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Thanks

We are enormously grateful to the staff of the **International Centre for Mathematical Sciences (ICMS)**, who are administering CT2019. We most especially thank **Johanna McBryde** and **Jane Walker** for their epic feats, and above all for working so hard and creatively to expand this conference to well beyond its anticipated size. Thanks also to **Robyn Stewart-Evans** and **Dawn Wasley**.

Crucial parts were played by our departments at the University of Edinburgh, the School of Informatics and the School of Mathematics, especially in the shape of **Iain Gordon** and **Chris Jowett**.

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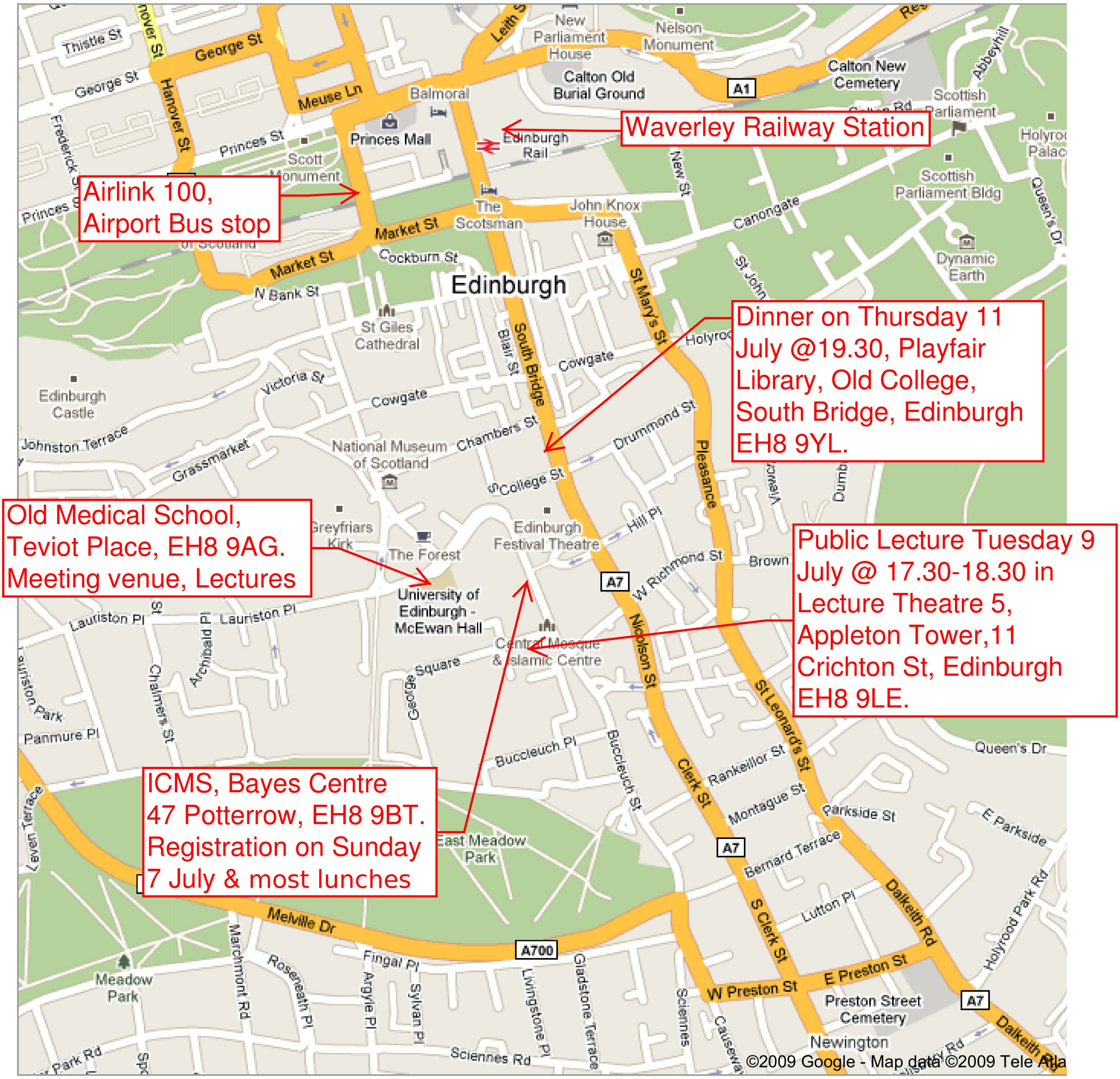
We are very grateful to the Scientific Committee for the enormous amount of work that they have done: its chair **Steve Awodey**, along with **Julie Bergner**, **Gabriella Böhm**, **John Bourke**, **Eugenia Cheng**, **Robin Cockett**, Chris Heunen, **Zurab Janelidze**, and **Dominic Verity**. Many thanks too to the other members of the Organizing Committee: **Steve Awodey**, **Richard Garner**, and **Christina Vasilakopoulou**.

The smooth running of the conference this week is made possible by our Local Experts: **Pablo Andrés-Martínez**, **Joe Collins**, **Pau Enrique Moliner**, **Martti Karvonen**, **Chad Nester**, and **Emily Roff**. Thanks to them for all their work, as well as gamely agreeing to walk around in bright orange.

Finally, thank **you** for participating. We were overwhelmed by the popularity of the conference: we'd predicted 120 participants, but nearly 170 are here, and unfortunately we had to turn some people away. Apologies to those who endured nail-biting weeks on the waiting list. One effect of the unexpectedly high level of participation is that the lecture room is completely packed. Another is that we've had to make some unusual arrangements, such as splitting both lunches and coffee breaks between two different locations. **Thank you** for your patience and understanding!

Chris Heunen (School of Informatics, University of Edinburgh)
Tom Leinster (School of Mathematics, University of Edinburgh)

Category Theory 2019



Emergencies *Ambulance, fire, police:* call 999 and alert an organizer
Conference administrator: Johanna.McBryde@icms.org.uk, +44 131 6509816
Academic organizers: Chris.Heunen@ed.ac.uk, Tom.Leinster@ed.ac.uk

Where's my lunch? Lunch is split between two locations: the MacMillan room (upstairs from talks) and ICMS (different building).

If you have a RED dot on your badge, your lunch is in the MacMillan room on MONDAY and ICMS other days.
If you have a YELLOW dot on your badge, your lunch is in the MacMillan room on TUESDAY and ICMS other days.
If you have a GREEN dot on your badge, your lunch is in the MacMillan room on THURSDAY and ICMS other days.
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We kindly ask for your cooperation in following this scheme, as we simply don't have room for everyone to eat in the same place.

Conference website http://conferences.inf.ed.ac.uk/ct2019

Category Theory 2019 programme

	Monday		Tuesday		Wednesday	Thursday		Friday		Saturday
9:00-9:30	welcome		Ghani		Shulman	Baez		Grandis		Sobral
9:30-10:00	Sobocinski									
10:00-10:30			Adamek		Campbell	Spivak		Santamaria		Gran
10:30-11:00	tea		tea		tea	tea		tea		tea
11:00-11:30	Riehl		Fiore		Paoli	Garner		Lack		Rodelo
11:30-12:00			Cockett			Berger		Bourke		Hoefnagel
12:00-12:30	Rasekh		Heunen		Paré	Szumiło		Cheng		Kock
12:30-13:00	lunch		lunch		photo	lunch		lunch		
13:00-13:30					excursion					
Room	*Meadows*	*Smith*	*Meadows*	*Smith*		*Meadows*	*Smith*	*Meadows*	*Smith*	
13:30-14:00	Tendas	Obradović	Duvieusart	Ho Thanh		Lo Monaco	Maehera	Nunes	Shen	
14:00-14:30	Fong	Hackney	Tholen	Uemura		Di Liberti	Lanari	Lucyshyn-Wright	Lai	
14:30-15:00	Frey	Zoethout	Trotta	Saville		Faul	Chu	Sousa	Cho	
15:00-15:30	Fujii	Sarazola	Parzygnat	Arkor		Menni	Dean	Clarke	Hofmann	
15:30-16:00	tea		tea			tea		tea		
16:00-16:30	Karvonen	van den Berg	MacAdam	Blechschmidt		Niefield	Rosolini	Campion	Horst	
16:30-17:00	Collins	Henry	Lemay	Bayeh		Pronk	Espindola	Stenzel	Renaud	
17:00-17:30	Sterck	den Besten				Dupont	Hutzler	Feller	Blain Levy	
reception			Cheng							
			drinks			dinner				

Reception: Sunday 17:30-19:30 in Bayes Centre

Drinks: Tuesday, after the public lecture 17:30-18:30 in Lecture Theatre 5, Appleton Tower

Dinner: Thursday 19:30 in Playfair Library, Old College

Jiři Adámek: Profinite monads and Reiterman's theorem
Nathanael Arkor: Algebraic simple type theory: a polynomial approach
John Baez: Structured cospans
Marzieh Bayeh: The category of orbit classes
Clemens Berger: Involutive factorisation systems and Dold–Kan correspondences
Ingo Blechschmidt: A general Nullstellensatz for generalised spaces
John Bourke: Accessible aspects of 2-category theory
Alexander Campbell: The model category of algebraically cofibrant 2-categories
Timothy Campion: Cubical approximation in simplicial and cubical homotopy theory
Eugenia Cheng: Coherence for tricategories via weak vertical composition
Eugenia Cheng (public lecture): Inclusion-exclusion in mathematics: who stays in, who falls out, why it happens and what we could do about it
Simon Cho: Categorical semantics of metric spaces and continuous logic
Hongyi Chu: Homotopy-coherent algebra via Segal conditions
Bryce Clarke: Internal lenses as monad morphisms
Robin Cockett: Hyperconnections
Joseph Collins: Hopf–Frobenius algebras
Christopher Dean: Higher modules and directed identity types
Martijn den Besten: A Quillen model structure for bigroupoids and pseudofunctors
Ivan di Liberti: The Scott adjunction
Benjamin Dupont: Coherence modulo and double groupoids
Arnaud Duvieusart: Simplicial objects and relative monotone-light factorizations in Mal'tsev categories
Christian Espíndola: Duality, definability and conceptual completeness for κ-pretoposes
Peter Faul: Artin glueings of frames and toposes as semidirect products
Matt Feller: New model structures on simplicial sets
Marcelo Fiore: Fast-growing clones
Brendan Fong: Graphical regular logic
Jonas Frey: A language for closed cartesian bicategories
Soichiro Fujii: A unified framework for notions of algebraic theory
Richard Garner: Fixpoint toposes
Neil Ghani: Compositional game theory
Marino Gran: Split extension classifiers in the category of cocommutative Hopf algebras
Marco Grandis: Adjunctions and limits for double and multiple categories
Philip Hackney: Right adjoints to operadic restriction functors
Simon Henry: A model 2-category of combinatorial model categories
Chris Heunen: Compact inverse categories
Michael Hoefnagel: $\mathcal{M}$-coextensivity and the strict refinement property
Dirk Hofmann: Coalgebras for enriched Hausdorff (and Vietoris) functors
Michael Horst: Cohomology of Picard categories
Cédric Ho Thanh: A type theory for opetopes
Matthias Hutzler: Internal language and classified theories of toposes in algebraic geometry
Martti Karvonen: Limits in dagger categories
Joachim Kock: Operadic categories, decalage, and decomposition spaces

Steve Lack: Weak adjoint functor theorems
Hongliang Lai: Continuous complete categories enriched over quantales
Edoardo Lanari: The generalized Homotopy Hypothesis
JS Pacaud Lemay: Tangent categories from the coalgebras of differential categories
Paul Blain Levy: What is a monoid?
Giulio Lo Monaco: Set-theoretic remarks on a possible definition of elementary ∞-topos
Fernando Lucatelli Nunes: Descent and monadicity
Rory Lucyshyn-Wright: Functional distribution monads and τ-additive Borel measures
Benjamin MacAdam: An enriched perspective on differentiable stacks
Yuki Maehara: Lax Gray tensor product for 2-quasi-categories
Matías Menni: One: a characterization of skeletal objects for the Aufhebung of Level 0 in certain toposes of spaces
Susan Niefield: Exponentiability in double categories and the glueing construction
Jovana Obradović: Combinatorial homotopy theory for operads
Simona Paoli: Segal-type models of weak n-categories
Robert Paré: Retro cells
Arthur Parzygnat: Non-commutative disintegrations and regular conditional probabilities
Dorette Pronk: Left cancellative categories and ordered groupoids
Nima Rasekh: Truncations and Blakers–Massey in an elementary higher topos
François Renaud: Double quandle coverings
Emily Riehl: A formal category theory for ∞-categories
Diana Rodelo: Intrinsic Schreier split extensions
Giuseppe Rosolini: Relating the effective topos to HoTT
Alessio Santamaria: A solution for the compositionality problem of dinatural transformations
Maru Sarazola: A recipe for black box functors
Philip Saville: A type theory for cartesian closed bicategories
Lili Shen: Quantale-valued dissimilarity
Michael Shulman: Internal languages of higher toposes
Pawel Sobocinski: Graphical linear algebra
Manuela Sobral: Non-semi-abelian split extensions in categorical algebra
Lurdes Sousa: Codensity monads and D-ultrafilters
David Spivak: Abelian calculi present abelian categories
Raffael Stenzel: Univalence and completeness of Segal objects
Florence Sterck: On the category of cocommutative Hopf algebras
Karol Szumiło: A constructive Kan–Quillen model structure
Giacomo Tendas: Enriched regular theories
Walter Tholen: Functorial decomposition of colimits
Davide Trotta: The existential completion
Taichi Uemura: A general framework for categorical semantics of type theory
Benno van den Berg: Uniform Kan fibrations in simplicial sets
Jetze Zoethout: Relative partial combinatory algebras over Heyting categories

Abstracts

PROFINITE MONADS AND REITERMAN'S THEOREM

Jiři Adámek
Czech Technical University Prague

Jan Reiterman characterized in 1980's **pseudovarieties** $\mathcal{D}$ of finite algebras, i.e., classes closed under finite products, subalgebras and quotients: they are precisely the classes that can be presented by equations between profinite terms, see [1]. A profinite term is an element of the **profinite monad** which is the codensity monad of the forgetful functor of $Pro\mathcal{D}$, the profinite completion of the given pseudovariety.

We have recently generalized this result to pseudovarieties of $\mathcal{T}$-algebras where $\mathcal{T}$ is a monad over a base category which can be an arbitrary locally finite variety of (possibly ordered) algebras, see [2]. But now we realize that a much more general result holds: the base category need not be a variety, it can be an arbitrary complete and wellpowered category $\mathcal{D}$ in which a full subcategory $\mathcal{D}_f$ (of objects called 'finite') is chosen so that

(i) $\mathcal{D}_f$ is closed under subobjects,

(ii) every finite object is strong quotient of a strongly projective object, and

(iii) all strong epimorphims in $\mathcal{D}_f$ are closed under cofiltered limits.

For every monad over $\mathcal{D}$ peserving strong epimorphisms we then introduce its profinite monad on $Pro\mathcal{D}_f$ and the corresponding concept of profinite equation. We then prove that a collection of finite algebras (i.e., $\mathcal{T}$-algebras carried by objects of $\mathcal{D}_f$) is a pseudovariety iff it can be presented by profinite equations. As a special case we obtain the result of Pin and Weil about pseudovarieties of first-order structures, see [3].

References

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[3]J.-E.Pin and P.Weil, A Reiterman theorem for pseudovarieties of finite first-order structures. Algebra Universalis 14 (1996), 577-595

Nathanael Arkor*

Department of Computer Science and Technology
University of Cambridge

Algebraic simple type theory: a polynomial approach

Algebraic type theory is the study of type theory qua the extension of universal algebra to richer settings (*e.g.* sorting, binding, polymorphism, dependency, and so on). In this context, this talk will focus on *simple type theories*, by which we mean ones with type structure consisting of finitary algebraic operators and with term structure consisting of binding operators, *cf.* [2]. Examples of simple type theories beyond algebraic theories are first-order logic, the untyped and simply-typed λ-calculi, and the computational λ-calculus.

