ACCESSIBLE ASPECTS OF 2-CATEGORY THEORY

JOHN BOURKE

Two-dimensional universal algebra is primarily concerned with categories equipped with structure and functors preserving that structure up to coherent isomorphism. The 2-category of monoidal categories and strong monoidal functors provides a good example. As understood by the Australian school in the 1980s such 2-categories admit all weak 2-categorical colimits, but not necessarily genuine colimits such as coequalisers. In particular, they are rarely locally presentable in the usual sense.

The theory of accessible categories was developed by Makkai and Pare around the same time. This weakens the theory of locally presentable categories by requiring only the existence of certain filtered colimits. Whilst the initial motivations come from model theory, accessible categories have since become an important tool in homotopy theory.

The present talk, which builds on discussions between Makkai and I, will explain the connection between two dimensional universal algebra and accessible categories. Using homotopical techniques, we will see that many 2-categories, such as the 2-category of monoidal categories mentioned above, are accessible, and that accessibility of a 2-category of categorical structures is intimately connected to the structures in question being sufficiently weak.

I also hope to mention connections with infinity-cosmoi, for which see also the related talk of Steve Lack.