category, that every braided monoidal category is equivalent to one of these. The proof closely follows Joyal and Kock's method of clique constructions. Joyal and Kock use train track diagrams to give just enough "rigidity" to the structure of points in 3-space, and they describe this as preventing the points from being able to simply commute past each other via an Eckmann–Hilton argument. We are aiming for a different axis of strictness and so instead of points in R^2 with cliques arising from train track diagrams, we use use points embedded in the interior of I^2 with cliques arising from horizontal "slides". This method generalises, by omitting the "slide" cliques, to prove the corresponding result for doubly degenerate Trimble 3-categories. There are several critical subtleties to this which is why we have to leave vertical associativity weak but can use fully doubly degenerate structures, where Joyal and Kock were able to have all associativity strict but could not use fully doubly degenerate structures. We also extend the result to totalities, exhibiting a biequivalence of appropriate bicategories.