First, we will be concerned with the abstract syntax of simple type theories. Their signatures will be introduced and their algebraic models defined. The approach, which is based on the theories of abstract syntax and variable binding [3] and of polynomial functors [4], is new. At its core is the general idea that type-theoretic rules are notation for polynomial functors and that the syntax generated by such rules is a free algebra, *cf.* [1]. Free algebra constructions will be discussed. Next, we will extend the theory to incorporate substitution. Algebraic models will be discussed and the preceding developments will be used to construct initial models with substitution. This relates to the introduction of substitution as a meta-operation in type theory and to the construction of a classifying category in category theory. Time permitting, we will touch upon the theory further encompassing equational presentations.

This work provides a basis and framework for our ongoing development of algebraic dependent type theory.

References:

[1] M. Fiore, Discrete generalised polynomial functors, *Proceedings of the 39th International Colloquium on Automata, Languages and Programming* (2012) 214–226.

[2] M. Fiore and C.-K. Hur, Second-order equational logic, *Proceedings of the 19th EACSL Annual Conference on Computer Science Logic* (2010) 320–335.

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*Joint work with Marcelo Fiore.

STRUCTURED COSPANS

JOHN BAEZ

Open systems of many kinds can be treated as morphisms in symmetric monoidal categories. Two complementary approaches can be used to work with such categories: props (which are more algebraic in flavor) and cospan categories (which are more geometrical). In this talk we focus on the latter. Brendan Fong's "decorated cospans" are a powerful tool for treating open systems as cospans equipped with extra structure. Recently Kenny Courser has found a simpler alternative, the theory of "structured cospans". We describe this theory and sketch how it has been applied to a variety of open systems, such as electrical circuits, Markov processes, chemical reactions and Petri nets.

University of California Riverside

THE CATEGORY OF ORBIT CLASSES

MARZIEH BAYEH

The concept of orbit class was introduced by Bayeh and Sarkar to study topological spaces endowed with an action of a topological group. In particular, this concept has been used to compute some bounds for the equivariant LS-category and the invariant topological complexity, and it has been proved that in some cases the orbit class is a good replacement for the orbit type.

In this talk I will introduce the category of orbit classes and study some aspects of this category.

INVOLUTIVE FACTORISATION SYSTEMS AND DOLD-KAN CORRESPONDENCES

CLEMENS BERGER

In the late 1950's Dold [3] and Kan [5] showed that a simplicial abelian group is completely determined by an associated chain complex and that this construction yields an equivalence of categories. This *Dold-Kan correspondence* is important in algebraic topology because it permits an explicit construction of topological spaces with prescribed homotopy groups (e.g. Eilenberg-MacLane spaces).

There have been several attempts to extend this kind of categorical equivalence to other contexts, most recently by Lack and Street [6]. We present here an approach based on the notion of *involutive factorisation system*, i.e. an orthogonal factorisation system $\mathcal{C} = (\mathcal{E}, \mathcal{M})$ equipped with a faithful, identity-on-objects functor $\mathcal{E}^{\mathrm{op}} \to \mathcal{M} : e \mapsto e^*$ such that $ee^* = 1$ and three other axioms are satisfied.

We show that for each category $\mathcal{C}$ equipped with such an involutive factorisation system $(\mathcal{E}, \mathcal{M}, (-)^*)$, there is an equivalence $[\mathcal{C}^{\mathrm{op}}, \mathcal{A}] \simeq [\Xi(\mathcal{C})^{\mathrm{op}}, \mathcal{A}]_*$ where $\mathcal{A}$ is any idempotent-complete *additive* category, and $\Xi(\mathcal{C})$ is the locally pointed category of *essential* $\mathcal{M}$-maps. Since in the simplex category Δ the only essential $\mathcal{M}$-maps are the last face operators $\epsilon_n : [n-1] \to [n]$, we get ordinary chain complexes in $\mathcal{A}$ on the right so that our equivalence specialises to Dold-Kan correspondence if $\mathcal{C} = \Delta$.

Our approach recovers several known equivalences, cf. [8, 4, 2]. An interesting *new* family is given by Joyal's cell categories Θ_n (cf. [1]) where our equivalence relates to n-th order Hochschild homology and E_n-homology (cf. [9, 7]).

This is joint work with Christophe Cazanave and Ingo Waschkies.

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A GENERAL NULLSTELLENSATZ FOR GENERALISED SPACES

INGO BLECHSCHMIDT

The Nullstellensatz of commutative algebra states that some algebraic truths are witnessed by explicit algebraic certificates, syntactical a priori reasons for why a given truth is to be expected.

Any geometric theory possesses a tautologous yet intriguing *generic model*. Topologically, any model can be obtained as a pullback of the generic one, and logically, it has exactly those properties which are shared by all models. Crucially however, this statement is only true for properties which can be put as geometric sequents. The generic model may enjoy additional first-order or higher-order properties which are not shared by all models.

The talk starts by reviewing this circle of ideals, in particular highlighting some of the applications of nongeometric properties in commutative algebra, where they provide new reduction techniques, and in algebraic geometry, where they allow for a synthetic development of the foundations. We then present a logical analogue of the algebraic Nullstellensatz, valid for the generic model of any geometric theory. This analogue replaces algebraic certificates by logical certificates – geometric proofs. It is a source of nongeometric sequents enjoyed by generic models, and it turns out that it is the universal such source.

ACCESSIBLE ASPECTS OF 2-CATEGORY THEORY

JOHN BOURKE

Two-dimensional universal algebra is primarily concerned with categories equipped with structure and functors preserving that structure up to coherent isomorphism. The 2-category of monoidal categories and strong monoidal functors provides a good example. As understood by the Australian school in the 1980s such 2-categories admit all weak 2-categorical colimits, but not necessarily genuine colimits such as coequalisers. In particular, they are rarely locally presentable in the usual sense.

The theory of accessible categories was developed by Makkai and Pare around the same time. This weakens the theory of locally presentable categories by requiring only the existence of certain filtered colimits. Whilst the initial motivations come from model theory, accessible categories have since become an important tool in homotopy theory.

The present talk, which builds on discussions between Makkai and I, will explain the connection between two dimensional universal algebra and accessible categories. Using homotopical techniques, we will see that many 2-categories, such as the 2-category of monoidal categories mentioned above, are accessible, and that accessibility of a 2-category of categorical structures is intimately connected to the structures in question being sufficiently weak.

I also hope to mention connections with infinity-cosmoi, for which see also the related talk of Steve Lack.

Alexander Campbell
Macquarie University

The model category of algebraically cofibrant 2-categories

As I discussed at CT2017 (see also [5]), a basic obstruction to the development of a purely **Gray**-enriched model for three-dimensional category theory is the fact that not every 2-category is cofibrant in Lack's model structure on **2-Cat** [4]. This obstruction can be overcome by the introduction of a new base for enrichment: the monoidal model category $\mathbf{2\text{-}Cat}_Q$ of *algebraically cofibrant* 2*-categories*, which is the subject of this talk.

This category $\mathbf{2\text{-}Cat}_Q$ can be defined as the category of coalgebras for the normal pseudofunctor classifier comonad on **2-Cat**, and is thus a non-full replete subcategory of **2-Cat** whose objects are the cofibrant 2-categories. (It can also be defined as the evident 2-categorical analogue of the category of simplicial computads studied by Riehl and Verity [6].) Using modern model category techniques [3, 2], I will show that the category $\mathbf{2\text{-}Cat}_Q$ admits an "injective" model structure, left-induced from (and Quillen equivalent to) Lack's model structure on **2-Cat** along the left-adjoint inclusion $\mathbf{2\text{-}Cat}_Q \longrightarrow \mathbf{2\text{-}Cat}$.

Remarkably, the category of bicategories and normal pseudofunctors is equivalent, via the normal strictification functor, to the full subcategory of $\mathbf{2\text{-}Cat}_Q$ consisting of the fibrant objects for the induced model structure. Moreover, like Lack's model structure on **2-Cat**, the induced model structure on $\mathbf{2\text{-}Cat}_Q$ is monoidal with respect to the (symmetric) Gray tensor product, but unlike Lack's model structure, the induced model structure is also cartesian.

Note that the word "normal" in the above definition of $\mathbf{2\text{-}Cat}_Q$ is crucial to these results: a simple argument shows that the category of coalgebras for the (non-normal) pseudofunctor classifier comonad on **2-Cat** fails the acyclicity condition, and therefore does not admit a model structure left-induced from Lack's model structure on **2-Cat**. This disproves a conjecture posed by Ching and Riehl [1].

References:

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CUBICAL APPROXIMATION IN SIMPLICIAL AND CUBICAL HOMOTOPY THEORY

TIMOTHY CAMPION

We formulate and prove cubical approximation theorems in cubical, simplicial, and topological settings, which are analogous to the familiar simplicial approximation theorem. To this end, we construct a cubical version of Kan's Ex^∞ functor and show that it is a functorial fibrant replacement with good properties on certain categories of cubical sets. We give several applications.

As a prerequisite, we systematically construct model structures on several categories of cubical sets (cubical sets with or without connections / symmetries / reversals), study their monoidal properties, and establish Quillen equivalences among them and to simplicial sets and topological spaces.

COHERENCE FOR TRICATEGORIES VIA WEAK VERTICAL COMPOSITION

EUGENIA CHENG

It is well known that every strict 2-category is equivalent to a weak one, but that the analogous result for 3-categories does not hold. Rather, coherence for weak 3-categories (tricategories) needs more nuance. One way of viewing this is that we need to take account of possible braidings that arise and cannot be strictified into symmetries. The original coherence result of Gordon–Power–Street says, essentially, that every tricategory is equivalent to one in which everything is strict except interchange. The intuition is that "braidings arise from weak interchange". However, from close observation of how the Eckmann-Hilton argument works, Simpson conjectured that weak units would be enough, and this result was proved for the case n=3 by Joyal and Kock. Their result involves a weak unit I in an otherwise completely strict monoidal 2-category. They showed that End(I) is naturally a braided monoidal category, and that every braided monoidal category is equivalent to a End(I) for some monoidal 2-category. Regarding this as a degenerate 3-category, this would mean that everything is strict except horizontal units. In this talk we will address a third case, in which everything is strict except vertical composition; this amounts to considering categories strictly enriched in the category of bicategories with strict functors, with respect to Cartesian product. We will show that every doubly degenerate such tricategory is naturally a braided monoidal category, that every braided monoidal category is equivalent to one of these. The proof closely follows Joyal and Kock's method of clique constructions. Joyal and Kock use train track diagrams to give just enough "rigidity" to the structure of points in 3-space, and they describe this as preventing the points from being able to simply commute past each other via an Eckmann–Hilton argument. We are aiming for a different axis of strictness and so instead of points in R^2 with cliques arising from train track diagrams, we use use points embedded in the interior of I^2 with cliques arising from horizontal "slides". This method generalises, by omitting the "slide" cliques, to prove the corresponding result for doubly degenerate Trimble 3-categories. There are several critical subtleties to this which is why we have to leave vertical associativity weak but can use fully doubly degenerate structures, where Joyal and Kock were able to have all associativity strict but could not use fully doubly degenerate structures. We also extend the result to totalities, exhibiting a biequivalence of appropriate bicategories.

INCLUSION-EXCLUSION IN MATHEMATICS: WHO STAYS IN, WHO FALLS OUT, WHY IT HAPPENS, AND WHAT WE COULD DO ABOUT IT

EUGENIA CHENG

The question of why women and minorities are under-represented in mathematics is complex and there are no simple answers, only many contributing factors. I will focus on character traits, and argue that if we focus on this rather than gender we can have a more productive and less divisive conversation. To this end I will introduce gender-neutral character adjectives "ingressive" and "congressive" as a new dimension to shift our focus away from masculine and feminine. I will share my experience of teaching congressive abstract mathematics to art students, in a congressive way, and the possible effects this could have for everyone in mathematics, not just women. I will present the abstract field of Category Theory as a particularly congressive subject area, accessible to bright school students, and contrast it with the types of maths that are often used to push or stimulate those students. Moreover I will show that it is applicable to working towards a more inclusive, congressive society in this politically divisive era. This goes against the assumption that abstract mathematics can only be taught to high level undergraduates and graduate students, and the accusation that abstract mathematics is removed from real life. No prior knowledge will be needed.

School of Art Institute of Chicago

CATEGORICAL SEMANTICS OF METRIC SPACES AND CONTINUOUS LOGIC

SIMON CHO

Using the category of metric spaces as a template, we develop a metric analogue of the categorical semantics of classical/intuitionistic logic, and show that the natural notion of predicate in this 'continuous semantics' is equivalent to the a priori separate notion of predicate in continuous logic, a logic which is independently well-studied by model theorists and which finds various applications. We show this equivalence by exhibiting the real interval $[0, 1]$ in the category of metric spaces as a 'continuous subobject classifier' giving a correspondence not only between the two notions of predicate, but also between the natural notion of quantification in the continuous semantics and the existing notion of quantification in continuous logic. Along the way, we formulate what it means for a given category to behave like the category of metric spaces, and afterwards show that any such category supports the aforementioned continuous semantics. As an application, we show that categories of presheaves of metric spaces are examples of such, and in fact even possess continuous subobject classifiers.

HOMOTOPY-COHERENT ALGEBRA VIA SEGAL CONDITIONS

HONGYI CHU AND RUNE HAUGSENG

In this talk I will introduce a general framework for homotopy-coherent algebraic structures defined by presheaves on certain infinity categories - so-called "algebraic patterns" - satisfying Segal-type limit conditions. This theory does not only recover well-known examples for infinity categorical structures such as (infinity, n)-categories, infinity-operads and infinity-properads, but also allows us to define enrichment on these objects as well as algebras for infinity-operads. I will discuss some applications of the theory of algebraic patterns such as providing a simple proof for the existence of operadic Kan extensions in the sense of Lurie. Moreover, every algebraic structure given by an algebraic pattern defines a polynomial monad on a functor category. At the end of this talk we will see that the infinity category of polynomial monads on functor categories is actually equivalent to the infinity category of algebraic patterns.

Bryce Clarke
Macquarie University

Internal lenses as monad morphisms

Lenses were originally introduced in [1] as a mathematical structure which captures the notion of synchronisation between a pair of sets. When extending this idea to consider synchronisation between a pair of categories [2], we obtain functors akin to Grothendieck opfibrations lacking the universal property. The purpose of this talk is to motivate the notion of synchronisation between a pair of internal categories in a category with pullbacks [3].

We recall that internal categories are monads in the bicategory of spans, and thus internal functors are colax monad morphisms whose underlying span has identity left leg. Dually, internal cofunctors are the corresponding lax monad morphisms, which may be represented as a span of internal functors whose left leg is an identity-on-objects functor and whose right leg is a discrete opfibration.

In this talk I will define an internal lens as a monad morphism which is both an internal functor and cofunctor. Specialising internal to **Set** clarifies the usual notion of asymmetric lens, while considering internal lenses in **Cat** between double categories of squares yields split Grothendieck opfibrations. I will also define internal symmetric lenses as a pair of internal Mealy functors [4] (or two-dimensional partial maps) and establish the relationship with spans of internal lenses, generalising the results in [5].

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Robin Cockett*
University of Calgary

Hyperconnections

Functors between join restriction categories admit a factorization into localic functors (which are bijective on objects and preserve meets) followed by hyperconnected functors (which are bijective on the locales of restriction idempotents) - for the basic localic/hyperconnected factorization on mere restriction categories see [5].

A partite category (see [3], for example, for source étale internal categories: a *partite* internal category has its objects and arrow partitioned into many objects) internal to a join restriction category $\mathbb{B}$ induces, by considering partial sections of the domain maps, an external join restriction category which sits over $\mathbb{B}$ by a hyperconnected functor. Conversely an external join restriction category over $\mathbb{B}$ (where the latter must be assumed to have all gluings [4]) induces a source étale partite category internal to $\mathbb{B}$. This correspondence may be completed to a Galois adjunction between join restriction categories over $\mathbb{B}$ and partite categories internal to $\mathbb{B}$ with cofunctors [1, 2]. The adjunction specializes to an equivalence between hyperconnections over $\mathbb{B}$ and source étale partite categories internal to $\mathbb{B}$.

