ADJUNCTIONS AND LIMITS FOR DOUBLE AND MULTIPLE CATEGORIES

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Based on a series of joint papers with R. Paré.

In a general 2-dimensional adjunction, the left adjoint is colax and the right adjoint is lax: they should not be composed, but organised as vertical and horizontal arrows of a double category. There are various examples of interesting adjunctions which can only be treated in this way, like the obvious pushout-pullback adjunction between spans and cospans, or the extension of an adjunction between abelian categories to their double categories of relations. Finally, a 2-dimensional adjunction lives in a strict double category of (small) weak double categories, with lax and colax functor as horizontal or vertical arrows, and suitable double cells. Another crucial point of interest of weak double categories is the existence of limits: while — for instance — the bicategory of spans of sets lacks most of them, the corresponding weak double category (with ordinary maps in the strict direction) has all limits. All this can be extended in infinite dimension, to weak and lax multiple categories.

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