UNIVALENCE AND COMPLETENESS OF SEGAL OBJECTS

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In this talk, we make precise an analogy between univalence and completeness that has been subject to informal discussions in the research community. More precisely, we give a definition of univalence and a definition of Rezk-completeness for Segal objects X in a large class of type theoretic model categories M. The former is a straightforward generalization of univalence in the type theoretic fibration category C of fibrant objects in M as treated in [3]. The latter is a generalization of Rezk's original definition of completeness for Segal spaces. Both conditions share the heuristic purpose to contract a respective object of internal equivalences associated to X over the object X_0 of points, turning that object of internal equivalences into a path object for X_0. A priori, these objects of internal equivalences do not necessarily coincide, so the goal of this talk is to show the following theorem.

Let X be a "sufficiently fibrant" Segal object in M. Then X is univalent if and only if its Reedy fibrant replacement RX is complete.

As a corollary we obtain that a fibration in C is univalent if and only if the Reedy fibrant replacement of its nerve in M is Rezk-complete (this comparison in fact is independent of its infinity-categorical version presented in [2]). This implies for instance that univalent completion of a Kan fibration as introduced in [1] is a special case of Rezk completion of its associated Segal space.

References

[1] Van den Berg, Moerdijk - Univalent Completion
[2] Rasekh - Complete Segal objects
[3] Shulman - Univalence for inverse diagrams and homotopy canonicity