This phenomenon occurs in many different places in mathematics (often specialized to groupoids). In particular, as all join restriction categories have a hyperconnected fundamental functor to the category of locales (with partial maps) one can conclude that join restriction categories (with join functors) correspond precisely to source étale partite categories internal to locales (with cofunctors). From algebraic geometry, considering schemes as a join restriction category with gluings, the identity functor on schemes induces an internal partite category: the object of morphisms from the affine scheme R to $\mathbb{Z}[x]$ is then exactly the structure sheaf of R.

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*Joint work with Richard Garner.

Hopf-Frobenius algebras

Joseph Collins[1] Ross Duncan[1,2]
joseph.collins@strath.ac.uk ross.duncan@strath.ac.uk
[1]Department of Computer and Information Sciences
University of Strathclyde
26 Richmond Street, Glasgow, United Kingdom
[2]Cambridge Quantum Computing Ltd
9a Bridge Street, Cambridge, United Kingdom

In the monoidal categories approach to quantum theory [1, 6] Hopf algebras [14] have a central role in the formulation of complementary observables [5]. In this setting, a quantum observable is represented as special commutative †-Frobenius algebra; a pair of such observables are called *strongly complementary* if the algebra part of the first and the coalgebra part of the second jointly form a Hopf algebra. In abstract form, this combination of structures has been studied under the name "interacting Frobenius algebras" [8] where it is shown that relatively weak commutation rules between the two Frobenius algebras produce the Hopf algebra structure. From a different starting point Bonchi et al [3] showed that a distributive law between two Hopf algebras yields a pair of Frobenius structures, an approach which has been generalised to provide a model of Petri nets [2]. Given the similarity of the two structures it is appropriate to consider both as exemplars of a common family of *Hopf-Frobenius algebras*.

In the above settings, the algebras considered are both commutative and cocommutative. However more general Hopf algebras, perhaps not even symmetric, are a ubiquitous structure in mathematical physics, finding application in gauge theory [12], condensed matter theory [13], quantum field theory [4] and quantum gravity [11]. We take the first steps towards generalising the concept of Hopf-Frobenius algebra to the non-commutative case, and opening the door to applications of categorical quantum theory in other areas of physics.

Loosely speaking, a Hopf-Frobenius algebra consists of two monoids and two comonoids such that one way of pairing a monoid with a comonoid gives two Frobenius algebras, and the other pairing yields two Hopf algebras, with the additional condition that antipodes are constructed from the Frobenius forms. Fundamental to the concept of a Hopf-Frobenius algebra is a particular pair of morphisms called an integral and a cointegral. We show that when these morphisms are 'compatible' in a particular sense, they produce structure similar to a Hopf-Frobenius algebra. It is from this that we produce necessary and sufficient conditions to extend a Hopf algebra to a Hopf-Frobenius algebra in a symmetric monoidal category. It was previously known that in $\mathbf{FVect_k}$, the category of finite dimensional vector spaces, every Hopf algebra carries a Frobenius algebra on both its monoid [10] and its comonoid [7, 9]; in fact we show that every Hopf algebra in $\mathbf{FVect_k}$ is Hopf-Frobenius. We are therefore able to find many examples of Hopf-Frobenius algebras that are not commutative or cocommutative. Finally, due to the fact that every Frobenius algebra is self dual, in a compact closed category we may find a natural isomorphism between the algebra and its dual. We use this isomorphism to construct a Hopf algebra on $H \otimes H$ that is isomorphic to the Drinfeld double.

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Higher Modules and Directed Identity Types

Christopher Dean
Department of Computer Science
University of Oxford

Virtual double categories provide an excellent framework for **formal category theory**. For many sorts of "category-like" object, there is a virtual double category of categories, profunctors and transformations between these things. The virtual double categories of interest typically contain canonical profunctors which behave like Hom-profunctors. The universal property characterizing these objects is an abstract version of the Yoneda Lemma. Equivalently, it says that we have **identity types** in a **directed type theory**.

The right adjoint of the functor which forgets such identity types is the monoids and modules construction on virtual double categories. Such constructions are ubiquitous throughout category theory and allow us to construct new virtual double categories of "category-like" objects. Ordinary, enriched and internal categories, together with the corresponding notions of profunctors and transformations, can be defined succinctly as the result of applying the monoids and modules construction to simpler virtual double categories. (See [3].) In this way we can construct all the data we need to "do category theory".

Our contribution is the extension of this discussion to **formal higher category theory**. Globular multicategories are to ω-categories what virtual double categories are to categories. Identity types can be defined in this setting. We call the right adjoint of the functor which forgets these identity types the **higher modules construction**. This allows us to construct notions of higher categories, higher profunctors, profunctors between profunctors, . . . and transformations between these things.

At first we obtain strict notions of all these things. However, the Batanin-Leinster approach (see [1], [5], [4]) to weakening globular operads can be extended to this setting and in this way we obtain weak notions of all these objects. There is also a notion of composition of weak transformations.

Thus, globular multicategories should provide a good setting for "doing higher category theory". As a first result, we observe that, discarding everything except the trivial modules, we obtain a weak ω-category of weak ω-categories in the sense of Batanin. Globular multicategories should also provide a natural semantics for (directed) type theories with identity types. In fact [2] can be seen as a first result in this direction.

Reference list: see programme online

A QUILLEN MODEL STRUCTURE FOR BIGROUPOIDS AND PSEUDOFUNCTORS

MARTIJN DEN BESTEN

We construct a Quillen model structure on the category of (small) bigroupoids and pseudofunctors. We show that the inclusion of the category of (small) 2-groupoids and 2-functors in the aforementioned category is the right adjoint part of a Quillen equivalence, with respect to the model structure provided by Moerdijk and Svensson (1993). To construct this equivalence, and in order to keep certain calculations of manageable size, we prove a coherence theorem for bigroupoids and a coherence theorem for pseudofunctors. These coherence theorems may be of independent interest as well.

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Accessible categories with directed colimits have proven to be a suitable framework to develop abstract model theory and generalize the notion of abstract elementary class, quite relevant in model theory. For every accessible category with directed colimits A, one can define its Scott topos S(A). This construction is the categorification of the Scott topology over a poset with directed unions, and thus provides a geometric understanding of accessible categories. S(A) represents also a candidate axiomatization of A, in the sense that the category of points of the Scott topos (i.e., the models of the theory that it classifies) is very often a relevant hull of A. During the talk we introduce the Scott construction and explain both its geometric and logical aspects.

Coherence modulo and double groupoids

Benjamin Dupont, Université Claude Bernard Lyon 1, Institut Camille Jordan,
bdupont@math.univ-lyon1.fr

This work is part of a research project aiming at developing constructive methods based on rewriting theory to study algebraic structures, and compute coherent presentations, linear bases and higher syzygies. In many situations presentations of algebraic structrues have a great complexity due to the number of relations, some of them being axioms of the structure itself. We present a categorical approach for rewriting modulo and a method to compute coherent presentations modulo axioms. We introduce a notion of polygraphs modulo as higher-dimensional rewriting systems presenting higher dimensional categories whose axioms are not considered as oriented rules, [2]. We define following [4] termination and confluence properties for these polygraphs, with confluence diagrams of the form

$$\begin{tikzcd} u \arrow[r, "f"] \arrow[d, "e"'] & u' \arrow[r, dotted, "f'"] & w \arrow[d, dotted, "e'"] \\ v \arrow[r, "g"'] & v' \arrow[r, dotted, "g'"'] & w' \end{tikzcd}$$

where the cells f,f',g and g' correspond to rewriting sequences and the cells e, e' correspond to equations derived from axioms. Following [3, 5], a coherent presentation of an n-category can be obtained from a presentation by a convergent $(n+1)$-polygraph P extended by $(n+2)$-cells associated to confluence diagrams of critical branchings. In this spirit, coherence modulo axioms can be formulated in terms of categories enriched in double groupoids where the horizontal cells correspond to rewriting sequences, and the vertical cells correspond to the congruence generated by the axioms.

Using the approach of [1] to construct free double categories, we introduce a notion of double $(n+2,n)$-polygraph as a data generating a free n-category enriched in double groupoids defined from a set of square cells on the pair of free n-categories on the axioms and on rewriting rules. We introduce a suited notion of coherent confluence modulo given by existence of a square cell in each confluence modulo diagram as follows:

$$\begin{tikzcd} u \arrow[r, "f"] \arrow[d, "e"'] & u' \arrow[r, dotted, "f'"] \arrow[d, phantom, "\Downarrow A"] & w \arrow[d, dotted, "e'"] \\ v \arrow[r, "g"'] & v' \arrow[r, dotted, "g'"'] & w' \end{tikzcd}$$

We prove that the coherent confluence of a polygraph modulo is equivalent to the coherent confluence of some critical branchings modulo. We define a coherent confluence modulo procedure that we apply to the computation of coherent presentations of commutative monoids and pivotal categories.

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Arnaud Duvieusart
Université catholique de Louvain

Simplicial objects and relative monotone-light factorizations in Mal'tsev categories

In [1], Brown and Janelidze introduced a Galois structure on the category of simplicial sets, by considering the category of groupoids as a reflective full subcategory and the class of Kan fibrations as extensions, and showed that Kan complexes are admissible for this Galois structure. Later, Chikhladze introduced the notion of *relative* factorization system, and showed that this Galois structure induces a relative monotone-light factorization system for Kan fibrations [2].

On the other hand, it is known [3, 4] that every simplicial object in a regular Mal'tsev category has the Kan property; and moreover exact Mal'tsev categories form a good setting for Categorical Galois Theory, as every Birkhoff subcategory is then admissible [5]. Groupoids themselves also play an important role in the study of Mal'tsev categories [6].

It seems then natural to ask whether internal groupoids can be seen as an admissible subcategory of simplicial objects in any exact Mal'tsev category. We will prove that it is in fact a Birkhoff subcategory, and that it also admits relative monotone-light factorizations for Kan fibrations (which in this context coincide with regular epimorphisms).

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Duality, definability and conceptual completeness for κ-pretoposes

Christian Espíndola

The κ-geometric toposes introduced in [2], for regular κ such that $\kappa^{<\kappa} = \kappa$ (or any regular κ if the Generalized Continuum Hypothesis holds), are associated to κ-geometric logic, an extension of geometric logic in which conjunction of less than κ many formulas as well as quantification of less than κ many variables are possible. They in fact occur as κ-classifying toposes, i.e., toposes with the obvious universal property applied to κ-geometric morphisms (those whose inverse image preserve all κ-small limits). When these toposes are in addition κ-separable, they turn out to have enough κ-points. We prove here that this completeness theorem has the surprising consequence that for $\lambda > \kappa$, the λ-classifying topos of any theory with at most κ axioms, expressible in κ-geometric logic, is in fact the topos of presheaves over the category of λ-presentable models of the theory.

As applications we get positive results on definability theorems for infinitary logic, conceptual completeness for κ-pretoposes, infinitary versions of Joyal's completeness theorem for infinitary intuitionistic logic, a Stone type duality in the form of a biequivalence arising from a syntax-semantics adjunction, the descent theorem for κ-pretoposes, and a characterization of categoricity for models of infinitary sentences. Time permitting we will show how this latter result provides a topos-theoretic approach to Shelah's eventual categoricity conjecture.

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Artin glueings of frames and toposes as semidirect products

Peter Faul

University of Cambridge, Cambridge, United Kingdom
peter@faul.io

Abstract

An Artin glueing [2] of two frames H and N is a frame G in which H and N are included as sublocales, with H open and N its closed complement. Artin glueings are not unique, but are determined by finite-meet preserving maps $f\colon H \to N$. This notion categorifies to the setting of toposes. The details of these constructions can be found in [1].

Compare this to a semidirect product G of two groups N and H. Both N and H are subgroups of G, with N being normal. They satisfy that $N \cap H = \{e\}$ and $NH = G$, which if thought of in terms of the lattice of subobjects of G, says that N and H are complements. Furthermore, just as with the Artin glueing, semidirect products of groups are not unique and are similarly determined by a map $f\colon H \to \mathrm{Aut}(N)$.

In order to make this connection precise we examine a link to extension problems. It is well known that the split extensions between any two groups N and H are precisely the semidirect products of N and H. We show that Artin glueings of frames are the solutions to a natural extension problem in the category **RFrm** of frames with finite-meet preserving maps, and that Artin glueings of toposes are the solutions to a natural extension problem in the category **RTopos** of toposes with finite-limit preserving maps.

Talking about extensions requires appropriate notions of kernels and cokernels. We say a chain $N \xrightarrow{m} G \xrightarrow{e} H$ is an extension when m is the kernel of e and e is the cokernel of m. In the case of groups it is the split extensions that are important and these satisfy the property that if s is a splitting of e, then the images of m and s together generate G. This is not so in **RFrm** and **RTopos** and will only occur when s is the right adjoint of e. This motivates restricting to adjoint split extensions.

We show that there is a natural way to view Artin glueings of frames and toposes as extensions and that every extension $N \xrightarrow{m} G \xrightarrow{e} H$ can be thought of as the glueing of H and N along m^*e_*.

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NEW MODEL STRUCTURES ON SIMPLICIAL SETS

MATT FELLER

Abstract. In the way Kan complexes and quasi-categories model up-to-homotopy groupoids and categories, can we find model structures on simplicial sets which give up-to-homotopy versions of more general objects? We investigate this question, with the particular motivating example of 2-Segal sets. Cisinski's work on model structures in presheaf categories provides abstract blueprints for these new model structures, but turning these blueprints into a satisfying description is a nontrivial task. As a first step, we describe the minimal model structure on simplicial sets arising from Cisinski's theory.

University of Virginia, Department of Mathematics, Charlottesville VA 22903
E-mail address: `feller@virginia.edu`

Marcelo Fiore

Department of Computer Science and Technology
University of Cambridge

Fast-growing clones

A concrete clone on a set is a family of finitary operations on the set that is closed under projections and composition. Abstract clones are the algebras for the $\mathbb{N}$-sorted equational presentation that axiomatises this structure. Categorically, they are Lawvere theories.

It is a standard result of universal algebra that every abstract clone can be represented by a concrete clone on a set [2]. This has been generalised to discrete enriched clones for enriching categories with enough structure [1], crucially infinitary structure in the form of colimits of ω-chains of sections.

Consideration of whether infinitary structure is in general necessary for clone representation, led us to show that there is no clone-representation result in the context of finite sets. This I will present as a by-product of technical developments of a logical, rather than algebraic or combinatorial, nature. The underlying categorical theory allows one to construct monads on finite sets with free algebras that asymptotically grow faster than every iterated exponential, for any natural height, on their generators.

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Brendan Fong and David Spivak
Massachusetts Institute of Technology

Graphical Regular Logic

Regular logic can be regarded as the *internal language* of regular categories, but the logic itself is generally not given a categorical treatment. In this talk, I'll give an overview of how to understand the syntax and proof rules of regular logic in terms of the free regular category $\mathsf{FRg}(\mathrm{T})$ on a set T. From this point of view, regular theories are certain monoidal 2-functors from a suitable 2-category of contexts—the 2-category of relations in $\mathsf{FRg}(\mathrm{T})$—to that of posets. Such functors assign to each context the set of formulas in that context, ordered by entailment. We refer to such a 2-functor as a *regular calculus* because it naturally gives rise to a graphical string diagram calculus in the spirit of Joyal and Street. The main theorem is that the category of regular categories is essentially reflective in that of regular calculi. Along the way, I'll demonstrate how to use this graphical calculus for regular logic.

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A LANGUAGE FOR CLOSED CARTESIAN BICATEGORIES

JONAS FREY

A *cartesian bicategory* [CW87; Car+08] is a bicategory $\mathcal{B}$ in which all hom-categories have finite products, and the full subcategory of left adjoints has finite bicategorical products which extend to a symmetric monoidal structure on $\mathcal{B}$ in a canonical way. We call a cartesian bicategory *closed*, if all pre- and postcomposition functors have right adjoints. The canonical example of a closed cartesian bicategory is the bicategory **Prof** of small categories and profunctors, where composition is given by coends, and the closed structure by ends. I present a natural-deduction style language for cartesian bicategories which controls the 'mixed variances' appearing in calculations with (co)ends by means of a syntactic condition on judgments. Specifically, the judgments of the language are of the form

$$\langle \vec{A}_0 \rangle \, x_1 : \varphi_1 \, \langle \vec{A}_1 \rangle \, \dots \, \langle \vec{A}_{n-1} \rangle \, x_n : \varphi_n \, \langle \vec{A}_n \rangle \vdash t : \psi$$

where the $\vec{A}_i$ are lists of variables (representing objects of categories in the interpretation in **Prof**), the φ_i and ψ are *formulas* (representing profunctors), and t is a term representing a 2-cell between a composition of the φ_i, and ψ.

