LEFT CANCELLATIVE CATEGORIES AND ORDERED GROUPOIDS

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One way to characterize etendues is as the categories of sheaves on a site with monic maps, or, in the language of [2], a left cancellative category with a Grothendieck topology.

In [2], Lawson introduces a notion of ordered groupoid and introduces functors between the category of left cancellative categories and that of ordered groupoids, but does not establish it as an equivalence of 2-categories, because it is not clear when two ordered groupoids should be equivalent.

In [3], Lawson and Steinberg introduce the notion of an Ehresmann topology on ordered groupoids and establish a correspondence between Grothendieck topologies on left cancellative categories and Ehresmann topologies on ordered groupoids, corresponding to each of their functors. They show that these correspondences give an equivalence between the induced categories of sheaves. The main result in this paper is a characterization of etendues as sheaves on an Ehresmann site. What is missing in this paper then equivalences of the 2-categories of left cancellative categories and ordered groupoids and a description of what morphisms between ordered groupoids give rise to geometric morphisms between the induced categories of sheaves on the Ehresmann sites.

By considering ordered groupoids as a special type of double categories we characterize what a weak equivalence of ordered groupoids is, and then use this to establish an adjoint equivalence of 2-categories between the 2-category of left cancellative categories and the 2-category of ordered groupoids.

We show how this extends to an equivalence between the appropriate categories of Grothendieck sites with monic maps and Ehresmann sites and as an application, we translate the comparison lemma for sites with monic maps given in [1], to a comparison lemma for Ehresmann sites, characterizing which functors between Ehresmann sites give rise to equivalences of etendues.

REFERENCES

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[3] Mark V. Lawson, Benjamin Steinberg, Ordered Groupoids and Etendues, Cahiers de topologie et géométrie différentielle catégoriques, 45 (2004), pp. 82-108