ENRICHED REGULAR THEORIES

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Regular and exact categories were first introduced by Michael Barr in 1971; since then, the theory has developed and found many applications in algebra, geometry, and logic. In particular, a small regular category determines a certain theory, in the sense of logic, whose models are the regular functors into Set.

In 1986 Barr showed that each small and regular category can be embedded in a particular category of presheaves; then in 1990 Makkai gave a simple explicit characterization of the essential image of the embedding, in the case where the original regular category is moreover exact. More recently Prest and Rajani, in the additive context, and Kuber and Rosicky, in the ordinary one, described a duality which connects an exact category with its (definable) category of models.

Considering a suitable base for enrichment, we define an enriched notion of regularity and exactness, and prove a corresponding version of the theorems of Barr, of Makkai, and of Prest-Rajani/Kuber-Rosicky.

Based on a Master Thesis supervised by Stephen Lack.