Mixed variance is controlled by enforcing a syntactic criterion which says that the formula φ_i may depend on $\vec{A}_0, \dots, \vec{A}_{i-1}$ only covariantly, and on $\vec{A}_i, \dots, \vec{A}_n$ only contravariantly. The right-hand-side formula ψ may depend covariantly on the variables $\vec{A}_0$, contravariantly on $\vec{A}_n$, and on the other variables not at all.

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A unified framework for notions of algebraic theory

Soichiro Fujii*

Abstract

Universal algebra uniformly captures various algebraic structures, by expressing them as equational theories or abstract clones. The ubiquity of algebraic structures in mathematics and related fields has given rise to several variants of universal algebra, such as theories of symmetric operads, non-symmetric operads, generalised operads, PROPs, PROs, and monads. These variants of universal algebra are called notions of algebraic theory. In this talk, we present a unified framework for them. The key observation is that each notion of algebraic theory can be identified with a monoidal category, in such a way that algebraic theories correspond to monoid objects therein. To incorporate semantics, we introduce a categorical structure called metamodel, which formalises a definition of models of algebraic theories. We also define morphisms between notions of algebraic theory, which are a monoidal version of profunctors. Every strong monoidal functor gives rise to an adjoint pair of such morphisms, and provides a uniform method to establish isomorphisms between categories of models in different notions of algebraic theory. A general structure-semantics adjointness result and a double categorical universal property of categories of models are also shown.

*Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606-8502, Japan. Email: s.fujii.math@gmail.com

Richard Garner
Macquarie University

Fixpoint toposes

If $F\colon \mathcal{E} \to \mathcal{E}$ is any functor, we can look at its category $\mathbf{Fix}(F)$ of fixpoints: objects $X \in \mathcal{E}$ endowed with an isomorphism $X \cong FX$. The first goal of this talk is to explain that, if $\mathcal{E}$ is an topos and F is a pullback-preserving endofunctor which generates a cofree comonad, then $\mathbf{Fix}(F)$ is again a topos. The proof builds on the material of [1].

Specific examples of this construction include the well-known Jonsson–Tarski topos, whose objects are sets endowed with an isomorphism $X \cong X \times X$; the generalised Jonsson–Tarski toposes of Leinster [2]; and the Kennison topos, whose objects are sets endowed with an isomorphism $X \cong X + X$. The second goal of this talk is to explain how such toposes give rise to objects of interest to algebraists, such as Cuntz–Kreiger C^*-algebras [3], Leavitt path algebras [4], and their associated étale groupoids [5].

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COMPOSITIONAL GAME THEORY

NEIL GHANI

Category theory is important to me as it provides a theory of structure and structure is my mechanism of choice to look at the world. This talk shows how this programme can be applied to Economic Game Theory as invented by John Nash to produce a compositional treatment of game theory. The categorical approach not only uncovers new ideas and concepts, but is also vital in taming the complexity of ensuing calculations.

University of Strathclyde

SPLIT EXTENSION CLASSIFIERS IN THE CATEGORY OF COCOMMUTATIVE HOPF ALGEBRAS

MARINO GRAN, GABRIEL KADJO AND JOOST VERCRUYSSE

The category $Hopf_{K,coc}$ of cocommutative Hopf algebras over an arbitrary field K has been recently shown to be semi-abelian [3,5] extending the classical theorem by Takeuchi saying that the category of commutative and cocommutative Hopf algebras is abelian. The category $Hopf_{K,coc}$ is also action representable in the sense of [2], so that the categorical notions of center, centralizer and commutator can be explored in this category. In particular, we shall explain that the categorical notion of center in $Hopf_{K,coc}$ turns out to coincide with the usual notion of center from Hopf algebra theory [1]. When K is an algebraically closed field of characteristic 0 it is possible to give an explicit description of the split extension classifier of a cocommutative Hopf algebra [4]. This universal construction can be seen as the natural counterpart in $Hopf_{K,coc}$ of both the automorphism group in the category of groups and of the algebra of derivations in the category of Lie algebras.

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ADJUNCTIONS AND LIMITS FOR DOUBLE AND MULTIPLE CATEGORIES

MARCO GRANDIS

Based on a series of joint papers with R. Paré.

In a general 2-dimensional adjunction, the left adjoint is colax and the right adjoint is lax: they should not be composed, but organised as vertical and horizontal arrows of a double category. There are various examples of interesting adjunctions which can only be treated in this way, like the obvious pushout-pullback adjunction between spans and cospans, or the extension of an adjunction between abelian categories to their double categories of relations. Finally, a 2-dimensional adjunction lives in a strict double category of (small) weak double categories, with lax and colax functor as horizontal or vertical arrows, and suitable double cells. Another crucial point of interest of weak double categories is the existence of limits: while — for instance — the bicategory of spans of sets lacks most of them, the corresponding weak double category (with ordinary maps in the strict direction) has all limits. All this can be extended in infinite dimension, to weak and lax multiple categories.

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University of Genova

RIGHT ADJOINTS TO OPERADIC RESTRICTION FUNCTORS

GABRIEL C. DRUMMOND-COLE AND PHILIP HACKNEY

Abstract. If $f : P \to Q$ is a morphism of operads, then there is a restriction functor from Q algebras to P algebras. This restriction functor generally admits a left adjoint. This restriction may or may not admit a right adjoint: if $G \to H$ is a group homomorphism, then the forgetful functor from H-sets to G-sets has a right adjoint, while there is no right adjoint to the functor from commutative algebras to associative algebras.

In this talk, we provide a concise necessary and sufficient condition for the existence of a right adjoint to the restriction functor, phrased in terms of the operad map f. We give a simple formula for this right adjoint, and examine the criterion in special cases. All of this is applicable over quite general ground categories. (Joint work with Gabriel C. Drummond-Cole)

Center for Geometry and Physics, Institute for Basic Science (IBS), Pohang 37673, Republic of Korea

Email address: `gabriel@ibs.re.kr`

Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA, United States of America

Email address: `philip@phck.net`

A MODEL 2-CATEGORY OF COMBINATORIAL MODEL CATEGORIES

SIMON HENRY

Model categories have been the most typical framework for categorical homotopy theory since their introduction 50 years ago. But the structure of the category of all model categories itself has remained relatively mysterious. The goal of this talk is to answer a relatively old question in the area: organizing 'combinatorial model categories' into a model categories. More precisely, we propose to construct a model 2-category, whose fibrant objects are the 'combinatorial model categories', with left Quillen functors between them, and weak equivalences being the Quillen equivalences. We will in fact construct several interconnected version of this model category. In the case of simplicial model categories one can even obtain a similar result but where furthermore every object is fibrant (so that one really has a model category structure on the category of combinatorial simplicial model category). Note that here 'model category' was used as a generic term for Quillen model category and various weakening of the notion (left semi-model category, weak model categories etc...), the precise statement will be detailed during the talk.

COMPACT INVERSE CATEGORIES

CHRIS HEUNEN

An *inverse category* is a category that comes with a contravariant involution $\dagger$ that acts as the identity on objects and satisfies $f = ff^\dagger f$ and $ff^\dagger gg^\dagger = gg^\dagger ff^\dagger$ on morphisms [2]. The one-object case, of *inverse monoids*, has been well-studied [4]. In particular, abelian inverse monoids obey a structure theorem [3]: any abelian inverse monoid is a semilattice of abelian groups. In the many-object case, any inverse category gives rise to a semilattice-shaped family of groupoids in a similar way, but not in a functorial way, and it is generally impossible to recover the inverse category from this family without a degree of commutativity.

From the perspective of computer science, inverse categories provide semantics for typed reversible programs. To model recursion, it would be desirable to have additional *compact closed* structure. We show that *compact inverse categories* generalise abelian inverse monoids to multiple objects, and extend the structure theorem: any compact inverse category is a semilattice of compact inverse groupoids. The latter are also known as *coherent 2-groups* or *crossed modules*, and have several characterisations [1]. This structure theorem crucially uses features inherent in compact categories such as traces and scalars.

Based on joint work with Robin Cockett.

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University of Edinburgh, United Kingdom
Email address: chris.heunen@ed.ac.uk

$\mathcal{M}$-coextensivity and the strict refinement property

Michael Hoefnagel

Abstract

Various refinement properties exist for direct-product decompositions of universal algebras, all of which give information about the uniqueness of such direct-product decompositions. The strongest of these, the so-called *strict refinement property* [2], implies that any isomorphism between a product of irreducible structures is uniquely determined by a family of isomorphisms between the factors, in such a way that the original isomorphism is the product of these. Examples of structures which possess the strict refinement property include any unitary ring, any centerless or perfect group, and any congruence distributive algebra. Almost all geometric structures possess the dual property, which is mainly due to the fact that almost all categories of geometric structures are *extensive* in the sense of [1]. In this talk we present an analysis of the relationship between the strict refinement property and extensivity. In particular, we introduce the notion of an $\mathcal{M}$-coextensive object, which brings these two properties together. We show how it links up with regular majority categories [3], and centerless objects in an exact Mal'tsev category. When $\mathcal{M}$ is the class of all product projections in a category, then an $\mathcal{M}$-coextensive object is called *projection-coextensive*. One of the main results shows that the poset of product projections of a projection-coextensive object is a Boolean lattice. Moreover, this is a characteristic property of projection-coextensivity. This generalizes a similar result for relational structures in [2].

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COALGEBRAS FOR ENRICHED HAUSDORFF (AND VIETORIS) FUNCTORS

DIRK HOFMANN

Starting with early studies in the nineties [11] until the introduction of uniform notions of *behavioural metric* in the last decade [3], several works investigate coalgebras over metric-like spaces and their respective limits. Existing work on coalgebras over metric spaces focus on four specific areas: (1) liftings of functors from the category Set to categories of metric spaces [3, 4, 12] (as a way of lifting state-based transition systems into transitions systems over categories of metric spaces); (2) results on the existence of final coalgebras and their computation [11, 3] (as a way of calculating the behavioural distance of two given states of a transition system); (3) the introduction of behavioural metrics with corresponding up-to techniques [3, 4] (as a way of easing the calculation of behavioural distances); (4) the development of coalgebraic logical foundations over metric spaces [2] (so that one can reason about transition systems in a quantitative way).

Our work aims at contributing to these lines of research. In continuation of our study [6], we investigate completeness properties of categories of coalgebras for "powerset-like" functors; moving now from ordered structures to quantale-enriched ones. As a starting point, we show that, for an extension of a Set-functor to a topological category X over Set which commutes with the forgetful functor, the corresponding category of coalgebras over X is topological over the category of coalgebras over Set and therefore cannot be "more complete". Secondly, based on a Cantor-like argument [5], we observe that Hausdorff functors [1, 9] on categories of quantale-enriched categories do not admit a final coalgebra. Motivated by these "negative" results, in this talk we combine quantale enriched categories and topology *à la* Nachbin [8, 10]. Besides studying some basic properties of these categories, we investigate functors which simultaneously encode the classical Hausdorff metric and Vietoris topology and, moreover, lead now to complete categories of coalgebras. Seemingly unrelated, we use these constructions to improve some of the duality results of [7].

This talk is based in joint work with Renato Neves (Minho University) and Pedro Nora (University of Aveiro).

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Full reference list: see programme online

COHOMOLOGY OF PICARD CATEGORIES

MICHAEL HORST

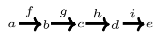
Picard cohomology is a categorification of group cohomology. After reviewing background information on Picard categories, we will present a construction for free Picard categories generated by groupoids and will do so by using a tensoring over groupoids to demonstrate the appropriate free-forgetful adjunction. We continue by discussing the framework for Picard cohomology, including categorical modules and derived functors, and close by using free Picard categories to form projective resolutions and compute specific examples of Picard cohomology.

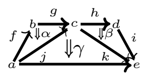
A TYPE THEORY FOR OPETOPES

PIERRE-LOUIS CURIEN, CDRIC HO THANH, AND SAMUEL MIMRAM

Introduction. Opetopes were originally introduced by Baez and Dolan in order to formulate a definition of weak ω-categories [BD98]. Their name reflects the fact that they encode the possible shapes for higher-dimensional operations: they are *ope*ration poly*topes*. They have been the subject of many investigations in order to provide good working definitions of opetopes allowing to explore their combinatorics [HMP00, Lei04]. One of the most commonly used nowadays is the formulation based on polynomial functors and the corresponding graphical representation using "zoom complexes" [KJBM10].

In order to grasp quickly the nature of opetopes, consider a sequence of four composable arrows as in figure (1). There are various ways we can compose them. For instance, we can compose f with g, as well as h with i, and then $g \circ f$ with $i \circ h$. Or we can compose f, g and h together all at once, and then the result with i. These two schemes for composing can respectively be pictured in (2) and (3). From there, the general idea of getting "higher-dimensional" is that we should take these compositions as "2-operations", which can themselves be composed. Opetopes describe all the ways in which these compositions can be meaningfully specified, in arbitrary dimension. We can expect (and it is indeed the case) that the combinatorics of these objects is not easy to describe.

(1)


(2)


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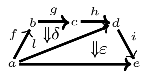


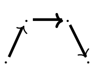
Constructing low dimensional opetopes. Opetopes are defined by induction on their dimension, the base cases being dimensions 0 and 1. The unique 0-opetope is written $\blacklozenge$ and called the *point*. The unique 1-opetope is written $\blacksquare$, called the *arrow*, and is graphically represented in (4). The arrow can be seen as a 1-dimensional "operation", taking a point as input, and outputting another point, whence the tree representation (5). Say that the unique input edge of that tree is $*$. Then the "source pasting scheme" of $\blacksquare$, i.e. the way its inputs are arranged, is completely described by the expression (6). An expression of this form is called a *preopetope*. In the case of $\blacksquare$, those three representations are a bit trivial, but those approaches will become of relevance in the higher dimensional cases.

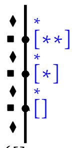
(4)


(5)


(6) $\{* \leftarrow \blacklozenge$

Now, the arrow can be used to create "1-pasting schemes", i.e. meaningful compositions of cells whose shapes are the arrow. An example of such a pasting scheme is given in (7), and the corresponding composition tree is represented in (8). As previously mentioned, the input edge of each $\blacksquare$ node is called $*$, and so we can associate an *address* to every node in that tree (in blue), which is a bracket-enclosed sequence of names giving "walking instructions from the root node". Syntactically, this can be expressed by the preopetope in (9), and since $\blacksquare$ is encoded by the preopetope $\{* \leftarrow \blacklozenge$, this expression can be further expanded as in (10).

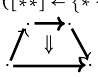
(7)


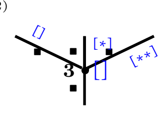
(8)


(9) $\left\{\begin{array}{l} [] \leftarrow \blacksquare \\ [*] \leftarrow \blacksquare \\ [**] \leftarrow \blacksquare \end{array}\right.$

(10) $\left\{\begin{array}{l} [] \leftarrow \{* \leftarrow \blacklozenge \\ [*] \leftarrow \{* \leftarrow \blacklozenge \\ [**] \leftarrow \{* \leftarrow \blacklozenge \end{array}\right.$

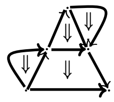
Now, the pasting scheme (7) on the left can be "filled" with a 2-cell representing its "compositor", as depicted in (11). This compositor, which we shall denote by $\mathbf{3}$, has three input cells, located at address $[]$, $[*]$, and $[**]$, and so we may represent it as a corolla (12), i.e., a tree consisting of a unique node labeled by $\mathbf{3}$, whose three input edges are named $[]$, $[*]$, and $[**]$, and labeled by the opetopes at those addresses, in this case, $\blacksquare$ for all three. Further, the output of $\mathbf{3}$ is the target in (11), i.e., an arrow, and thus the root edge is labeled by $\blacksquare$ as well. This process can be iterated by forming n-pasting schemes, and filling them in order to obtain $(n+1)$-opetopes, that can in turn by assembled into $(n+1)$-pasting schemes. Figure (13) is an example of a 2-pasting scheme, and its tree representation (the 2-opetopes $\mathbf{1}$ and $\mathbf{2}$ are defined similarly to $\mathbf{3}$) is given in (14). As before, the pasting scheme can be expressed as a preopetope (15), and fully expanded as in (16).

