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*Fast-growing clones*

A concrete clone on a set is a family of finitary operations on the set that is closed under projections and composition. Abstract clones are the algebras for the  $\mathbb{N}$ -sorted equational presentation that axiomatises this structure. Categorically, they are Lawvere theories.

It is a standard result of universal algebra that every abstract clone can be represented by a concrete clone on a set [2]. This has been generalised to discrete enriched clones for enriching categories with enough structure [1], crucially infinitary structure in the form of colimits of  $\omega$ -chains of sections.

Consideration of whether infinitary structure is in general necessary for clone representation, led us to show that there is no clone-representation result in the context of finite sets. This I will present as a by-product of technical developments of a logical, rather than algebraic or combinatorial, nature. The underlying categorical theory allows one to construct monads on finite sets with free algebras that asymptotically grow faster than every iterated exponential, for any natural height, on their generators.

References:

- [1] M. Fiore. On the concrete representation of discrete enriched abstract clones. In *Tbilisi Math J* 10(3):297-328, 2017.
- [2] W. Taylor. Abstract clone theory. In I. Rosenberg and G. Sabidussi, editors, *Algebras and Orders*, volume 389 of *NATO ASI Series C: Mathematical and Physical Sciences*, pages 507–530, 1993.