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Internal lenses as monad morphisms

Lenses were originally introduced in [1] as a mathematical structure which captures the notion of synchronisation between a pair of sets. When extending this idea to consider synchronisation between a pair of categories [2], we obtain functors akin to Grothendieck opfibrations lacking the universal property. The purpose of this talk is to motivate the notion of synchronisation between a pair of internal categories in a category with pullbacks [3].

We recall that internal categories are monads in the bicategory of spans, and thus internal functors are colax monad morphisms whose underlying span has identity left leg. Dually, internal cofunctors are the corresponding lax monad morphisms, which may be represented as a span of internal functors whose left leg is an identity-on-objects functor and whose right leg is a discrete opfibration.

In this talk I will define an internal lens as a monad morphism which is both an internal functor and cofunctor. Specialising internal to **Set** clarifies the usual notion of asymmetric lens, while considering internal lenses in **Cat** between double categories of squares yields split Grothendieck opfibrations. I will also define internal symmetric lenses as a pair of internal Mealy functors [4] (or two-dimensional partial maps) and establish the relationship with spans of internal lenses, generalising the results in [5].

References:

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