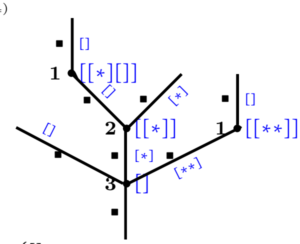
(11)


(12)


If $n \geq 2$, the set $\mathbb{P}_n$ of n-preopetopes is no longer a singleton as in dimension 0 and 1. Consequently, edge labelings in $(n+1)$-pasting schemes are not trivial, and dictate which opetope can be adjacent to which. Since preopetopes do not keep track of edge labels, some of them describe pasting schemes that are not "well-formed", and thus do not correspond to an actual opetope. First, some preopetopes do not even describe a tree: $\{[*] \leftarrow \{* \leftarrow \blacklozenge$ does not have a root node (that would necessarily be located at address $[]$). But, more importantly, from dimension 3 and higher, some compatibility conditions have to be verified when building pasting diagrams. For example, the corolla associated with (14) has four inputs, one of them being decorated by $\mathbf{2}$.. When attaching another corolla to that corolla at this particular input, we must make sure that the target of the opetope decorating the node ot the attached corolla is $\mathbf{2}$.

Syntax. The set $\mathbb{A}_n$ of addresses (or more precisely, *n-addresses*) locating n-opetopes in n-pasting schemes, is inductively defined as follows: $\mathbb{A}_0 = \{*\}$, and $\mathbb{A}_{n+1}$ is the set of finite words over the alphabet $\mathbb{A}_n$, written with enclosing brackets. For example, $[[[**][*]][][[*][]]] \in \mathbb{A}_3$, while the empty address $[]$ is in $\mathbb{A}_n$ for all $n \geq 1$. The set $\mathbb{P}_n$ of *n-preopetopes* is also defined inductively: $\mathbb{P}_0 = \{\blacklozenge\}$, and $\mathbb{P}_{n+1}$ is the set of expression of the form $\left\{\begin{array}{l}[p_1]\leftarrow \mathbf{p}_1 \\ \vdots \\ [p_k]\leftarrow \mathbf{p}_k\end{array}\right.$ and $\{\!\{\mathbf{q}$, where $[p_1], \dots, [p_k] \in \mathbb{A}_n$ are distinct n-addresses, $\mathbf{p}_1, \dots, \mathbf{p}_k \in \mathbb{P}_n$, and $\mathbf{q} \in \mathbb{P}_{n-1}$ if $n \geq 2$. The expression $\{\!\{\mathbf{q}$ describes an empty n-pasting scheme, i.e. a tree with no nodes and a unique edge labeled by the $(n-1)$-preopetope $\mathbf{q}$.

(13)


(14)


(15) $\left\{\begin{array}{l} [] \leftarrow \mathbf{3} \\ [[*]] \leftarrow \mathbf{2} \\ [[*][]] \leftarrow \mathbf{1} \\ [[**]] \leftarrow \mathbf{1} \end{array}\right.$

(16) $\left\{\begin{array}{l} [] \leftarrow \left\{\begin{array}{l} * \leftarrow \{* \leftarrow \blacklozenge \\ [*] \leftarrow \{* \leftarrow \blacklozenge \\ [**] \leftarrow \{* \leftarrow \blacklozenge \end{array}\right. \\ [[*]] \leftarrow \left\{\begin{array}{l} [] \leftarrow \{* \leftarrow \blacklozenge \\ [*] \leftarrow \{* \leftarrow \blacklozenge \end{array}\right. \\ [[*][]] \leftarrow \{[] \leftarrow \{* \leftarrow \blacklozenge \\ [[**]] \leftarrow \{[] \leftarrow \{* \leftarrow \blacklozenge \end{array}\right.$

As previously mentioned, not all preopetopes correspond to an opetope. We shall present a derivation system named $\text{O}\textsc{pt}^?$ that characterizes opetopes among preopetopes as those that are derivable. The inference rules of the system essentially follow the construction procedure presented above: rule `point` derives the 0-preopetope $\blacklozenge$ with no prior assumption; rule `degen` takes a n-preopetope $\mathbf{q}$ and constructs the empty pasting scheme $\{\!\{\mathbf{q}$; rule `shift` takes a n-preopetope $\mathbf{p}$ and considers it as the unique cell of a n-pasting scheme $\{[] \leftarrow \mathbf{p}$; and finally, rule `graft` takes a $(n+1)$-preopetope $\mathbf{r} = \left\{\begin{array}{l}[p_1]\leftarrow \mathbf{p}_1 \\ \vdots \\ [p_k]\leftarrow \mathbf{p}_k\end{array}\right.$ as above, an n-address $[p_{k+1}]$, and a n-preopetope $\mathbf{p}_{k+1}$, and, extends $\mathbf{r}$ as the pasting scheme $\left\{\begin{array}{l}[p_1]\leftarrow \mathbf{p}_1 \\ \vdots \\ [p_{k+1}]\leftarrow \mathbf{p}_{k+1}\end{array}\right.$ obtained from $\mathbf{r}$ by adding the cell $\mathbf{p}_{k+1}$ at address $[p_{k+1}]$, embodying the well-formedness property as side conditions.

Further developments. Higher addresses are an extremely convenient tool when dealing with the combinatorics of opetopes. They allow for a succinct and precise formalism of preopetopes, and are a cornerstone to the definition of $\mathbb{O}$, the category of opetopes, based on the intuition carried by the graphical representations of pasting schemes. In the preprint [CHM18], we also present another syntax for opetopes, using variables instead of higher addresses, and a corresponding derivation system $\text{O}\textsc{pt}^!$, is presented. The latter system is more user-friendly and easy to read, but does not lend itself so nicely to a mathematical treatment, especially when it come to organizing opetopes into a category. The results developed in thhe latter approach have been submitted elsewhere. Variations of systems $\text{O}\textsc{pt}^!$ and $\text{O}\textsc{pt}^?$, designed for syntactical representations of finite opetopic sets (finite presheaves over $\mathbb{O}$), are also presented in [CHM18], and a Python implementation of all of them is available in [Ho 18]. Together with an adequate formulation of opetopic higher categories, it is our hope that this work will be used productively for mechanical proofs of coherence in opetopic ω-categories or opetopic ω-groupoids.

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INTERNAL LANGUAGE AND CLASSIFIED THEORIES OF TOPOSES IN ALGEBRAIC GEOMETRY

MATTHIAS HUTZLER

Topos theory originated from the desire to compute the cohomology of schemes in algebraic geometry. Only after that it was noticed that toposes carry a rich structure providing an internal language and that they 'are logical theories' just as much as they 'are spaces'.

The theory of classifying toposes was started in [2], when the notion of a geometric theory was not developed yet. The initial insight being that, similarly to classifying spaces in algebraic topology, some structures in toposes correspond to geometric morphisms into a special classifying topos for that sort of structures. The first example, given in [2], was that of the big Zariski topos, which classifies local rings.

In [3], there is a remark that many toposes from algebraic geometry should be classifying toposes of reasonable geometric theories. However, not much of this vision seems to have been developed since. We give an answer to the question about the classified theory for the big infinitesimal topos. For this, we extensively utilize the concept of a theory of presheaf type, applying different theorems from [1].

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LIMITS IN DAGGER CATEGORIES

CHRIS HEUNEN AND MARTTI KARVONEN

A dagger category is a category equipped with a dagger: a contravariant involutive identity-on-objects endofunctor. Such categories are used to model quantum computing and reversible computing, amongst others. The philosophy when working with dagger categories is that all structure in sight should cooperate with the dagger. This causes dagger category theory to differ in many ways from ordinary category theory. Standard theorems have dagger analogues once one figures out what "cooperation with the dagger" means for each concept, but often this is not just an application of formal 2-categorical machinery or a passage to (co)free dagger categories.

We discuss limits in dagger categories. To cooperate with the dagger, limits in dagger categories should be defined up to a unique unitary isomorphism (instead of only up to iso), that is, an isomorphism whose inverse is its dagger. We exhibit a definition that achieves this and generalises known cases of dagger limits. Moreover, we discuss connections to polar decomposition, applications to ordinary category theory and time permitting, address commutativity of dagger limits with dagger colimits.

OPERADIC CATEGORIES, DECALAGE, AND DECOMPOSITION SPACES

JOACHIM KOCK

Batanin and Markl introduced the notion of operadic category as a machinery used to prove the duoidal Deligne conjecture. Operadic categories C have algebras called C-operads. For example, ordinary symmetric operads are algebras for the terminal operadic category F, the category of finite sets, in the spirit of viewpoints tracing back to Day-Street and Barwick. An operadic category is a small category equipped with a notion of cardinality, a notion of abstract fibre, and a choice of local terminal objects, and this data is subject to 9 axioms, mimicking the way these notions behave in F. At CT 2014, Lack gave a more conceptual reinterpretation of the notion of operadic category in terms of skew monoidal categories. In this talk I will explain a different reinterpretation of the notion, based on the decalage comonad D. Operadic categories are exhibited as algebras for a monad on the category of D-coalgebras sliced over F, and that monad is itself induced by D. One benefit of the approach is that it reveals relationships with decomposition spaces (2-Segal spaces).

This is joint work with Richard Garner and Mark Weber.

WEAK ADJOINT FUNCTOR THEOREMS

STEVE LACK

The General Adjoint Functor Theorem of Freyd has a straightforward extension to the enriched setting. There is also a Weak Adjoint Functor Theorem, due to Kainen, and providing a sufficient condition for the existence of a weak left adjoint, where weakness refers to the existence but not uniqueness of factorizations.

I will report on recent joint work with John Bourke and Lukas Vokrinek, in which we prove a weak adjoint functor theorem in the enriched context. This actually contains the other three theorems as special cases. Our base for enrichment will be a monoidal model category.

Our motivating example involves simplicially enriched categories, where the base category of simplicial sets is equipped with the Joyal model structure. The theorem then has applications to the Riehl-Verity theory of infinity-cosmoi.

Continuous complete categories enriched over quantales

Hongliang Lai, Dexue Zhang

Ordered sets are often viewed as thin categories, and on the other hand, categories are regarded as generalized ordered structures. For instance, the complete distributivity [4, 5] and the continuity [1, 2] of categories are investigated, like that of ordered sets.

Form the viewpoint of Lawvere[3], categories enriched over a monoidal closed category, especially over a quantale, can also be regarded as ordered sets in the sense of "quantitative logics". Hence, quantale enriched categories are studied as quantitative ordered sets in the quantitative domain theory.

The main objective of this paper is to contribute to the study of "generalized" Scott's continuous lattices based on quantale enriched categories.

The characterization of continuous posets is concerned with the relation between a poset P and the posets $\mathsf{Idl}(P)$ of ideals of P. For all $p \in P$, $\downarrow p = \{x \in P : x \leq p\}$ defines an embedding

$$\downarrow : P \longrightarrow \mathsf{Idl}(P).$$

A poset P is directed complete if $\downarrow$ has a left adjoint

$$\sup \dashv \downarrow : P \longrightarrow \mathsf{Idl}(P)$$

and a directed complete poset P is continuous if there is a string of adjunctions

$$\twoheaddownarrow \dashv \sup \dashv \downarrow : P \longrightarrow \mathsf{Idl}(P).$$

In a locally small category $\mathcal{E}$, ind-objects, or equivalently, the presheaves generated by ind-objects, play the role of ideals in posets. Let $\mathsf{Ind}\text{-}\mathcal{E}$ be the category of all presheaves of $\mathcal{E}$ generated by ind-objects in $\mathcal{E}$, then $\mathcal{E}$ has small filtered colimits if the Yoneda embedding $\mathsf{y} : \mathcal{E} \longrightarrow \mathsf{Ind}\text{-}\mathcal{E}$ has a left adjoint

$$\mathrm{colim} \dashv \mathsf{y} : \mathcal{E} \longrightarrow \mathsf{Ind}\text{-}\mathcal{E}$$

and it is further continuous if there is a string of adjunctions

$$w \dashv \mathrm{colim} \dashv \mathsf{y} : \mathcal{E} \longrightarrow \mathsf{Ind}\text{-}\mathcal{E}.$$

Email addresses: hllai@scu.edu.cn (Hongliang Lai), dxzhang@scu.edu.cn (Dexue Zhang)

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For continuation of abstract and reference list, see programme online

The generalized Homotopy Hypothesis

Edoardo Lanari

June 28, 2019

Abstract

The homotopy hypothesis roughly states that weak n-groupoids are algebraic models for homotopy n-types. In this talk I will introduce Grothendieck (weak) n-groupoids for $0 \leq n \leq \infty$ and describe how to obtain a homotopy theory for these objects based on a notion of homotopy groups. I will then discuss the main obstruction to a proof of the homotopy hypothesis, which essentially lies in the validity of a technical lemma about the invariance of the homotopy type of a given n-groupoid after having attached a new cell to it along its source. Next, I will introduce truncated and coskeletal models and prove several equivalences between ∞-categories of such objects, which will culminate with the result that if a left semi-model structure for n-groupoids (called the *canonical* one) exists, then the generalized homotopy hypothesis is valid. If time permits, I will spend some word on a possible strategy to tackle this problem, and the way our main result was proven.

This is joint work with S. Henry.

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Tangent Categories from the Coalgebras of Differential Categories

JS Pacaud Lemay
Joint work with: Robin Cockett and Rory B. B. Lucyshyn-Wright

Differential categories [1] were introduced to provide categorical models of differential linear logic and study the algebraic foundations of differentiation. Following the pattern from linear logic, the coKleisli category of a differential category is well studied: it is a Cartesian differential category [2], whose differential structure formalize the directional derivative and provides the semantics of the differential λ-calculus. What then is the coEilenberg-Moore category of a differential category? The answer, which is the subject of this talk, is that it is a tangent category! Briefly, a tangent category [3, 4] is a category equipped with an endofunctor that formalizes the basic properties of the tangent bundle functor on the category of smooth manifolds. In this talk, we will explain how under a mild limit assumption (and thanks to an adjoint existence theorem of Butler's [5]) the coEilenberg-Moore category of a differential category is a tangent category whose tangent bundle functor is a right adjoint to a sort of infinitesimal extension on coalgebras. Key examples of such tangent categories include the opposite category of commutative rings and the opposite category of $\mathcal{C}^\infty$-rings, whose tangent category structures respectfully capture fundamental aspects of algebraic geometry and synthetic differential geometry.

This work extends on previous work by Lucyshyn-Wright [6, 7].

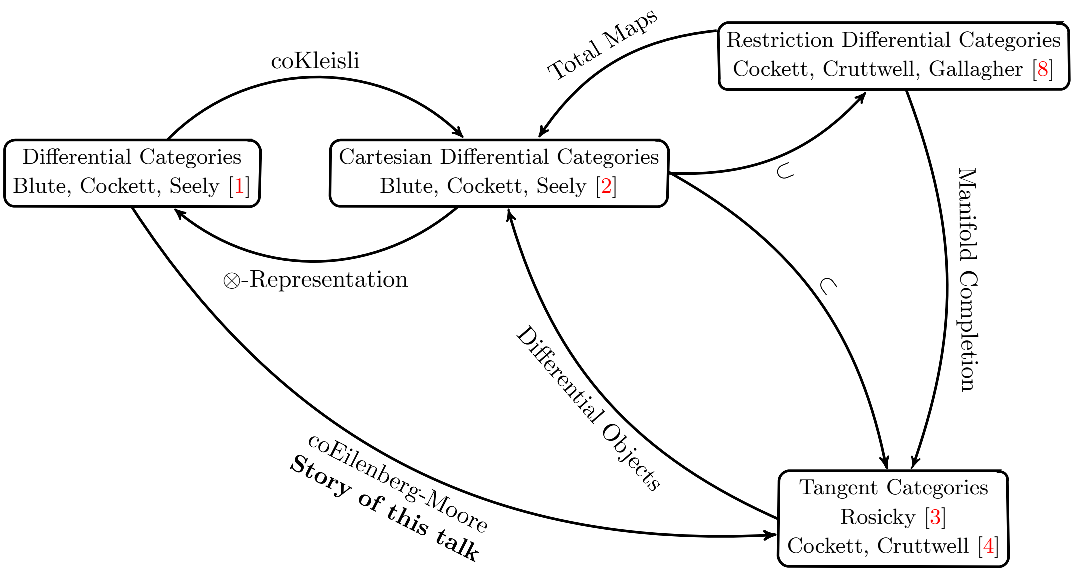

Figure 1: The world of differential categories and how it's all connected.

