PROFINITE MONADS AND REITERMAN'S THEOREM

Jiři Adámek
Czech Technical University Prague

Jan Reiterman characterized in 1980's **pseudovarieties** $\mathcal{D}$ of finite algebras, i.e., classes closed under finite products, subalgebras and quotients: they are precisely the classes that can be presented by equations between profinite terms, see [1]. A profinite term is an element of the **profinite monad** which is the codensity monad of the forgetful functor of $Pro\mathcal{D}$, the profinite completion of the given pseudovariety.

We have recently generalized this result to pseudovarieties of $\mathcal{T}$-algebras where $\mathcal{T}$ is a monad over a base category which can be an arbitrary locally finite variety of (possibly ordered) algebras, see [2]. But now we realize that a much more general result holds: the base category need not be a variety, it can be an arbitrary complete and wellpowered category $\mathcal{D}$ in which a full subcategory $\mathcal{D}_f$ (of objects called 'finite') is chosen so that

(i) $\mathcal{D}_f$ is closed under subobjects,

(ii) every finite object is strong quotient of a strongly projective object, and

(iii) all strong epimorphims in $\mathcal{D}_f$ are closed under cofiltered limits.

For every monad over $\mathcal{D}$ peserving strong epimorphisms we then introduce its profinite monad on $Pro\mathcal{D}_f$ and the corresponding concept of profinite equation. We then prove that a collection of finite algebras (i.e., $\mathcal{T}$-algebras carried by objects of $\mathcal{D}_f$) is a pseudovariety iff it can be presented by profinite equations. As a special case we obtain the result of Pin and Weil about pseudovarieties of first-order structures, see [3].

References

[1] Jan Reiterman, The Birkhoff theorem for finite algebras. Algebra Universalis 14 (1982), 1-10

[2] Liang-Ting Chen, Jiří Adámek, Stefan Milius and Henning Urbat, Profinite Monads, Profinite Equations and Reiterman's Theorem. Lecture Notes Comput. Sci. (ARCoSS) 9634 (2016), 531-547

[3]J.-E.Pin and P.Weil, A Reiterman theorem for pseudovarieties of finite first-order structures. Algebra Universalis 14 (1996), 577-595