INVOLUTIVE FACTORISATION SYSTEMS AND DOLD-KAN CORRESPONDENCES

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In the late 1950's Dold [3] and Kan [5] showed that a simplicial abelian group is completely determined by an associated chain complex and that this construction yields an equivalence of categories. This *Dold-Kan correspondence* is important in algebraic topology because it permits an explicit construction of topological spaces with prescribed homotopy groups (e.g. Eilenberg-MacLane spaces).

There have been several attempts to extend this kind of categorical equivalence to other contexts, most recently by Lack and Street [6]. We present here an approach based on the notion of *involutive factorisation system*, i.e. an orthogonal factorisation system $\mathcal{C} = (\mathcal{E}, \mathcal{M})$ equipped with a faithful, identity-on-objects functor $\mathcal{E}^{\mathrm{op}} \to \mathcal{M} : e \mapsto e^*$ such that $ee^* = 1$ and three other axioms are satisfied.

We show that for each category $\mathcal{C}$ equipped with such an involutive factorisation system $(\mathcal{E}, \mathcal{M}, (-)^*)$, there is an equivalence $[\mathcal{C}^{\mathrm{op}}, \mathcal{A}] \simeq [\Xi(\mathcal{C})^{\mathrm{op}}, \mathcal{A}]_*$ where $\mathcal{A}$ is any idempotent-complete *additive* category, and $\Xi(\mathcal{C})$ is the locally pointed category of *essential* $\mathcal{M}$-maps. Since in the simplex category Δ the only essential $\mathcal{M}$-maps are the last face operators $\epsilon_n : [n-1] \to [n]$, we get ordinary chain complexes in $\mathcal{A}$ on the right so that our equivalence specialises to Dold-Kan correspondence if $\mathcal{C} = \Delta$.

Our approach recovers several known equivalences, cf. [8, 4, 2]. An interesting *new* family is given by Joyal's cell categories Θ_n (cf. [1]) where our equivalence relates to n-th order Hochschild homology and E_n-homology (cf. [9, 7]).

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