Reference list: see programme online

What is a monoid?

Paul Blain Levy, University of Birmingham

May 2, 2019

In many situations one encounters a notion that resembles that of a monoid. It consists of a carrier and two operations that resemble a unit and a multiplication, subject to three equations that resemble associativity and left and right unital laws. The question then arises whether this notion in fact that of a monoid in a suitable sense.

Category theorists have answered this question by providing a notion of monoid in a monoidal category. In many of the above situations, one chooses an appropriate monoidal category $\mathcal{C}$ and then the notion of interest is precisely that of monoid in $\mathcal{C}$. But sometimes the desired $\mathcal{C}$ is not a monoidal category but some kind of “generalized monoidal category”.

For example, $\mathcal{C}$ may be a multicategory, where a morphism goes from a list of objects to an object. Hermida [2] showed that a multicategory with tensors corresponds to a monoidal category, but sometimes the multicategory we want does not have tensors, e.g. for size reasons. And sometimes the tensors exist but are complicated.

In other cases, $\mathcal{C}$ may be a left-skew monoidal category or a right-skew monoidal category, in the sense of Szlanchanyi [3]. Here—by contrast with the definition of monoidal category—the associator and unitors are not required to be isomorphisms. Bourke and Lack [1] have further generalized left-skew monoidal category to left-skew multicategory, and (symmetrically) right-skew monoidal category to right-skew multicategory.

However, there are situations where even these are not general enough. One example is the notion of “staggered category” that arose in studying a fragment of call-by-push-value (which is a kind of λ-calculus). In this talk we give a notion of *bi-skew multicategory* that subsumes both left-skew and right-skew multicategory. Then we define a notion of monoid in a bi-skew multicategory. By choosing appropriate bi-skew multicategories, we recover the notion of staggered category (with fixed object structure) and other examples.

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SET-THEORETIC REMARKS ON A POSSIBLE DEFINITION OF ELEMENTARY OO-TOPOS

GIULIO LO MONACO

In axiomatizing the significance of oo-toposes in homotopy type theory, one realizes that dependent products and sums should be required to be recoverable within suitable classes of morphisms, namely some classes for which there exists a classifier. However, asking this of dependent products turns out to be a quite strong request. In fact, once we fix a Grothendieck universe, the possibility of finding classes of morphisms in an oo-topos which both have a classifier and are closed under dependent products will be proven to be equivalent to the universe being 1-inaccessible, a condition which is strictly stronger than merely being inaccessible.

Fernando Lucatelli Nunes*
University of Coimbra

Descent and Monadicity

In [3], given a 2-category A, under suitable hypotheses, we give the semantic factorization of a morphism p that has the codensity monad via descent. This specializes to a new connection between monadicity and descent theory, which can be seen as a counterpart account to the celebrated Bénabou-Roubaud Theorem [1]. It also leads in particular to a (formal) monadicity theorem.

The result is new even in the case of the Eilenberg-Moore factorization of a functor that has a left adjoint in Cat. In this talk, we shall give a sketch of the ideas and constructions involved in this particular case. We give focus on the monadicity theorem. If time allows, we talk about applications in the context of [2, 4].

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*lucatellinunes@uc.pt

Rory Lucyshyn-Wright[*]
Brandon University

Functional distribution monads and τ-additive Borel measures

The article [1] defines a categorical framework for algebraic dualization processes that give rise to restricted double-dualization monads called *functional distribution monads*, which specialize to yield various spaces of measures, distributions, filters, closed subsets, compacta, and so forth. Functional distribution monads are canonically induced by a given enriched algebraic category $\mathscr{A}$ and a suitable object of $\mathscr{A}$ acting as 'dualizer'. These given data can be formulated equivalently as a suitable *algebraic dual adjunction* [2], comprising a given pair of algebraic $\mathscr{V}$-categories and a contravariant adjunction that captures the relevant dualization processes and the relation between an algebraic theory and its commutant [3] relative to the dualizer.

In this talk, we will show that by taking as $\mathscr{V}$ the category of convergence spaces and as $\mathscr{A}$ the category of convex spaces internal to $\mathscr{V}$, with the unit interval as dualizer, the induced functional distribution monad gives rise to the notion of *τ-additive* (or *τ-smooth*) *probability measure* on Tikhonov spaces. For locally compact Hausdorff spaces, bounded τ-additive measures coincide with bounded Radon measures, so this generalizes an earlier result of the speaker (announced in [5]). But bounded τ-additive measures on Tikhonov spaces also generalize bounded Borel measures on Polish spaces, so the resulting functional distribution monad captures a wide class of settings in topological measure theory. In proving our result, we establish a connection between τ-additive measures and continuous convergence, and we establish integral representation theorems for bounded τ-additive measures that are formulated in terms the cartesian closed structure of $\mathscr{V}$.

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AN ENRICHED PERSPECTIVE ON DIFFERENTIABLE STACKS

BENJAMIN MACADAM AND JONATHAN GALLAGHER

In this talk, we will apply the theory of tangent categories to study the tangent structure of differentiable stacks. A stack is a (2,1)-sheaf on a site (X,J), using the theory of locally presentable categories one may see this is equivalent to an internal groupoid in Sh(X,J). A differentiable stack is certain kind of (2,1)-sheaf on the category of smooth manifolds. The difficulty in defining a tangent bundle [1] for a differentiable stack is similar to those that arise when defining the tangent bundle in a smooth topos. When studying synthetic differential geometry [7], one restricts their attention to sheaves which satisfy a microlinearity condition, we will apply this technique to differentiable stacks. Following the work of Garner and Leung [2, 3], one may regard a tangent category [4, 5] as a kind of E-enriched category (where E is the category of microlinear presheaves on Weil algebras [6]). Then one may replace a sheaf satisfying a microlinearity condition with an enriched sheaf. In this talk we will extend this technique to stacks: we will consider a notion of strict tangent (2,1)-category as a certain kind of Gpd(E)-enriched category. Then we may lift a tangent category to a (2,1)-tangent category and consider Gpd(E)-presheaves.

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LAX GRAY TENSOR PRODUCT FOR 2-QUASI-CATEGORIES

YUKI MAEHARA

The *Gray tensor product* [2, §4] plays a crucial role in classical 2-category theory. It is a "weaker" or "less commutative" kind of product, and there are two versions (up to duality) depending on whether one wants the comparison 2-cell between $(f \otimes 1)(1 \otimes g)$ and $(1 \otimes g)(f \otimes 1)$ to be invertible or not; the former is the *pseudo* version while the latter is called *lax*.

Our ultimate goal is to "do 2-category theory" in 2-quasi-categories which are a model of $(\infty, 2)$-categories introduced by Ara in [1]. In particular, it requires extending the definition of Gray tensor product to the 2-quasi-categorical context. Luckily, their geometric nature means the usual categorical product of 2-quasi-categories models the pseudo Gray tensor product. However, constructing the lax Gray tensor product and proving it is "homotopically well-behaved" (*i.e.* left Quillen) is a non-trivial task, and this is what we will present in this talk.

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Menni, M.
Conicet and Universidad Nacional de La Plata

One: a characterization of skeletal objects for the Aufhebung of Level 0 in certain toposes of spaces.

In his 1990 *Thoughts on the future of category theory* Lawvere says that "It seems that a significant portion of algebraic geometry and differential geometry does not depend so much on the particular algebraic theory used to construct models for it but is of a more fundamental conceptual nature. One-dimensional, like connected, is actually a philosophical concept, related to the minimal Hegelian level of figures which must be considered within an arbitrary space in order to determine that space's connectedness." He then proposes to consider the Aufhebung relation between levels (or essential localizations) in a category. "The basic idea is simply to identify dimensions with levels and then try to determine what the general dimensions are in particular examples." This determination is well-understood only in a few cases. Namely, in those examples worked out by Lawvere in his theory of graphic toposes [4], those discussed by Lawvere and Kelly [1], and the non-graphic 'combinatorial' pre-cohesive presheaf toposes analyzed in [2]. Starting from an arbitrary local hyperconnected geometric morphism (whose centre is thought of as a level 0) I will define what it means to be 'naively 1-dimensional'. I will then show that for many pre-cohesive presheaf toposes (including the known examples) the naively 1-dimensional objects coincide with the skeletal objects for the Aufhebung of level 0.

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EXPONENTIABILITY IN DOUBLE CATEGORIES AND THE GLUEING CONSTRUCTION

SUSAN NIEFIELD

Adjunctions between double categories have been considered by Grandis and Paré in several settings. Since left adjoints are oplax and right adjoints are lax, the pair is an orthogonal adjunction in the double category $\mathbb{D}$bl, whose horizontal and vertical morphisms are lax and oplax functors, respectively. If the left adjoint is lax, then it is an adjunction in the 2-category **LxDbl**, whose morphisms are lax functors. If both adjoints are pseudo functors, then it is an adjunction in the 2-category **PsDbl**, whose morphisms are pseudo functors.

A double category $\mathbb{D}$ is *pre-cartesian*, if the diagonal $\Delta\colon \mathbb{D} \to \mathbb{D} \times \mathbb{D}$ has a right adjoint in **LxDbl**. We say $\mathbb{D}$ is *pre-cartesian closed*, if the lax functor $- \times Y\colon \mathbb{D} \to \mathbb{D}$ has a right adjoint in **LxDbl**, for every Y. If the right adjoints in question are pseudo-functors, we say $\mathbb{D}$ is *cartesian closed*. Examples of the latter include the double categories $\mathbb{R}$el and $\mathbb{S}$pan, whose objects are sets, horizontal morphisms are functions, and vertical morphisms are relations and spans, respectively.

In this talk, we consider *pre-exponentiable* objects Y in a pre-cartesian double category $\mathbb{D}$, i.e., objects for which $- \times Y$ has a right adjoint in **LxDbl**. Such an object is necessarily (classically) exponentiable in the horizontal category $\mathbb{D}_0$. Since the right adjoints we consider are merely lax, even when the left adjoints are pseudo-functors, the setting for this talk is **LxDbl**, rather than **PsDbl**.

For double categories $\mathbb{D}$ satisfying a generalization of the glueing construction from topos theory, we show that Y is pre-exponentiable in $\mathbb{D}$ if and only if it is exponentiable in $\mathbb{D}_0$. Applications include the double categories $\mathbb{C}$at, $\mathbb{P}$os, $\mathbb{S}$paces, $\mathbb{L}$oc, and $\mathbb{T}$opos, whose objects are small categories, posets, topological space, locales, and toposes, respectively. Thus, $\mathbb{C}$at and $\mathbb{P}$os are pre-cartesian closed, and the pre-exponentiable objects in $\mathbb{S}$paces, $\mathbb{L}$oc, and $\mathbb{T}$opos are precisely those we know are exponentiable in the classical sense.

Jovana Obradović
Institute of Mathematics, Czech Academy of Sciences

Combinatorial homotopy theory for operads

We introduce an explicit combinatorial characterization of the minimal model of the coloured operad encoding non-symmetric operads, introduced in [3]. The polytopes of our characterization are hypergraph polytopes [1, 2] whose hypergraphs arise in a certain way from rooted trees – we refer to them as operadic polytopes. In particular, each operadic polytope displays the homotopy relating different ways of composing the nodes of the corresponding rooted tree. In this way, our operad structure generalizes the structure of Stasheff's topological A_∞-operad: the family of associahedra corresponds to the suboperad determined by linear rooted trees. We then further generalize this construction into a combinatorial resolution of the coloured operad encoding non-symmetric cyclic operads.

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SEGAL-TYPE MODELS OF WEAK n-CATEGORIES

SIMONA PAOLI

The theory of higher categories is a very active area of research which has penetrated diverse fields of science. Different approaches to higher categories have been developed over the years. In this talk I present a new approach to working with higher categories: this is based on n-fold categories as well as on a new paradigm to weaken higher categorical structures, which is the idea of weak globularity. I will illustrate how the new model, called weakly globular n-fold categories, is suitably equivalent to a model of higher categories that has been studied in great depth, the one introduced by Tamsamani and further studied by Simpson. This comparison is achieved by developing a larger context of 'Segal-type models of weak n-categories', based on multi-simplicial structures, of which both the Tamsamani model and weakly globular n-fold categories are special cases.

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University of Leicester

RETRO CELLS

ROBERT PARÉ

An important property of bicategories is biclosedness, i.e. the functors giving composition have right adjoints in each variable. Most good bicategories are part of naturally occurring double categories, and this point of view often simplifies things considerably. However the internal homs, those right adjoints just mentioned, are not functorial at this level. The problem is that in the domain variable they are neither covariant nor contravariant, but a combination of both. To remedy this failure we introduce retro cells. We will discuss their properties and point out some open questions.

NON-COMMUTATIVE DISINTEGRATIONS AND REGULAR CONDITIONAL PROBABILITIES

ARTHUR PARZYGNAT

Classical regular conditional probabilities and disintegrations can be formulated diagrammatically in a category of stochastic maps. Combining this with the functor taking stochastic maps to positive maps on C*-algebras, the notion of non-commutative disintegration and regular conditional probability can be extended to states on C*-algebras. Unlike the classical case, the existence of non-commutative disintegrations is not guaranteed even on finite-dimensional matrix algebras. We will state some general existence and uniqueness results as well as examples. This is joint work with Benjamin P. Russo (Farmingdale State College SUNY).

LEFT CANCELLATIVE CATEGORIES AND ORDERED GROUPOIDS

DORETTE PRONK AND DARIEN DEWOLF

One way to characterize etendues is as the categories of sheaves on a site with monic maps, or, in the language of [2], a left cancellative category with a Grothendieck topology.

In [2], Lawson introduces a notion of ordered groupoid and introduces functors between the category of left cancellative categories and that of ordered groupoids, but does not establish it as an equivalence of 2-categories, because it is not clear when two ordered groupoids should be equivalent.

In [3], Lawson and Steinberg introduce the notion of an Ehresmann topology on ordered groupoids and establish a correspondence between Grothendieck topologies on left cancellative categories and Ehresmann topologies on ordered groupoids, corresponding to each of their functors. They show that these correspondences give an equivalence between the induced categories of sheaves. The main result in this paper is a characterization of etendues as sheaves on an Ehresmann site. What is missing in this paper then equivalences of the 2-categories of left cancellative categories and ordered groupoids and a description of what morphisms between ordered groupoids give rise to geometric morphisms between the induced categories of sheaves on the Ehresmann sites.

By considering ordered groupoids as a special type of double categories we characterize what a weak equivalence of ordered groupoids is, and then use this to establish an adjoint equivalence of 2-categories between the 2-category of left cancellative categories and the 2-category of ordered groupoids.

We show how this extends to an equivalence between the appropriate categories of Grothendieck sites with monic maps and Ehresmann sites and as an application, we translate the comparison lemma for sites with monic maps given in [1], to a comparison lemma for Ehresmann sites, characterizing which functors between Ehresmann sites give rise to equivalences of etendues.

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TRUNCATIONS AND BLAKERS–MASSEY IN AN ELEMENTARY HIGHER TOPOS

NIMA RASEKH

We study truncated and connected objects in an elementary higher topos. In particular, we show that they have the same behavior as in spaces, construct a universal truncation functor using natural number objects and show all modalities, and in particular connected maps, satisfy the statement of the Blakers–Massey Theorem.

Double quandle coverings

Francois Renaud

April 26, 2019

IRMP Université catholique de Louvain

Quandles were introduced in D. Joyce's PhD thesis [5] and capture the algebraic theory of group conjugation with applications in geometry (as the intrinsic structure of symmetric spaces) and knot theory (as a complete invariant for oriented knots).

