

# A GENERAL NULLSTELLENSATZ FOR GENERALISED SPACES

INGO BLECHSCHMIDT

The Nullstellensatz of commutative algebra states that some algebraic truths are witnessed by explicit algebraic certificates, syntactical a priori reasons for why a given truth is to be expected.

Any geometric theory possesses a tautologous yet intriguing \*generic model\*. Topologically, any model can be obtained as a pullback of the generic one, and logically, it has exactly those properties which are shared by all models. Crucially however, this statement is only true for properties which can be put as geometric sequents. The generic model may enjoy additional first-order or higher-order properties which are not shared by all models.

The talk starts by reviewing this circle of ideals, in particular highlighting some of the applications of nongeometric properties in commutative algebra, where they provide new reduction techniques, and in algebraic geometry, where they allow for a synthetic development of the foundations. We then present a logical analogue of the algebraic Nullstellensatz, valid for the generic model of any geometric theory. This analogue replaces algebraic certificates by logical certificates – geometric proofs. It is a source of nongeometric sequents enjoyed by generic models, and it turns out that it is the universal such source.