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Hyperconnections

Functors between join restriction categories admit a factorization into localic functors (which are bijective on objects and preserve meets) followed by hyperconnected functors (which are bijective on the locales of restriction idempotents) - for the basic localic/hyperconnected factorization on mere restriction categories see [5].

A partite category (see [3], for example, for source étale internal categories: a *partite* internal category has its objects and arrow partitioned into many objects) internal to a join restriction category $\mathbb{B}$ induces, by considering partial sections of the domain maps, an external join restriction category which sits over $\mathbb{B}$ by a hyperconnected functor. Conversely an external join restriction category over $\mathbb{B}$ (where the latter must be assumed to have all gluings [4]) induces a source étale partite category internal to $\mathbb{B}$. This correspondence may be completed to a Galois adjunction between join restriction categories over $\mathbb{B}$ and partite categories internal to $\mathbb{B}$ with cofunctors [1, 2]. The adjunction specializes to an equivalence between hyperconnections over $\mathbb{B}$ and source étale partite categories internal to $\mathbb{B}$.

This phenomenon occurs in many different places in mathematics (often specialized to groupoids). In particular, as all join restriction categories have a hyperconnected fundamental functor to the category of locales (with partial maps) one can conclude that join restriction categories (with join functors) correspond precisely to source étale partite categories internal to locales (with cofunctors). From algebraic geometry, considering schemes as a join restriction category with gluings, the identity functor on schemes induces an internal partite category: the object of morphisms from the affine scheme R to $\mathbb{Z}[x]$ is then exactly the structure sheaf of R.

References:

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[4] M. Grandis, *Cohesive categories and manifolds*, Ann. Mat. Pura Appl. 157, 199-244 (1990)

[5] J.R.B. Cockett and R. Garner *Restriction categories as enriched categories*, TCS, Vol. 523, 37-55 (2014)

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