The category Qnd of quandles admits the category Set of sets as a subvariety. The left adjoint $\pi_0 \colon \mathsf{Qnd} \to \mathsf{Set}$ of the inclusion functor $\mathsf{Set} \to \mathsf{Qnd}$ sends a quandle to its set of connected components. This adjunction was shown to be admissible (in the sense of [4]) by V. Even in [3] where he showed that central extensions defined through Galois Theory [4] coincide with quandle coverings defined by M. Eisermann in [2]. Such coverings form a reflective subcategory of the category of surjective homomorphisms of quandles [1].

We show that this adjunction is in turn admissible, and this gives rise to a notion of double central extension of quandles for which we provide an algebraic characterisation. Both centrality conditions in dimension 1 and 2 may be expressed in terms of a new notion of commutator defined for quandle congruences.

These results provide new tools to study quandles, as well as a new context of application for higher Galois theory.

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A FORMAL CATEGORY THEORY FOR ∞-CATEGORIES

EMILY RIEHL

"Formal category theory" is category theory applied to itself. In this talk, we explain how the notion of a 2-topos, as axiomatized by Weber and Street, provides a framework for defining standard categorical structures—adjunctions, limits, cartesian fibrations, the Yoneda embedding, pointwise Kan extensions—in which the expected interrelationships between these notions can be proven. We then describe joint work in progress with Dominic Verity to adapt the 2-topos framework to develop a model-independent formal category theory of $(\infty, 1)$-categories.

JOHNS HOPKINS UNIVERSITY

INTRINSIC SCHREIER SPLIT EXTENSIONS

DIANA RODELO

In the context of regular unital categories, under conditions satisfied by all Jónsson-Tarski varieties of universal algebras, we introduce an intrinsic version of the notion of a "Schreier split epimorphism", originally considered for monoids.

We show that such split epimorphisms satisfy the same homological properties as Schreier split epimorphisms of monoids do. This gives rise to new examples of S-protomodular categories, and allows us to better understand the homological behaviour of monoids from a categorical perspective.

Relating the Effective Topos to HoTT

Steve Awodey, Jonas Frey, Giuseppe Rosolini

The effective topos **Eff** was introduced by Martin Hyland in [4] and proved to be a very useful category where to test computational properties of constructive theories, see [9]. In the talk we present a way to see **Eff** as part of a model of Homotopy Type Theory [6].

The presentations of **Eff** as an exact completion and of its full subcategory **Asm** on the assemblies as a regular completion in [2] suggested that the topos might be obtained as a homotopy quotient of some appropriate category, see also [7]. This is understood in a very rough sense, based on the construction of the exact completion via the pseudo-equivalence relations of Aurelio Carboni as in [1].

By considering the category of the pseudo-equivalence relations in **Asm** (with graph homomorphisms), we can show that **Eff** is a full subcategory of the homotopy quotient $\mathrm{Ho}(\mathrm{Kan}([\mathcal{C}^{\mathrm{op}}, \mathbf{Asm}]))$ of the category of Kan fibrant cubical assemblies, see [3, 5].

In fact, we obtain this from the stronger result that the extensional realizability topos **Ext** of [8], into which **Eff** embeds fully, is a full subcategory of $\mathrm{Ho}(\mathrm{Kan}([\mathcal{C}^{\mathrm{op}}, \mathbf{Asm}]))$.

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A Solution for the Compositionality Problem of Dinatural Transformations

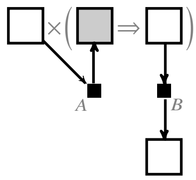
Guy McCusker[*] Alessio Santamaria[†]

Dinatural transformations are a generalisation of the well-known *natural transformations*, as such they are ubiquitous in Mathematics and Computer Science. They appeared for the first time in [18] in the context of Algebraic Topology, where the notion of *(co)end* of a functor was introduced; lately they were formally defined in [3]. Given functors $F, G \colon \mathbb{C}^{\mathrm{op}} \times \mathbb{C} \to \mathbb{D}$, a dinatural transformation $\varphi \colon F \to G$ is a family $\big(\varphi_A \colon F(A,A) \to G(A,A)\big)_{A \in \mathbb{C}}$ such that for all $f \colon A \to B$ in $\mathbb{C}$ the following hexagon commutes:

$$\begin{tikzcd}
 & F(A,A) \arrow[r, "\varphi_A"] & G(A,A) \arrow[dr, "G(1,f)"] & \\
F(B,A) \arrow[ur, "F(f,1)"] \arrow[dr, "F(1,f)"'] & & & G(A,B) \\
 & F(B,B) \arrow[r, "\varphi_B"'] & G(B,B) \arrow[ur, "G(f,1)"'] &
\end{tikzcd}$$

A classical example is the family of evaluation maps $\big(eval_{A,B} \colon A \times (A \Rightarrow B) \to B\big)$ in any cartesian closed category $\mathbb{C}$: the transformation $eval$ is natural in B and dinatural in A.

Dinatural transformations, however, suffer from a troublesome shortcoming: they do not compose. This remarkable problem was already known to their discoverers: many studies have been conducted about them [1, 2, 5, 6, 9, 11, 12, 14, 15, 16, 17], and many attempts have been made to find a proper calculus for dinatural transformations, but only *ad hoc* solutions have been found and, ultimately, they have remained poorly understood. We present a sufficient and essentially necessary condition for two arbitrary, consecutive dinatural transformations φ and ψ for the composite $\psi \circ \varphi$ to be dinatural, thus solving the compositionality problem of dinatural transformations in its full generality [10]. We were inspired by the work of Eilenberg and Kelly on *extranatural transformations* [4], which are less general than dinaturals and also fail to compose: the authors associated to each extranatural a graph, the archetype of a *string diagram*, that captures their naturality properties. We extended such graphical calculus to dinatural transformations; for example, consider the transformation $eval$ as above: its domain is the functor $T \colon \mathbb{C} \times \mathbb{C}^{\mathrm{op}} \times \mathbb{C} \to \mathbb{C}$ where $T(X,Y,Z) = X \times (Y \Rightarrow Z)$, while the codomain is $id_{\mathbb{C}}$. The graph of $eval$ is:

$\Gamma(eval) =$ 

The three upper boxes correspond to the arguments of T, while the lower one to $id_{\mathbb{C}}$. Graphs of consecutive dinatural transformations $\varphi \colon F \to G$ and $\psi \colon G \to H$ can be composed by "glueing" them together along the G-boxes. Our result asserts that if the composite graph $\Gamma(\psi) \circ \Gamma(\varphi)$ is *acyclic*, then $\psi \circ \varphi$ is indeed dinatural. The proof exploits the theory of *Petri Nets* [13], of which these graphs are a particular example, by translating the dinaturality property of $\psi \circ \varphi$ into a reachability problem for the Petri Net $\Gamma(\psi) \circ \Gamma(\varphi)$. We can now finally define a generalised functor category $\{\mathbb{C}, \mathbb{D}\}$ of mixed-variance functors and (partially) dinatural transformations; this is the first step towards the formalisation of a generalised Godement calculus as sought by Kelly in [7] in order to describe coherence problems abstractly [8].

[*]University of Bath, G.A.McCusker@bath.ac.uk
[†]University of Bath, A.Santamaria@bath.ac.uk

Reference list: see programme online

A RECIPE FOR BLACK BOX FUNCTORS

BRENDAN FONG AND MARU SARAZOLA

The task of constructing compositional semantics for network-style diagrammatic languages, such as electrical circuits or chemical reac tion networks, has been dubbed the black boxing problem, as it gives semantics that describes the properties of each network that can be observed externally, by composition, while discarding the internal structure. One way to solve these problems is to formalise the diagrams and their semantics using hypergraph categories, with semantic interpretation a hypergraph functor, called the black box functor, between them. Reviewing a principled method for constructing hypergraph categories and functors, known as decorated corelations, in this paper we construct a category of *decorating data*, and show that the decorated corelations method is itself functorial, with a universal property characterised by a left Kan extension. We then argue that the category of decorating data is a good setting in which to construct any hypergraph functor, giving a new construction of Baez and Pollard's black box functor for reaction networks as an example.

Philip Saville*

Department of Computer Science and Technology
University of Cambridge

A type theory for cartesian closed bicategories

I will introduce an internal language for cartesian closed bicategories [2], explaining its underlying design principles. Firstly, I will begin with a type theory for bicategories synthesised from a bicategorification of the notion of abstract clone from universal algebra. The result is a 2-dimensional type theory (in the style of Hilken [3]) with a form of explicit substitution capturing an 'up to isomorphism' composition operation. Next, I shall show how semantic considerations give rise to the addition of product and exponential type structures. The resulting type theory generalises the Simply-Typed Lambda Calculus and its syntactic models satisfy a suitable 2-dimensional freeness universal property, thereby lifting the Curry-Howard-Lambek correspondence to the bicategorical setting. If time permits, I will conclude by sketching a bicategorical generalisation of the categorical normalisation-by-evaluation argument of Fiore [1] to prove a conjectured coherence result for cartesian closed bicategories.

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*Joint work with Marcelo Fiore.

Quantale-valued dissimilarity

Hongliang Lai, Lili Shen, Yuanye Tao and Dexue Zhang

Abstract

Motivated by the theory of apartness relations of Scott [3], a positive theory of *dissimilarity* valued in an involutive quantale

$$\mathsf{Q} = (\mathsf{Q}, \&, k, {}^\circ)$$

is established without the aid of negation. The notion of Q-valued dissimilarity dualizes that of Q-*valued set* (i.e., a set equipped with a Q-valued similarity) in the sense of Höhle–Kubiak [2], whose prototype comes from the theory of Ω-*sets* of Fourman–Scott [1].

It is demonstrated that sets equipped with a Q-valued dissimilarity are symmetric categories enriched in the quantaloid

$$\mathbf{B}(\mathsf{Q})$$

of *back diagonals* of Q [4]. Moreover, it is shown that similarities and dissimilarities are interdefinable if Q is a *Girard quantale*, in which case there is an isomorphism

$$\mathbf{D}(\mathsf{Q}) \cong \mathbf{B}(\mathsf{Q})$$

of quantaloids, where $\mathbf{D}(\mathsf{Q})$ is the quantaloid of *diagonals* of Q [5]. In the case that Q is a commutative quantale, it is proved that the above isomorphism holds if, and only if, Q is a Girard quantale.

Keywords: dissimilarity, similarity, back diagonal, diagonal, quantale, quantaloid

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INTERNAL LANGUAGES OF HIGHER TOPOSES

MICHAEL SHULMAN

The internal language of toposes, a form of higher-order logic, is well-established as a powerful tool for internalizing mathematics in many geometric contexts. The corresponding internal language for *higher* toposes has long been conjectured to be a form of dependent type theory satisfying Voevodsky's univalence principle. There are now many such "homotopy type theories", but the original and simplest is Martin-Löf's original intensional type theory, with univalence and "higher inductive types" added using axioms; this is known as "Book HoTT" after the 2013 book "Homotopy Type Theory".

About a decade ago, Voevodsky constructed an interpretation of Martin-Löf type theory with univalence in the basic $(\infty, 1)$-topos of ∞-groupoids, using simplicial sets. Extending this to an interpretation of all of Book HoTT in all $(\infty, 1)$-toposes involves a number of coherence issues, many of which have been resolved piecemeal since then. Earlier this year I announced a solution to the largest remaining gap: every (Grothendieck–Lurie) $(\infty, 1)$-toposes can be presented by a model category that contains strict univalent universes. Thus, Book HoTT can now be used confidently as an internal language for all $(\infty, 1)$-toposes.

In this talk I will motivate the problem of internal languages for higher toposes, sketch the construction of strict universes, and describe some applications. For simplicity and wider familiarity I will focus on (2,1)-toposes (categories of stacks of ordinary 1-groupoids), where many of the important issues and ideas already arise.

University of San Diego

GRAPHICAL LINEAR ALGEBRA

PAWEL SOBOCINSKI

Linear algebra is unreasonably effective in engineering and computer science. From classical models of electrical circuits, signal flow graphs and Petri nets through more recent applications in quantum computing, data science and machine learning, much of the underlying computation and analysis is linear algebraic.

In this tutorial we will take a fresh look at the basic concepts of string algebra using a string diagrammatic language. The language arose from joint work with Filippo Bonchi and Fabio Zanasi on a presentation of the prop of linear relations — those relations that are also linear subspaces.

The talk will not be technical and will rather focus on how (1) the graphical language exposes the beautiful symmetries of linear algebra and (2) how it serves as a compositional calculus for various applications, since they, like the string diagrammatic formalism, are often diagrammatic in nature.

University of Southampton

NON-SEMI-ABELIAN SPLIT EXTENSIONS IN CATEGORICAL ALGEBRA

MANUELA SOBRAL

Various results of protomodular and semi-abelian algebra of split extensions have been recently generalized in a way that allows one to apply them to the category of monoids, semirings and other algebraic structures. In particular, the so-called Schreier split extensions of monoids have been re-introduced and studied categorically. The purpose of the talk will be to discuss this and briefly indicate a further step of generalization, from monoids to unitary magmas.

First part is joint work with D. Bourn, N. Martins-Ferreira and A. Montoli and the second one with M. Gran and G. Janelidze.

University of Coimbra

Codensity monads and D-ultrafilters

Lurdes Sousa[1,2] *

[1] CMUC, University of Coimbra, Portugal
[2] Polytechnic Institute of Viseu, Portugal
sousa@estv.ipv.pt

Let $\mathcal{A}$ be a small full subcategory of a complete category $\mathcal{K}$. $\mathcal{A}$ is said to be codense in $\mathcal{K}$ if every object X of $\mathcal{K}$ is the limit of the canonical diagram $X/\mathcal{A} \to \mathcal{K}$ sending every $X \xrightarrow{a} A$ to its codomain. Recently, Tom Leinster has drawn attention to codensity monads [2]. As he observed, the codensity monad of the inclusion $E : \mathcal{A} \hookrightarrow \mathcal{K}$, which is the identity precisely when $\mathcal{A}$ is codense in $\mathcal{K}$, may be regarded as a measure of how 'far away' $\mathcal{A}$ is from being codense in $\mathcal{K}$.

Let $\mathcal{K}$ have a cogenerator D contained in $\mathcal{A}$. We describe the codensity monad of $E : \mathcal{A} \hookrightarrow \mathcal{K}$ as a submonad of the monad $\mathbb{S}$ induced by the adjunction $\mathcal{K}(-, D) \dashv D^{-} : \mathsf{Set}^{\mathrm{op}} \to \mathcal{K}$, in terms of an intersection of subobjects of SX, for every $X \in \mathcal{K}$. In the case where $\mathcal{K}$ is a symmetric, closed monoidal category, the codensity monad is characterized as a certain submonad of the double-dualization monad given by $(-)^{**} = [[-, D], D]$.

It is known that the codensity monad of the embedding of finite sets into the category of sets is the ultrafilter monad [1], and the codensity monad of the embedding of finite-dimensional vector spaces into the category of vector spaces over a field is the double-dualization monad [2]. We introduce the concept of D-ultrafilter on an object and prove that the codensity monad assigns to every object an object representing all D-ultrafilters on it. By taking the expected cogenerator D, we obtain the D-ultrafilters for several examples of embeddings of the full subcategory of finitely presentable objects into a locally finitely presentable category, including posets, semilattices and graphs. For the embedding of all finite sober spaces into the category of topological spaces the codensity monad is the prime filter monad.

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*This is joint work with Jiří Adámek

ABELIAN CALCULI PRESENT ABELIAN CATEGORIES

DAVID I. SPIVAK* AND BRENDAN FONG

Abelian categories are "good places to do computation," e.g. homological algebra. The category of (finitely generated) abelian groups, of vector spaces, and of chain complexes are examples. The definition of abelian category is:

(*) a category with a zero object, finite products and coproducts, a kernel and cokernel for every morphism, and with the property that each monic is a kernel and each epic is a cokernel.

From these simple axioms follow many interesting consequences: each abelian category A has all finite limits and colimits, its finite products and coproducts coincide, its opposite is abelian, and—most interesting for this talk—it is a regular category.

