COHERENCE MODULO AND DOUBLE GROUPOIDS

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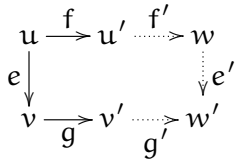
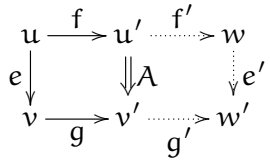
This work is part of a research project aiming at developing constructive methods based on rewriting theory to study algebraic structures, and compute coherent presentations, linear bases and higher syzygies. In many situations presentations of algebraic structrues have a great complexity due to the number of relations, some of them being axioms of the structure itself. We present a categorical approach for rewriting modulo and a method to compute coherent presentations modulo axioms. We introduce a notion of polygraphs modulo as higher-dimensional rewriting systems presenting higher dimensional categories whose axioms are not considered as oriented rules, [2]. We define following [4] termination and confluence properties for these polygraphs, with confluence diagrams of the form



where the cells f,f',g and g' correspond to rewriting sequences and the cells e, e' correspond to equations derived from axioms. Following [3, 5], a coherent presentation of an n-category can be obtained from a presentation by a convergent $(n+1)$-polygraph P extended by $(n+2)$-cells associated to confluence diagrams of critical branchings. In this spirit, coherence modulo axioms can be formulated in terms of categories enriched in double groupoids where the horizontal cells correspond to rewriting sequences, and the vertical cells correspond to the congruence generated by the axioms.

Using the approach of [1] to construct free double categories, we introduce a notion of double $(n+2, n)$-polygraph as a data generating a free n-category enriched in double groupoids defined from a set of square cells on the pair of free n-categories on the axioms and on rewriting rules. We introduce a suited notion of coherent confluence modulo given by existence of a square cell in each confluence modulo diagram as follows:



We prove that the coherent confluence of a polygraph modulo is equivalent to the coherent confluence of some critical branchings modulo. We define a coherent confluence modulo procedure that we apply to the computation of coherent presentations of commutative monoids and pivotal categories.

REFERENCES

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