Relating the Effective Topos to HoTT

Steve Awodey, Jonas Frey, Giuseppe Rosolini

The effective topos **Eff** was introduced by Martin Hyland in [4] and proved to be a very useful category where to test computational properties of constructive theories, see [9]. In the talk we present a way to see **Eff** as part of a model of Homotopy Type Theory [6].

The presentations of **Eff** as an exact completion and of its full subcategory **Asm** on the assemblies as a regular completion in [2] suggested that the topos might be obtained as a homotopy quotient of some appropriate category, see also [7]. This is understood in a very rough sense, based on the construction of the exact completion via the pseudo-equivalence relations of Aurelio Carboni as in [1].

By considering the category of the pseudo-equivalence relations in **Asm** (with graph homomorphisms), we can show that **Eff** is a full subcategory of the homotopy quotient $\mathrm{Ho}(\mathrm{Kan}([\mathcal{C}^{\mathrm{op}}, \mathbf{Asm}]))$ of the category of Kan fibrant cubical assemblies, see [3, 5].

In fact, we obtain this from the stronger result that the extensional realizability topos **Ext** of [8], into which **Eff** embeds fully, is a full subcategory of $\mathrm{Ho}(\mathrm{Kan}([\mathcal{C}^{\mathrm{op}}, \mathbf{Asm}]))$.

References

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