The fact that each abelian category A is regular implies that it has a "nice" theory of relations, a monoidal 2-category $\mathbb{A}$, whose monoidal structure is inherited from A and from which A can be recovered as the category of left adjoints. The relations in an abelian category can be considered as formulas for a kind of regular logic called *abelian logic*, where roughly speaking, new formulas can be built from old using existential quantification ($\exists$), meet ($\wedge$), true, equals ($=$), join ($\vee$), zero (0), and sum ($+$). For example, if $R(y, z)$ and $S(z, z')$ have type $Y \times Z$ and $Z \times Z$ respectively, then $\exists z. \left(R(y, z) \wedge S(z, z') \wedge z + z = z'\right) \vee (y = 0)$ has type $Y \times Z$.

In this talk we will discuss a new presentation language—a syntax we call abelian calculus—for abelian categories, that looks nothing like (*). Let Mat denote the category of matrices with integer coefficients (the full subcategory of finitely-generated abelian groups spanned by the free ones); it is a regular category. Let $\mathbb{M}\mathsf{at}$ denote the monoidal 2-category of relations in Mat, and let $\mathbb{P}\mathsf{oset}$ denote the monoidal 2-category of posets under Cartesian product, monotone maps, and natural transformations. An *abelian calculus with one type* is a lax monoidal 2-functor

$$C \colon \mathbb{M}\mathsf{at} \to \mathbb{P}\mathsf{oset}$$

such that each lax coherence map has both a left and a right adjoint. We abbreviate the condition "C is a bi-adjoint lax monoidal 2-functor" as "C is *bi-ajax*." An abelian calculus is a generalization (T, C) of the above, where T is a set, $\mathsf{Mat}_T := \coprod_T \mathsf{Mat}$ is the T-indexed coproduct prop, and $C \colon \mathbb{M}\mathsf{at}_T \to \mathbb{P}\mathsf{oset}$ is bi-ajax.

From any abelian category A, one obtains an abelian calculus $\mathrm{Rel}(A)$ as the relations in A, i.e. take $T := \mathrm{Ob}(A)$ and $C(t_1, \ldots, t_n) := \mathrm{Sub}(t_1 \times \cdots \times t_n)$. Each such poset is in fact a lattice, and its meet and join are roughly where the bi-ajax condition arises. Conversely, given an abelian calculus (T, C), one can form its *syntactic category* $\mathrm{Syn}(T, C)$ and prove that it is abelian. It is in this sense that abelian calculi form a presentation language for abelian categories: there is an adjunction

$$\mathrm{Syn} : \mathsf{AbCalc} \leftrightarrows \mathsf{AbCat} : \mathrm{Rel}$$

which is an essential reflection in the sense that, for every abelian category A, the functor $\mathrm{Syn}(\mathrm{Rel}(A)) \to A$ is an equivalence.

For continuation of abstract, see programme online

UNIVALENCE AND COMPLETENESS OF SEGAL OBJECTS

RAFFAEL STENZEL

In this talk, we make precise an analogy between univalence and completeness that has been subject to informal discussions in the research community. More precisely, we give a definition of univalence and a definition of Rezk-completeness for Segal objects X in a large class of type theoretic model categories M. The former is a straightforward generalization of univalence in the type theoretic fibration category C of fibrant objects in M as treated in [3]. The latter is a generalization of Rezk's original definition of completeness for Segal spaces. Both conditions share the heuristic purpose to contract a respective object of internal equivalences associated to X over the object X_0 of points, turning that object of internal equivalences into a path object for X_0. A priori, these objects of internal equivalences do not necessarily coincide, so the goal of this talk is to show the following theorem.

Let X be a "sufficiently fibrant" Segal object in M. Then X is univalent if and only if its Reedy fibrant replacement RX is complete.

As a corollary we obtain that a fibration in C is univalent if and only if the Reedy fibrant replacement of its nerve in M is Rezk-complete (this comparison in fact is independent of its infinity-categorical version presented in [2]). This implies for instance that univalent completion of a Kan fibration as introduced in [1] is a special case of Rezk completion of its associated Segal space.

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On the category of cocommutative Hopf algebras

Florence Sterck *

Université catholique de Louvain and Université Libre de Bruxelles

In [1] we prove that the category of cocommutative Hopf algebras over any field is semi-abelian [2]. The aim of this talk is to give an idea of the proof of this result and to explain some of its consequences.
In particular, we deduce the classical theorem by Takeuchi saying that the category of commutative and cocommutative Hopf algebras is abelian. We can then show that the categories of Hopf crossed modules in the sense of Majid [3] and the one of internal crossed modules in the sense of Janelidze [4] coincide in $\mathsf{Hopf}_{\mathsf{K,coc}}$ (see also [5] for related results in a general context). Finally, the possibility of introducing and studying the notion of crossed squares of cocommutative Hopf algebras will also be discussed.

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*Joint work with Marino Gran (UCLouvain) and Joost Vercruysse (ULB)

A Constructive Kan–Quillen Model Structure

Nicola Gambino Christian Sattler Karol Szumiło

April 30, 2019

The Kan–Quillen model structure refers to the model structure on the category of simplicial sets, introduced by Quillen, comprising weak homotopy equivalences, Kan fibrations and monomorphisms. It has become one of the foundation stones of modern homotopy theory and many different proofs and treatments have been developed. None of them was fully constructive, however, which became a major obstacle for its use as a model of Homotopy Type Theory.

In joint research with Nicola Gambino and Christian Sattler, we have proven the following theorem (first obtained independently by Simon Henry).

Theorem (constructive logic)**.** *The category of simplicial sets carries a cofibrantly generated, cartesian, proper model structure where*

- *weak equivalences are weak homotopy equivalences;*
- *fibrations are Kan fibrations;*
- *cofibrations are Reedy decidable inclusions.* □

The fundamental reason why the standard approaches fail to be constructive is that they all rely on the classically trivial statement "a simplex of a simplicial set is either degenerate or non-degenerate".

In order to adapt this theory to the constructive setting, we modify the notion of a cofibration. A *decidable inclusion* is, in categorical terms, a map of sets $i\colon A \to B$ such that there is a map $C \to B$ that together with i exhibits B as the coproduct of A and C. In logical terms, "$x \in A$" (where x is a variable of type B) is a decidable proposition. Then a cofibration is a *Reedy decidable inclusion*, i.e., a simplicial map $A \to B$ such that for all $m \in \mathbb{N}$, the relative latching map $A_m \sqcup_{L_m A} L_m B \to B_m$ is a decidable inclusion. Logically, this means that the proposition "$x \in A$ or x is degenerate" (for x varying over B_m) is decidable. Replacing monomorphisms with cofibrations in this sense fixes the constructivity issues of the standard approaches, but it introduces new difficulties since not all simplicial sets are cofibrant any more. In particular, weak homotopy equivalences need to be defined with great care.

In our work we have obtained two different proofs of the theorem above. One is inspired by type theoretic developments and using techniques such as the "Forbenius property" and the "equivalence extension property". The other is based on classical methods of simplicial homotopy theory, including Kan's Ex functor and Quillen's Theorem A which need to be redeveloped carefully to ensure the constructivity of all arguments. Both treatments are given in a categorical language which has an advantage that it is potentially interpretable in categories other than that of simplicial sets. In particular, the basic theory of decidable maps relies solely on the fact the category of sets is extensive. In this talk I will present the latter approach.

ENRICHED REGULAR THEORIES

GIACOMO TENDAS

Regular and exact categories were first introduced by Michael Barr in 1971; since then, the theory has developed and found many applications in algebra, geometry, and logic. In particular, a small regular category determines a certain theory, in the sense of logic, whose models are the regular functors into Set.

In 1986 Barr showed that each small and regular category can be embedded in a particular category of presheaves; then in 1990 Makkai gave a simple explicit characterization of the essential image of the embedding, in the case where the original regular category is moreover exact. More recently Prest and Rajani, in the additive context, and Kuber and Rosicky, in the ordinary one, described a duality which connects an exact category with its (definable) category of models.

Considering a suitable base for enrichment, we define an enriched notion of regularity and exactness, and prove a corresponding version of the theorems of Barr, of Makkai, and of Prest-Rajani/Kuber-Rosicky.

Based on a Master Thesis supervised by Stephen Lack.

Functorial Decomposition of Colimits

George Peschke[1] and Walter Tholen[2*]

[1] Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Canada
gepe@ualberta.ca
[2] Department of Mathematics and Statistics, York University, Toronto, Canada
tholen@mathstat.yorku.ca

This talk is centred around the following decomposition formula for colimits:

$$\mathrm{colim}_{k\in\mathcal{K}} Xk \cong \mathrm{colim}_{d\in\mathcal{D}}(\mathrm{colim}_{i\in\mathcal{I}_d} X_d i);$$

here $X : \mathcal{K} \longrightarrow \mathcal{X}$ is a diagram in the cocomplete category $\mathcal{X}$, where the diagram scheme $\mathcal{K}$ is itself a colimit of a diagram $D : \mathcal{D} \longrightarrow \mathbf{Cat}$, $d \mapsto \mathcal{I}_d$, in the category of small categories, with colimit injections $K_d : \mathcal{I}_d \longrightarrow \mathcal{K}$, producing the restricted diagrams $X_d = XK_d$ in $\mathcal{X}$. Its formal dualization gives a recomposition formula for limits in the complete category $\mathcal{X}$. The proof may be cast advantageously within the framework of Grothendieck fibrations. Their 2-categorical expansions (as given by Hermida and Buckley) lead to 2-categorical generalizations of the 1-dimensional formulae that were presented by the first author in a talk in Louvain-la-Neuve in May 2017.

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*Presenter

THE EXISTENTIAL COMPLETION

DAVIDE TROTTA

We determine the existential completion of a primary doctrine, and we prove that the 2-monad obtained from it is lax-idempotent, and that the 2-category of existential doctrines is isomorphic to the 2-category of algebras for this 2-monad. We also show that the existential completion of an elementary doctrine is again elementary. Finally we extend the notion of exact completion of an elementary existential doctrine to an arbitrary elementary doctrine.

A General Framework for Categorical Semantics of Type Theory

Taichi Uemura

University of Amsterdam, Amsterdam, the Netherlands
t.uemura@uva.nl

Dybjer [4] introduced *categories with families* as a notion of a model of basic dependent type theory. Extending categories with families, one can define notions of models of dependent type theories such as Martin-Löf type theory [5], two-level type theory [1] and cubical type theory [3]. The way to define a model of a dependent type theory is by adding algebraic operations corresponding to type and term constructors, and it is a kind of routine. However, as far as the author knows, there are no general notions of a “type theory” and a “model of a type theory” that include all of these examples. In this talk, we propose abstract notions of a type theory and a model of a type theory to unify semantics of type theories based on categories with families.

Steve Awodey [2] pointed out that a category with families is the same thing as a *representable map* of presheaves and that type and term constructors are modeled by algebraic operations on presheaves. Inspired by this work, we define a type theory to be a category equipped with a class of morphisms called representable morphisms and a model of a type theory to be a functor into a presheaf category that carries representable morphisms to representable maps.

With these definitions, we establish basic properties of the semantics of type theory. We give a simple and uniform way to construct the *bi-initial model* of a type theory. We give a formal definition of the *internal language* of a model of a type theory $\mathbb{T}$, yielding a 2-functor from the 2-category of models of $\mathbb{T}$ to a suitable (locally discrete) 2-category of *theories over* $\mathbb{T}$. This 2-functor has a left bi-adjoint and induces a bi-equivalence between the 2-category of theories over $\mathbb{T}$ and a full sub-2-category of the 2-category of models of $\mathbb{T}$.

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UNIFORM KAN FIBRATIONS IN SIMPLICIAL SETS

BENNO VAN DEN BERG

One of the cornerstones of homotopy type theory is the construction by Voevodsky of a model of Martin-Loef type theory with a univalent universe in which the types are the ∞-groupoids. (Very) roughly speaking, to build a model of type theory we need a category and a distinguished class of maps (the fibrations) to model the type dependencies. For his model, Voevodsky chose simplicial sets and the Kan fibrations and he heavily exploited the Kan-Quillen model structure on simplicial sets.

After the work by Voevodsky, much time and energy has been spent of trying to prove the same result in a constructive metatheory. Bezem, Coquand and Parmann identified several obstacles, which led Coquand and collaborators to turn to cubical sets. There they could define a suitable notion of fibration for which they managed to prove constructively that it gave them a model of Martin-Loef type theory with a univalent universe. Later, Sattler managed to prove (constructively) that there is a model structure as well. Many variations on these results have since appeared (by using different variants of cubical sets and different notions of fibration, etc), but, as far as I can see, so far none of these model structures have been shown to be equivalent to the standard Kan-Quillen model structure on simplicial sets.

In a way, Coquand et al made two changes: they changed the underlying category and they started adding uniformity conditions to the notion of a fibration (compatibility conditions for the fillers). Building on ideas by Coquand et al, Gambino and Sattler and Sattler returned to simplicial sets and showed that one can get quite far by adding similar uniformity conditions in simplicial sets. In fact, they almost managed to recover the Kan-Quillen model structure in a constructive metatheory. One key difficulty is that (constructively) it is not clear whether their notion of Kan fibration is "local", in the sense that it is not clear that a map will be a uniform Kan fibration if all its pullbacks with representable codomain are. This is important for constructing universes.

In joint work with Eric Faber I have starting investigating two notions of a uniform Kan fibration, both of which are stronger than the one used by Gambino and Sattler and both of which are local. This is all still very much work in progress, but right now I am convinced that one obtains with both our notions of a uniform fibration and within a constructive metatheory (say, Aczel's CZF with some universes) a model of type theory with universes (including identity and Pi-types) and a model structure on the fibrant objects. We are still working on univalence and extending the model structure to the entire category.

Besides emphasising that this is still joint work in progress with Eric Faber, I should also stress that, while we try to do all our maths constructively, our project forces us to prove results which are new and (I believe) interesting in the classical setting as well.

Relative Partial Combinatory Algebras over Heyting Categories

Jetze Zoethout, Utrecht University

Abstract

A partial combinatory algebra (PCA) is an abstract model of computation that generalizes the classical notion of computability on the set of natural numbers. More precisely, it is a nonempty set equipped with a binary partial operation that satisfies an abstract version of the Smn-theorem. These models can be studied from the point of view of category theory. Every PCA A gives rise to a category of assemblies $\mathsf{Asm}(A)$, which may be viewed as the category of all data types that can be implemented in A. A category of the form $\mathsf{Asm}(A)$ is always a quasitopos, and quasitoposes are closed under slicing. However, categories of assemblies for a PCA are not in general closed under slicing. Therefore, we wish to investigate what categories of the form $\mathsf{Asm}(A)/X$, where X is an assembly, look like.

The answer to this question is provided by W. Stekelenburg's PhD thesis [Ste13], which generalizes the notion of a PCA in two ways. Firstly, the category of sets, which plays a crucial role in the construction of $\mathsf{Asm}(A)$, is replaced by a general Heyting category $\mathcal{H}$. Secondly, the notion of computability is relativized by selecting a set of privileged subobjects of A that count as 'realizing sets'. We therefore call these objects relative PCAs constructed over a Heyting category, or HPCA's for short. The construction of $\mathsf{Asm}(A)$ can be generalized to HPCAs A, and if X is an assembly, then $\mathsf{Asm}(A)/X$ is of the form $\mathsf{Asm}(A')$ for some HPCA A'. We describe this A' explicitly in terms of A and X and use this description to compute a number of examples of categories of the form $\mathsf{Asm}(A)/X$, where A is a PCA.

PCAs are the objects of a preorder-enriched category (see, e.g., [Lon94] and [vO08]). Similarly, HPCAs constructed over a given Heyting category $\mathcal{H}$ can be made into a preorder-enriched category $\mathsf{PCA}_{\mathcal{H}}$. In this talk, we construct a larger 2-category PCA, which is the total category of an opfibration over the category of Heyting categories and whose fiber above a Heyting category $\mathcal{H}$ is precisely $\mathsf{PCA}_{\mathcal{H}}$. We investigate the structure of these categories, showing that PCA has small products, and that each of the fibers $\mathsf{PCA}_{\mathcal{H}}$ has finite (pseudo-)coproducts. Moreover, we extend the construction of $\mathsf{Asm}(A)$ to a 2-functor from PCA into the category of categories. We characterize the image of this 2-functor, thereby generalizing Longley's work from [Lon94].

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