Functorial Decomposition of Colimits

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This talk is centred around the following decomposition formula for colimits:

$$\mathrm{colim}_{k\in\mathcal{K}} Xk \cong \mathrm{colim}_{d\in\mathcal{D}}(\mathrm{colim}_{i\in\mathcal{I}_d} X_d i);$$

here $X : \mathcal{K} \longrightarrow \mathcal{X}$ is a diagram in the cocomplete category $\mathcal{X}$, where the diagram scheme $\mathcal{K}$ is itself a colimit of a diagram $D : \mathcal{D} \longrightarrow \mathbf{Cat}$, $d \mapsto \mathcal{I}_d$, in the category of small categories, with colimit injections $K_d : \mathcal{I}_d \longrightarrow \mathcal{K}$, producing the restricted diagrams $X_d = XK_d$ in $\mathcal{X}$. Its formal dualization gives a recomposition formula for limits in the complete category $\mathcal{X}$. The proof may be cast advantageously within the framework of Grothendieck fibrations. Their 2-categorical expansions (as given by Hermida and Buckley) lead to 2-categorical generalizations of the 1-dimensional formulae that were presented by the first author in a talk in Louvain-la-Neuve in May 2017.

References

M. Buckley: Fibrations of 2-categories and bicategories. *Journal of Pure and Applied Algebra* 218(6):1034–1074, 2014.

C. Hermida: Some properties of **Fib** as a fibred 2-category. *Journal of Pure and Applied Algebra* 134:83–109, 1999.

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